Understanding Math

Southern Nevada
Regional Professional Development Program

Negative and Zero Exponents

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Exponents—they're a basic component of mathematical notation. As addition is used to convey repeated counting and multiplication is used to convey repeated addition, exponents are used to convey repeated multiplication. However, the laws involving exponents are often accepted and applied without question. Some of these rules are rarely explored and are presented without any analysis as to why they work. In this edition of *Understanding Math*, we will do the frequently skipped exploration and analysis of selected rules.

Let us first go back to the definition of an exponent: a shorthand notation for repeated multiplication. We define the exponent *n*, where *n* is a whole number, in the expression b^n as meaning the multiplication of *n* factors of *b*. For example, $5^3 = 5 \cdot 5 \cdot 5$.

Having established that, we move on to the law for addition of exponents. Adding to our previous example, what is the product $5^3 \cdot 5^2$? Well, $5^3 \cdot 5^2 = (5 \cdot 5 \cdot 5) \cdot (5 \cdot 5) = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 5^5$.

Generalizing this idea, since b^m has *m* factors of *b* and b^n has *n* factors of *b*, it follows that $b^m \cdot b^n$ has $m+n$ factors of *b*, or $b^m \cdot b^n = b^{m+n}$. Usually, we go through this with students and they seem to have a good understanding of this rule and its corollary, $b^m \div b^n = b^{m-n}$.

One rule that students don't often understand is $b^{-n} = \frac{1}{b^n}$. This is often the result of the lack of connection to what they already know. Assuming students grasp $b^m \cdot b^n = b^{m+n}$, then they should be able to simplify $5^3 \cdot 5^{-2}$ and get $5^3 \cdot 5^{-2} = 5^{3+{}^-2} = 5^1 = 5$. But what is 5^{-2}? For the moment, let's solve the equation $5^3 \cdot x = 5$. Solving, $5^3 \cdot x = 5 \Rightarrow 125x = 5$, so $x = \frac{1}{25}$ or $x = \frac{1}{5^2}$. Now, compare $5^3 \cdot 5^{-2} = 5$ with $5^3 \cdot x = 5$. We can see that $x = 5^{-2} = \frac{1}{25}$! The meaning of the negative exponent is a little clearer now.

The last rule we'll look at is for an exponent of zero. We often say, "By definition, $b^0 = 1$ if $b \neq 0$." Students accept it, but don't really get why. Let's use the same process as in the previous paragraph. Solve $5^3 \cdot x = 1$. If $5^3 \cdot x = 1$, then $125x = 1$ or $x = \frac{1}{125} = \frac{1}{5^3} = 5^{-3}$.

Substituting *x* back into the original equation $5^3 \cdot x = 1$, $5^3 \cdot 5^{-3} = 1 \Rightarrow 5^{3+{}^-3} = 1$, or $5^0 = 1$.

Generalizing this with $b^n \cdot x = 1$ would eventually lead us to $b^0 = 1$.

A firm understanding of each step in the process—what exponents mean, addition of exponents, negative exponents, exponents of zero—leads to comprehending the next. Here we have done it with only integral exponents. These same principles can be also applied to laws regarding fractional exponents. That is an exercise currently left to the reader