Properties of Polynomial Functions

I: Investigating the End Behavior and Turning Points

Using a graphing calculator, adjust the window as shown. Graph each function and complete the table.

a) $f(x) = 9x^2 - 8x - 2$

b) $f(x) = -x^4 - 3x^3 + 3x^2 + 8x + 5$

c) $f(x) = 2x^6 - 13x^4 + 15x^2 + x - 17$

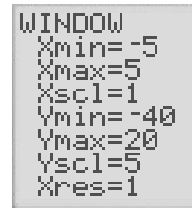
d) $f(x) = -2x^4 - 4x^3 + 3x^2 + 6x + 9$

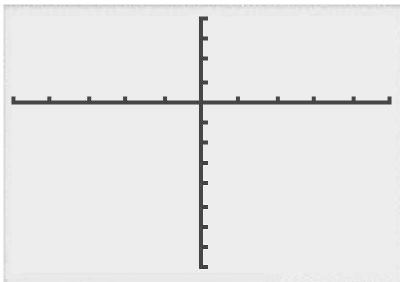
e) $f(x) = x^3 - 5x^2 + 3x + 4$

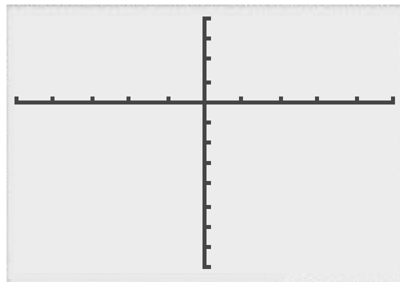
f) $f(x) = 2x^5 + 7x^4 - 3x^3 - 18x^2 - 20$

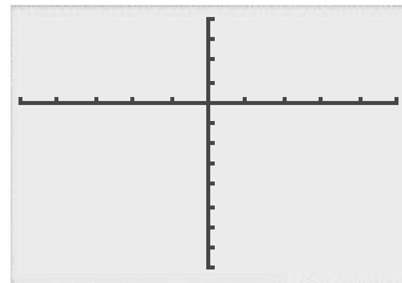
g) $f(x) = -x^7 + 8x^5 - 16x^3 + 8x$

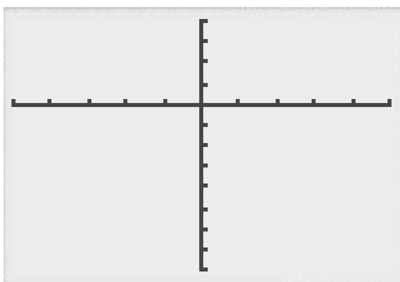
h) $f(x) = -2x^3 + 8x^2 - 5x + 3$

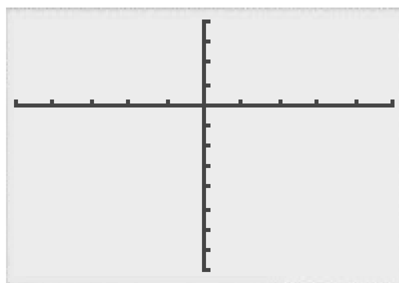


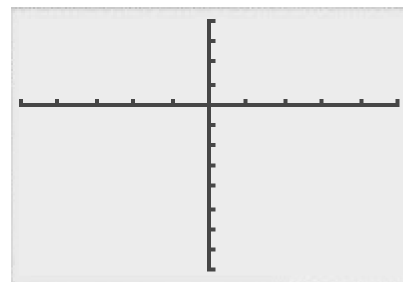


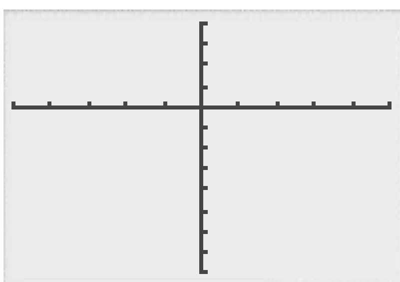


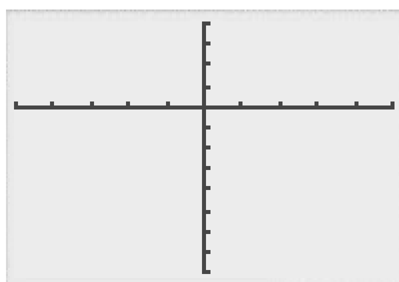












Function	Degree	Number of Turning Points	Leading Coefficient: + or - ?	Degree: even or odd	End Behavior	
					$x \to -\infty$	$x \to +\infty$
a)						
b)						
c)						
d)						
e)						
f)						
g)						
h)						

Observations

The maximum number of turning points in the graph of a polynomial function of degree 6 is ___________.

The maximum number of turning points in the graph of a polynomial function of degree 7 is ___________.

The maximum number of turning points in the graph of a polynomial function of degree *n* is ___________.

Polynomials with EVEN degree have end behaviours that are ___________________________

Polynomials with ODD degree have end behaviours that are ____________________________

State the **end behaviours** of a function with a degree that is:

a) **even** and has a **positive leading coefficient** ____

b) **even** and has a **negative leading coefficient** ____

c) **odd** and has a **positive leading coefficient** ____

d) **odd** and has a **negative leading coefficient** ____

II: Investigating the Number of Zeros

Using a graphing calculator, adjust the window as shown below. Graph each function and complete the table.

a) $f(x) = x^3 - 2x^2 - 4x + 8$

b) $f(x) = x^3 + x^2 - 2x - 7$

c) $f(x) = x^3 + 2x^2 - 3x - 5$

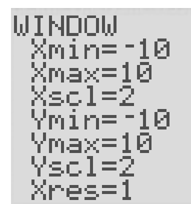
d) $f(x) = x^4 + 2x^3 - x^2 - 2x$

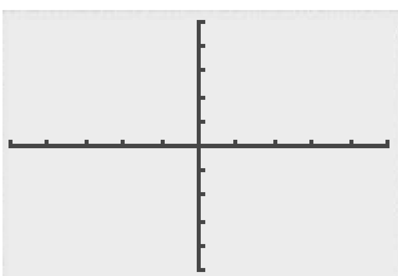
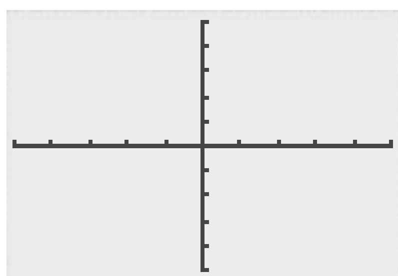
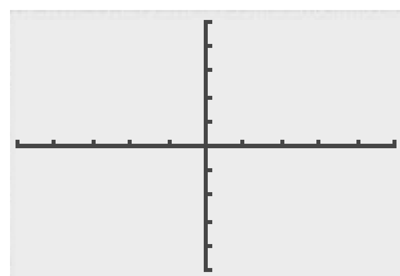
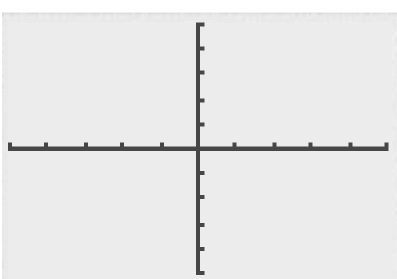
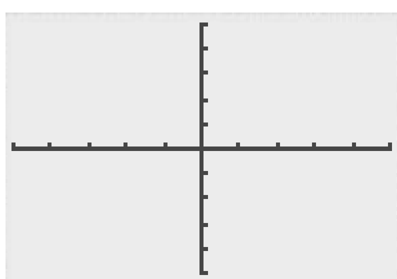
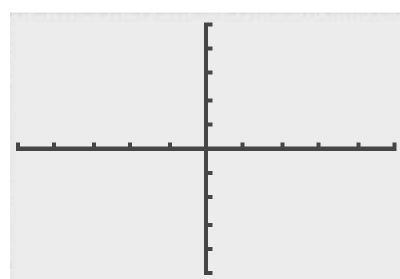
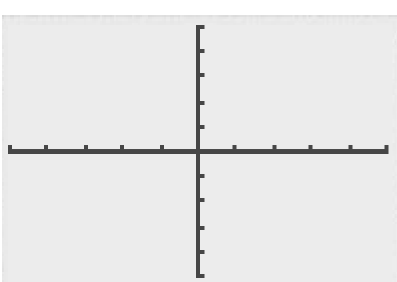
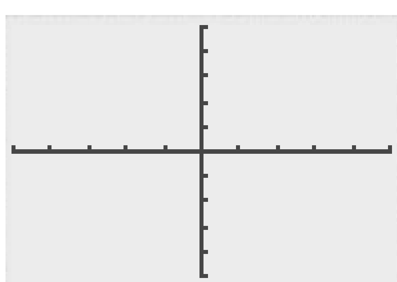
e) $f(x) = -x^4 + 2x^3 + x^2 + 2x$

f) $f(x) = 2x^4 - 6x^3 + x^2 + 4x + 5$

g) $f(x) = -x^4 - x^3 + 3x^2 + x - 2$

h) $f(x) = x^4 + x^3 + x + 1$



Function	Degree	Number of Zeroes
a)		
b)		
c)		
d)		
e)		
f)		
g)		
h)		

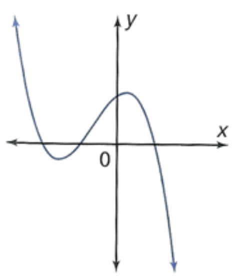
<u>Observations</u>

Degree	Minimum number of zeros	Maximum number of zeroes
3		
4		
5		
6		
n		

III: Exercises

1. Refer to the graphs of the following polynomial functions to complete the chart below.

a)



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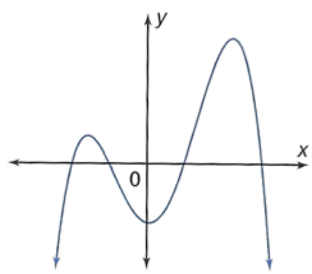
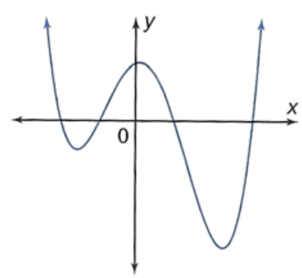
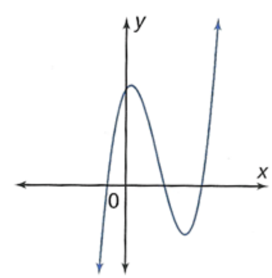


c)



d)



Function	Number of Turning Points	End Behavior		Leading Coefficient: + or - ?	Degree
		$x \to -\infty$	$x \to +\infty$		
a)					
b)					
c)					
d)					

2. Explain why **odd-degree** polynomial functions can have only ***local*** maximums and local minimums, but **even-degree** polynomial functions can have an ***absolute*** maximum or minimum.

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____

3. Describe the end behavior of each polynomial function by referring to the degree and the leading coefficient.

Function	End Behavior	
	$x \to -\infty$	$x \to +\infty$
a) $f(x) = 2x^2 - 3x + 5$		
b) $f(x) = -3x^3 + 2x^2 + 5x + 1$		
c) $f(x) = 5x^3 - 2x^2 - 2x + 6$		
d) $f(x) = -2x^4 + 5x^3 - 2x^2 + 3x - 1$		
e) $f(x) = 0.5x^4 + 2x^2 - 6$		
f) $f(x) = -3x^5 + 2x^3 - 4x$		

4. Sketch the graph of a polynomial function that satisfies each set of conditions.

a) degree 4, positive leading coefficient, 3 zeroes, 3 turning points

b) degree 4, negative leading coefficient, 2 zeroes, 1 turning point

c) degree 4, positive leading coefficient, 1 zero, 3 turning points

d) degree 3, negative leading coefficient, 1 zero, no turning point

e) degree 3, positive leading coefficient, 2 zeroes, 2 turning points