Understanding Math

Southern Nevada
Regional Professional Development Program

Laws of Exponents

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Exponents—they're a basic component of mathematical notation. However, the laws involving exponents are often accepted and applied without question. Some of these rules are rarely explored and are presented without any analysis as to why they work. In this edition of *Understanding Math*, we will do the exploration and analysis of selected rules that are so frequently skipped.

Let us first go to one purpose of an exponent, a shorthand notation for repeated multiplication. We **define** the exponent n, where n is a whole number, in the expression b^n as meaning the multiplication of n factors of the base b. For example, $5^3 = 5\cdot5\cdot5$.

Having established the definition, we move on to *the **law** of the product of two powers with equal bases*. Building on the previous example, we can address the product $5^3\cdot5^2$. Expanding the expression using the definition of an exponent, $5^3\cdot5^2 = (5\cdot5\cdot5)\cdot(5\cdot5) = 5\cdot5\cdot5\cdot5\cdot5 = 5^5$. It's not a great leap to prove to students that since b^m has m factors of b and b^n has n factors of b, it follows that $b^m\cdot b^n$ has $m + n$ factors of b. Thus, $b^m\cdot b^n = b^{m+n}$. Similarly, they are also fairly comfortable with $5^5 \div 5^2 = \frac{5\cdot5\cdot5\cdot5\cdot5}{5\cdot5} = \frac{\cancel{5}\cdot\cancel{5}\cdot5\cdot5\cdot5}{\cancel{5}\cdot\cancel{5}} = 5^3$, leading to $b^m \div b^n = b^{m-n}$, *the **law** of the quotient of two powers with equal bases*.

One concept that students don't often understand is $b^{-n} = \frac{1}{b^n}$, *the **definition** of negative exponents*. It's tough to imagine a negative number of factors of a number—how does one conceptualize negative three factors of five (5^{-3})? Perhaps as repeated division? However, connecting it to what they already know can help students along.

Assume that students understand $b^m \div b^n = b^{m-n}$ and ask what is the quotient $5^3 \div 5^5$? By the law, $5^3 \div 5^5 = 5^{3-5} = 5^{-2}$. But they also know that $5^3 \div 5^5 = \frac{5\cdot5\cdot5}{5\cdot5\cdot5\cdot5\cdot5} = \frac{1}{5^2}$. Thus, $5^{-2} = \frac{1}{5^2}$.

What do zero factors of a number look like? Hmmm. We can answer this question about *the **definition** of a zero exponent*, $b^0 = 1$, with an approach similar to the one above. Using $b^m \div b^n = b^{m-n}$ and the definition of a negative exponent, we can ask students to consider the expression $5^3 \div 5^3$. Using the laws of exponents, $5^3 \div 5^3 = 5^{3+{}^-3} = 5^0$. But, $5^3 \div 5^3 = 125 \div 125 = 1$. It must be that $5^0 = 1$.

Both the definition of negative exponents and zero exponent require that $b \neq 0$. Why? In the both cases, division by zero would be implied, and we know that's a mathematical "no-no."

It must be noted that b^0 doesn't equal zero…ever!

A firm understanding of each step in the process—what exponents mean, addition and subtraction of exponents, negative exponents, exponents of zero—leads to comprehending the next. It's not enough to accept them on faith. If students can understand from where the laws and definitions come, they're more apt to remember them.