Lesson 7: Ordering Integers and Other Rational Numbers

Student Outcomes

- Students write, interpret, and explain statements of order for rational numbers in the real-world.
- Students recognize that if $a < b$, then $-a > -b$, because a number and its opposite are equal distances from zero; students also recognize that moving along the horizontal number line to the right means the numbers are increasing.

Classwork

Opening Activity (6 minutes): Guess My Integer and Guess My Rational Number

MP.3

For this verbal warm-up game, students arrive at a rational number based on parameters provided by the teacher and their own questioning. Students are allowed to ask *no more than 3 questions,* which will help them figure out the number the teacher is thinking of. Students must raise their hand and wait to be called upon to ask a question. This is a whole group activity, and once students know what the number is, they should raise their hand. After the activity is modeled several times by the teacher with the class, students in groups of two or three could try the activity amongst themselves, with one person thinking of a number, and the others asking questions and guessing the number.

The game should increase in difficulty, so that the first round involves an integer number, the second round involves a non-integer positive rational number, and the third round involves a non-integer negative rational number.

A sample dialogue for two rounds of the game is stated below:

Round 1: Guess My Integer

- I am thinking of an integer between -7 and 0. (The teacher is thinking of the number -1.)
    - *Student 1: Is the number bigger than -6?*
- Yes.
    - *Student 2: Is the number smaller than -3?*
- No.
    - *Student 3: Is the number -2?*
- No.

- You have asked me 3 questions. Raise your hand if you know the integer I am thinking of.
    - -1
- Yes!

Round 2: Guess My Positive Non-Integer Rational Number

- I am thinking of a positive rational number between 1 and 2. (The teacher is thinking of the number $\frac{5}{4}$.)

    - *Student 1*: *Is the number greater than* $1\frac{1}{2}$*?*
- No.

    - *Student 2*: *Is the number smaller than 1.4?*
- Yes.

    - *Student 3*: *Is the number smaller than 1.2?*
- No.

- You have asked me 3 questions. Raise your hand if you know the integer I am thinking of.

    - 1.3
- No.

    - 1.35
- No.

    - 1.25
- Yes!

Discussion (3 minutes)

- If we had two rational numbers that we wanted to graph on the same number line, how would we describe to someone where each number lies in comparison to the other number?

    - *The lesser number would be to the left of the greater number. (Or, the greater number would be to the right of the lesser number.) Negative numbers are to the left of zero, and positive numbers are to the right of zero.*
- Describe the location on the number line of a number and its opposite.

    - *They are both the same distance from zero, but on opposite sides of zero.*
- If we were to list the numbers from least to greatest, which would be farthest to the left on the number line? Why? Which would be farthest to the right? Why?

    - *The numbers on the number line decrease as you move to the left and increase as you move to the right. So the number that is the least would be furthest left, and the number that is the greatest would be furthest right.*

Exercise 1 (5 minutes)

Allow time for a whole-group discussion to follow completion of the exercise.

Exercise 1

1.

a. **Graph 7 and its opposite on the number line. Graph 5 and its opposite on the number line.**

b. **Where does 7 lie in relation to 5 on the number line?**

On the number line, 7 is 2 units to the right of 5.

c. **Where does the opposite of 7 lie on the number line in relation to the opposite of 5?**

On the number line, −7 is 2 units to the left of −5.

d. **I am thinking of 2 numbers. The first number lies to the right of the second number on a number line. What can you say about the location of their opposites? (If needed, refer to your number line diagram.)**

On the number line, the opposite of the second number must lie to the right of the opposite of the first number. If we call the first number f, and the second number s, then −f and −s will have the opposite order of f and s because −f and −s are opposites of f and s; so, they lie on the opposite side of zero.

Example 1 (4 minutes)

MP.2

Example 1

The record low temperatures for a town in Maine for January and February are −20 and −19 degrees Fahrenheit respectively. Order the numbers from least to greatest. Explain how you arrived at the order.

- *Read: January: −20 and February: −19*

- *Draw: Draw a number line model.*

- *Write: Since −20 is furthest below zero, and −19 is above −20 on the vertical number line, −20 is less than −19.*

- *Answer: −20, −19*

0

−19
−20

Exercises 2–4 (8 minutes)

Exercises 2–4

For each problem, order the rational numbers from least to greatest. First read the problem, then draw a number line diagram, and finally, write/explain the answer. (Allow time for whole-group presentations.)

2. Jon's time for running the mile in gym class is 9.2 minutes. Jacky's time is 9.18 minutes. Who ran the mile in less time?

    $9.18, 9.2$
    I drew a number line and graphed 9.2 and 9.18; 9.2 is to the right of 9.18. So, 9.18 is less than 9.2, which means Jacky ran the mile in less time than Jon.

3. Mrs. Rodriguez is a teacher at Westbury Middle School.   She gives bonus points on tests for outstanding written answers and deducts points for answers that are not written correctly.   She uses rational numbers to represent the points.   She wrote the following on the students' papers: Student A: -2 points, Student B: -2.5 points. Did Student A or Student B perform worse on the test?

    $-2.5, -2$
    I drew a number line and -2 and -2.5 are both to the left of zero, but -2.5 is to the left of -2. So -2.5 is less than -2. That means student B did worse.

4. A carp is swimming approximately $8\frac{1}{4}$ feet beneath the water's surface, and a sunfish is swimming approximately $3\frac{1}{2}$ feet beneath the water's surface. Which fish is swimming further beneath the water's surface?

    $-8\frac{1}{4}, -3\frac{1}{2}$
    I drew a vertical number line and $-8\frac{1}{4}$ is further below zero than $-3\frac{1}{2}$. So, $-8\frac{1}{4}$ is less than $-3\frac{1}{2}$, which means the carp is swimming further beneath the water's surface.

Example 2 (3 minutes)

Example 2

Henry, Janon, and Clark are playing a card game. The object of the game is to finish with the most points. The scores at the end of the game are: Henry: -7, Janon: 0, and Clark: -5. Who won the game? Who came in last place? Use a number line model and explain how you arrived at your answer.

- *Read: Henry: -7, Janon: 0 and Clark: -5*
- *Draw:*

- *Explain: $-7, -5, 0$*
- *Janon won the game and Henry came in last place. I ordered the numbers on a number line and -7 is furthest to the left. That means -7 is the least of the three numbers, so Henry came in last place. Next on the number line is -5, which is to the right of -7, but to the left of 0. Farthest to the right is 0; so, 0 is the greatest of the three numbers. This means Janon won the game.*

Exercises 5–6 (6 minutes)

Exercises 5–6

For each problem, order the rational numbers from least to greatest by first reading the problem, then drawing a number line diagram, and finally, explaining your answer.

5. Henry, Janon, and Clark are playing another round of the card game. Their scores this time are as follows: Clark: -1, Janon: -2, and Henry: -4. Who won? Who came in last place?

    $-4, -2, -1.$

    Clark won the game, and Henry came in last place. I ordered the numbers on a number line, and -4 is farthest to the left. That means -4 is the least of the three numbers; so, Henry lost. Next on the number line is -2, which is to the right of -4, and to the left of -1. Farthest to the right is -1, which is the greatest of the three negative numbers; so, Clark won the game.

6. Represent each of the following elevations using a rational number. Then order, the numbers from least to greatest.

Cayuga Lake	122 meters above sea level
Mount Marcy	1,629 meters above sea level
New York Stock Exchange Vault	15.24 meters below sea level

    $-15.24;\ 122;\ 1,629$

    I drew a number line and -15.24 is the only number to the left of zero; so, it is the least (because as you move to the right, the numbers increase.) Next on the number line is 122, which is to the right of zero. Last on the number line is $1,629$, which is to the right of 122; so, $1,629$ meters is the greatest elevation.

Closing Exercise (5 minutes): What Is the Value of Each Number and Which Is Larger?

This Closing Exercise is a verbal game similar to the Opening Exercise. However, instead of asking questions, students will use the two clues the teacher provides and, as a visual model, the number line diagram in their materials. Using the clues and the number line, students will determine the value of each number and which number is greater. When a student raises his hand and is called upon, he states the values and which is greater.

Scaffolding:
- Some students will need to rely on the number line for this activity. Others will not, as they have moved from the pictorial to the abstract stage.
- Adjust the wait time and difficulty level of the questions to meet the needs of the learners.

Listed below are five rounds of potential questions.

- **Round 1:** The first number is 3 units to the right of -1. The second number is 5 units to the left of 4. What is the value of each number, and which is greater?
    - *First Number: 2; Second Number: -1; 2 is greater than -1.*

- **Round 2:** The first number is neither positive nor negative. The second number is 1 unit to the left of -1. What is the value of each number, and which is greater?
    - *First Number: 0; Second Number: -2; 0 is greater than -2.*

- **Round 3:** The first number is $8\frac{1}{2}$ units to the right of -5. The second number is 3 units to the right of 0. What is the value of each number, and which is greater?
   - *First Number: 3.5; Second Number: 3; 3.5 is greater than 3.*

- **Round 4:** The first number is $\frac{1}{4}$ unit to the left of -7. The second number is 8 units to the left of 1. What is the value of each number, and which is greater?
    - *First Number: -7.25; Second Number: -7; -7 is greater than -7.25.*
- **Round 5:** The opposite of the first number is 2 units to the right of 3. The opposite of the second number is 2 units to the left of -3. What is the value of each number, and which is greater?
    - *First Number: -5; Second Number: 5; 5 is greater than -5.*

Closing Exercise: What Is the Value of Each Number and Which Is Larger?

Use your teacher's verbal clues and this number line to determine which number is larger.
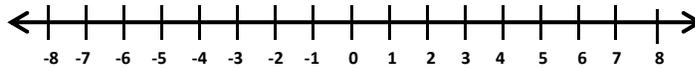
Exit Ticket (5 minutes)

Name _____   Date_____

Lesson 7: Ordering Integers and Other Rational Numbers

Exit Ticket

In math class, Christina and Brett are debating the relationship between two rational numbers. Read their claims below, and then write an explanation of who is correct. Use a number line model to support your answer.

Christina's Claim: "I know that 3 is greater than $2\frac{1}{2}$.   So -3 must be greater than $-2\frac{1}{2}$."

Brett's Claim: "Yes, 3 is greater than $2\frac{1}{2}$, but when you look at their opposites, their order will be opposite. So that means $-2\frac{1}{2}$ is greater than -3."

Exit Ticket Sample Solutions

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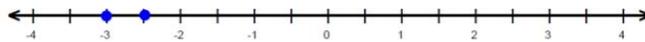
Brett is correct. I graphed the numbers on the number line, and -3 is to the left of $-2\frac{1}{2}$. The numbers increase as you move to the right; so, $-2\frac{1}{2}$ is greater than -3.

Problem Set Sample Solutions

1. In the table below, list each set of rational numbers in order from least to greatest. Then list their opposites. Then list the opposites in order from least to greatest. The first example has been completed for you.

Rational Numbers	Ordered from Least to Greatest	Opposites	Opposites ordered from Least to Greatest
$-7.1, -7.25$	$-7.25, -7.1$	$7.25, 7.1$	$7.1, 7.25$
$\frac{1}{4}, -\frac{1}{2}$	$-\frac{1}{2}, \frac{1}{4}$	$\frac{1}{2}, -\frac{1}{4}$	$-\frac{1}{4}, \frac{1}{2}$
$2, -10$	$-10, 2$	$10, -2$	$-2, 10$
$0, 3\frac{1}{2}$	$0, 3\frac{1}{2}$	$0, -3\frac{1}{2}$	$-3\frac{1}{2}, 0$
$-5, -5.6$	$-5.6, -5$	$5.6, 5$	$5, 5.6$
$24\frac{1}{2}, 24$	$24, 24\frac{1}{2}$	$-24, -24\frac{1}{2}$	$-24\frac{1}{2}, -24$
$-99.9, -100,$	$-100, -99.9$	$100, 99.9$	$99.9, 100$
$-0.05, -0.5$	$-0.5, -0.05$	$0.5, 0.05$	$0.05, 0.5$
$-0.7, 0$	$-0.7, 0$	$0.7, 0$	$0, 0.7$
$100.02, 100.04$	$100.02, 100.04$	$-100.02, -100.04$	$-100.04, -100.02$

2. For each row, what pattern do you notice between the numbers in the 2nd and 4th columns? Why is this so?

For each row, the numbers in the 2nd and 4th columns are opposites, and their order is opposite. This is because on the number line, as you move to the right, numbers increase. But as you move to the left, the numbers decrease. So, when comparing 5 and 10, 10 is to the right of 5; so, 10 is greater than 5. However, -10 is to the left of -5, so, -10 is less than -5.