Understanding Math

Southern Nevada
Regional Professional Development Program

Radicals – Imaginary Numbers

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This issue of *Understanding Math*, is a newsletter about a mathematical topic often taught, but may need a little polish.

Does $\sqrt{a}\sqrt{b}=\sqrt{ab}$? Always? After all, it is a corollary to the product of powers $a^n b^n = (ab)^n$, in this case $a^{1/2} b^{1/2} = (ab)^{1/2}$. In Algebra I and II, we drill on this rule; $\sqrt{2}\sqrt{3}=\sqrt{?}$, $\sqrt{?}\sqrt{3}=\sqrt{12}$, etc. Students get a pretty clear picture that $\sqrt{a}\sqrt{b}=\sqrt{ab}$. But, it's not always so.

For example, what is $\sqrt{-4}\sqrt{-9}$? Is it 6? After all $(-4)(-9)=36$ and $\sqrt{36}=6$. The question can be answered by going back to the definitions of the imaginary unit and the square root of a negative number.

Definition: The imaginary unit, denoted by *i*, is equal to $\sqrt{-1}$, and $i^2=-1$.

Definition: $\sqrt{-a}=i\sqrt{a}$, where $a > 0$.

Going back to the original question, $\sqrt{-4}\sqrt{-9}$, one can see the correct result fall into place because of the definitions. $\sqrt{-4}\sqrt{-9}=i\sqrt{4}\cdot i\sqrt{9}=i\cdot 2\cdot i\cdot 3=6i^2=6(-1)=-6$.

Try this one: $\sqrt{-4}\sqrt{9}$.

Yes, the result is $6i$. But, did you get it by $\sqrt{-4}\sqrt{9}=2i\cdot 3=6i$, or by $\sqrt{-4}\sqrt{9}=\sqrt{-36}=6i$? Apparently, it doesn't matter here. Evidently, our old friend $\sqrt{a}\sqrt{b}=\sqrt{ab}$ works so long as *a* and *b* are not *both* negative.

So, do we now have two different rules for radicals? In a sense, yes. But, many times in algebra and analysis there are rules for real numbers that do not apply to complex ones. We need to remain aware of that fact.

When in doubt, order of operations can help. Since $\sqrt{a}\sqrt{b}$ can be written as $a^{1/2}b^{1/2}$, by order of operations the exponential portions of the expression are evaluated first, before multiplying. Thus, in the case of $\sqrt{-4}\sqrt{-9}=(-4)^{1/2}(-9)^{1/2}$, the radicals/exponentials simplify to $2i$ and $3i$. Multiplying the resulting factors is then done: $2i\cdot 3i=6i^2=(6)(-1)=-6$.

So, once again, the values of *a* and *b* do indeed make a difference in the rule $\sqrt{a}\sqrt{b}=\sqrt{ab}$. This is one of the finer points that is often lost on students and should be pointed out.