Understanding Math

Southern Nevada
Regional Professional Development Program

Rules of Exponents

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One area in which students have difficulty in algebra is simplifying expressions with exponents. For some reason, they just don't quite understand the rules of exponents. In this issue of *Understanding Math*, we'll discuss why they don't get it and how we can address the problem.

What exponentiation is: $a^m = a \cdot a \cdot a \cdots a$ (m factors)

This is the heart of the matter. If students don't get this, we can write off any understanding of the rule of exponents. So what should we expect of students in this regard? Not only should students be able to plug an expression such as 5^4 into a calculator and read 625, but they also must demonstrate that when given 5^4 it is equal to $5 \cdot 5 \cdot 5 \cdot 5$ and vice-versa. Students should communicate that exponentiation is repeated multiplication. The same is true of expressions with variables: understanding that $x^3 = x \cdot x \cdot x$, , and must be demonstrated. $3^x = \underbrace{3 \cdot 3 \cdot 3 \cdots 3}_{x \text{ factors of } 3}$ $\quad a^x = \underbrace{a \cdot a \cdot a \cdots a}_{x \text{ factors of } a}$

Product of Powers with Equal Bases: $a^m \cdot a^n = a^{m+n}$

This is probably the first rule that students learn and therefore the first with which they have trouble. Very often, students make the mistake of multiplying the exponents instead of adding, as in $3^2 \cdot 3^4 = 3^8$ rather than the correct $3^2 \cdot 3^4 = 3^6$. Why? It goes back to either a lack of comprehension of what exponentiation is or not having conceptual understanding of the rule.

Before the rule is introduced, students should be required to break down the expression then re-write it in exponential form. For example, $3^2 \cdot 3^4 = \underbrace{3 \cdot 3}_{2 \text{ factors}} \cdot \underbrace{3 \cdot 3 \cdot 3 \cdot 3}_{4 \text{ factors}} = 3^6$. Ask, "How many factors of three?" A number of these exercises will help form an idea that exponents add when multiplying with like bases. Then, more abstract expressions can be introduced: $3^m \cdot 3^n = \underbrace{3 \cdot 3 \cdots 3}_{m \text{ factors}} \cdot \underbrace{3 \cdot 3 \cdots 3}_{n \text{ factors}} = \underbrace{3 \cdot 3 \cdots 3 \cdot 3}_{m+n \text{ factors}} = 3^{m+n}$. All of this takes place *before* we present the rule $a^m \cdot a^n = a^{m+n}$. If the rule is taught in isolation, first, without any conceptual development of its function, students are less likely to remember it and more likely to make mistakes.

Powers of a Power: $\left(a^m\right)^n = a^{m \cdot n}$

Again, *before* this rule is introduced, students should be required to work exercises using only the meaning of exponentiation. For instance, $\left(3^2\right)^4 = \underbrace{3^2 \cdot 3^2 \cdot 3^2 \cdot 3^2}_{4 \text{ factors of } 3^2} = \underbrace{3 \cdot 3} \cdot \underbrace{3 \cdot 3} \cdot \underbrace{3 \cdot 3} \cdot \underbrace{3 \cdot 3} = 3^8$ as a concrete problem, $\left(3^m\right)^4 = 3^m \cdot 3^m \cdot 3^m \cdot 3^m = \underbrace{3 \cdots 3}_{m \text{ factors}} \cdot \underbrace{3 \cdots 3}_{m \text{ factors}} \cdot \underbrace{3 \cdots 3}_{m \text{ factors}} \cdot \underbrace{3 \cdots 3}_{m \text{ factors}} = 3^{4m}$ as a more abstract one. Once students begin to make the connection that the exponents are multiplied the rule may be formally presented.

There are many more rules of exponents. All of them should be introduced conceptually *first*, going back to the basic definition of exponentiation, $a^m = a \cdot a \cdot a \cdots a$ (m factors). Then, after students have an inkling of where the development is leading, the rule can be defined and committed to memory. This process develops understanding of exponentiation as a body of knowledge rather than a set of disjointed rules.