



Lesson 12: The Relationship Between Absolute Value and Order

Student Outcomes

- Students understand that the order of positive numbers is the same as the order of their absolute values.
- Students understand that the order of negative numbers is the opposite order of their absolute values.
- Students understand that negative numbers are always less than positive numbers.

Lesson Notes

Prior to lesson prepare sticky notes containing a balanced variety of positive and negative integers ranging from -50 to 50 . Each pair of students will require a set of ten integers including five negative values, zero, and four positive values.

Classwork

Opening Exercise (5 minutes)

Divide students into pairs. Provide each pair with a set of ten integers including five negative values, zero, and four positive values written on sticky notes. Instruct student groups to order their integers from least to greatest based on prior knowledge. When the integers are in the correct order, students record the integer values in order in their student materials.

Opening Exercise

Record your integer values in order from least to greatest in the space below.

$-12, -9, -5, -2, -1, 0, 2, 5, 7, 8$

- Have one pair of students post their sticky notes to the wall in the specified order. Ask another pair of students to explain the reasoning for the order.
 - *The integers are in the same order in which they would be found located from left to right on the number line.*

Example 1 (8 minutes): Comparing Order of Integers to the Order of their Absolute Values

Students use the integer values from the opening exercise to compare the order of integers to the order of their absolute values.

Example 1: Comparing Order of Integers to the Order of their Absolute Values

Write an inequality statement relating the ordered integers from the Opening Exercise. Below each integer write its absolute value.

$-12 < -9 < -5 < -2 < -1 < 0 < 2 < 5 < 7 < 8$
 12 9 5 2 1 0 2 5 7 8

Scaffolding for ELL Learners:
 In place of the words ascending and descending, describe numbers as increasing from left to right or decreasing from left to right.

- Are the absolute values of your integers in order? Explain.
 - No. The absolute values of the positive integers listed to the right of zero are still in ascending order but the absolute values of the negative integers listed to the left of zero are now in descending order.

Instruct students to circle the absolute values that are in increasing numerical order and their corresponding integers, and then describe the circled values.

$-12 < -9 < -5 < -2 < -1 < 0 < 2 < 5 < 7 < 8$
 12 9 5 2 1 0 2 5 7 8

The circled integers are all positive values except zero. The positive integers and their absolute values have the same order.

Rewrite the integers that are not circled in the space below. How do these integers differ from the ones you circled?

$-12, -9, -5, -2, -1$; They are all negative integers.

Rewrite the negative integers in ascending order and their absolute values in ascending order below them.

$-12 < -9 < -5 < -2 < -1$
 1 2 5 9 12

Describe how the order of the absolute values compares to the order of the negative integers.

The orders of the negative integers and their corresponding absolute values are opposite.

MP.7

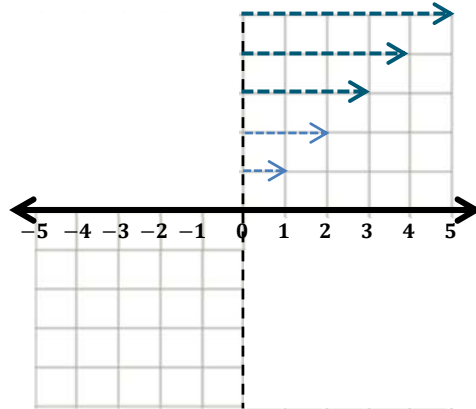
Example 2 (8 minutes): The Order of Negative Integers and their Absolute Values

Students examine the lengths of arrows corresponding with positive and negative integers on the number line and use their analysis to understand why the order of negative integers is opposite the order of their absolute values. Teacher monitors the room and provides guidance as needed, then guides whole class discussion with questions.

MP.2

Example 2: The Order of Negative Integers and their Absolute Values

Draw arrows starting at the dashed line (zero) to represent each of the integers shown on the number line below. The arrows that correspond with 1 and 2 have been modeled for you.



MP.2

- Which integer is represented by the longest arrow that you drew? Why?
 - 5 and -5 because, of the integers shown, they are furthest from zero on the number line.
- Which nonzero integer is represented by the shortest arrow? Why?
 - 1 and -1 because, of the integers shown, they are furthest from zero on the number line.
- Is there an arrow for the integer 0?
 - The length of such an arrow would be 0 so we could not see the arrow. We could call it an arrow with zero length, but we could not draw it.
- Do these “arrows” start at the same place on the number line? Why or why not?
 - They all start at zero on the number line because 0 is the reference point for all numbers on the number line.
- Do all arrows point in the same direction? Why or why not?
 - All arrows do not point in the same direction because some have opposite signs. The lengths of the arrows get shorter as you approach zero from the left or from the right, which means the absolute values decrease as you approach zero from the left or the right.

Direct students to complete the statements in their student materials by filling in the blanks.

As you approach zero from the left on the number line, the integers increase, but the absolute values of those integers decrease. This means that the order of negative integers is opposite the order of their absolute values.

Discussion (2 minutes)

- Think of a real world example that shows why the order of negative integers and the order of their absolute values are opposite.
 - *Answers will vary. An example follows: Alec, Benny, and Charlotte have cafeteria charge account balances of -3 , -4 , and -5 dollars respectively. The number that represents Alec’s cafeteria charge account balance is the greatest, because he has the least in debt. Alec owes the least amount of money and is the closest to having a positive balance. His balance of -3 is furthest right on the number line of the three balances.*

Exercise 1 (5 minutes)

Students separate positive and negative rational numbers and order them according to their absolute values. Students then order the given set of positive and negative rational numbers using the orders of their absolute values.

Exercise 1

Complete the steps below to order these numbers: $\{2.1, -4\frac{1}{2}, -6, 0.25, -1.5, 0, 3.9, -6.3, -4, 2\frac{3}{4}, 3.99, -9\frac{1}{4}\}$.

- Separate the set of numbers into positive and negative values and zero in the top cells below.
- Write the absolute values of the rational numbers (order does not matter) in the bottom cells below.

<p>Negative Rational Numbers</p> <p>$-4\frac{1}{2}$ -6 -6.3</p> <p>-4 $-9\frac{1}{4}$ -1.5</p>	Zero	<p>Positive Rational Numbers</p> <p>2.1 0.25 3.9</p> <p>$2\frac{3}{4}$ 3.99</p>
0	0	
<p>Absolute Values</p> <p>$4\frac{1}{2}$ 6 6.3</p> <p>4 $9\frac{1}{4}$ 1.5</p>		<p>Absolute Values</p> <p>2.1 0.25 3.9</p> <p>$2\frac{3}{4}$ 3.99</p>

- Order each subset of absolute values.

$1.5, 4, 4\frac{1}{2}, 6, 6.3, 9\frac{1}{4}$	0	$0.25, 2.1, 2\frac{3}{4}, 3.9, 3.99$
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- Order each subset of rational numbers.

$-9\frac{1}{4}, -6.3, -6, -4\frac{1}{2}, -4, -1.5$	0	$0.25, 2.1, 2\frac{3}{4}, 3.9, 3.99$
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- Order the whole given set of rational numbers.

$-9\frac{1}{4}, -6.3, -6, -4\frac{1}{2}, -4, -1.5, 0, 0.25, 2.1, 2\frac{3}{4}, 3.9, 3.99$

MP.2

Exercise 2 (8 minutes)

Students independently complete the following problems. The teacher monitors student progress and provides guidance as needed.

Exercise 2

- a. Find a set of four integers such that their order and that of the order of their absolute values is the same.
Answers will vary. An example follows: 4, 6, 8, 10
- b. Find a set of four integers such that their order and the order of their absolute values are opposite.
Answers will vary. An example follows: -10, -8, -6, -4
- c. Find a set of four non-integer rational numbers such that their order and the order of their absolute values is the same.
Answers will vary. An example follows: $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, $5\frac{1}{2}$
- d. Find a set of four non-integer rational numbers such that their order and the order of their absolute values are opposite.
Answers will vary. An example follows: $-5\frac{1}{2}$, $-4\frac{1}{2}$, $-3\frac{1}{2}$, $-2\frac{1}{2}$
- e. Order all of your numbers from parts (a)–(d) in the space below. This means you should be ordering 16 numbers from least to greatest.
Answers will vary. An example follows:
-10, -8, -6, $-5\frac{1}{2}$, $-4\frac{1}{2}$, -4, $-3\frac{1}{2}$, $-2\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, $5\frac{1}{2}$, 6, 8, 10

Closing (4 minutes)

- Below are the absolute values of a set of rational numbers in increasing order.

$\{0.4, 1, 2\frac{1}{2}, 4, 4.3, 7, 9.9\}$

Can you determine the order of the rational numbers themselves? If so, explain why and write the numbers in increasing order. If not, explain why?

- If you are told that the numbers above represent the absolute values of positive rational numbers, can you determine the order of the rational numbers? If so, explain why and write the numbers in increasing order. If not, explain why?
- If you are told that the numbers above represent the absolute values of negative rational numbers, can you find the order of the rational numbers? If so, explain why and write the numbers in increasing order. If not, explain why?

Lesson Summary

The absolute values of positive numbers will always have the same order as the positive numbers themselves. Negative numbers, however, have exactly the opposite order as their absolute values. The absolute values of numbers on the number line increase as you move away from zero in either direction.

Exit Ticket (5 minutes)

Exit Ticket Sample Solutions

1. Bethany writes a set of rational numbers in increasing order. Her teacher asks her to write the absolute values of these numbers in increasing order. When her teacher checks Bethany's work, she is pleased to see that Bethany has not changed the order of her numbers. Why is this?

All of Bethany's rational numbers are positive or 0. The positive rational numbers have the same order as their absolute values. If any of Bethany's rational numbers are negative then the order would be different.

2. Mason was ordering the following rational numbers in math class: -3.3 , -15 , $-8\frac{8}{9}$.

- a. Order the numbers from least to greatest.

-15 , $-8\frac{8}{9}$, -3.3

- b. List the order of their absolute values.

3.3 , $8\frac{8}{9}$, 15

- c. Explain why the orderings in parts (a) and (b) are different.

Since these are all negative numbers, when I ordered them from least to greatest, the one farthest away from zero (farthest to the left on the number line) came first. This number is -15 . Absolute value is the numbers' distance from zero, and so the number farthest away from zero has the greatest absolute value, so 15 will be greatest in the list of absolute values, and so on and so forth.

Problem Set Sample Solutions

1. Micah and Joel each have a set of five rational numbers. Although their sets are not the same, their sets of numbers have absolute values that are the same. Show an example of what Micah and Joel could have for numbers. Give the sets in order and the absolute values in order.

If Micah had 1, 2, 3, 4, 5, then his order of absolute values would be the same: 1, 2, 3, 4, 5. If Joel had the numbers -5 , -4 , -3 , -2 , -1 , then his order of absolute values would also be 1, 2, 3, 4, 5.

Enrichment Extension: Show an example where Micah and Joel both have positive and negative numbers.

If Micah had the numbers: -5 , -3 , -1 , 2, 4, his order of absolute values would be 1, 2, 3, 4, 5. If Joel had the numbers -4 , -2 , 1, 3, 5, then the order of his absolute values would also be 1, 2, 3, 4, 5.

2. For each pair of rational numbers below, place each number in the Venn diagram based on how it compares to the other.

- a. $-4, -8$
- b. $4, 8$
- c. $7, -3$
- d. $-9, 2$
- e. $6, 1$
- f. $-5, 5$
- g. $-2, 0$

