



OBJECTIVE #: G.CO.13

OBJECTIVE

Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

BIG IDEA (Why is this included in the curriculum?)

- Students are expected to construct equilateral triangles, squares, and circle-inscribed regular hexagons. This means they need to know the theorems and rules about these shapes, specifically that each shape is composed of congruent sides and congruent angles.

This objective will be implemented throughout the year.

PREVIOUS KNOWLEDGE (What skills do they need to have to succeed?)

- Students will need to know how to construct congruent line segments and congruent angles.
- Students will need to be able to draw (construct) perpendicular lines.
- Basic properties of regular polygons

VOCABULARY USED IN THIS OBJECTIVE (What terms will be essential to understand?)

PREVIOUS VOCABULARY (Terms used but defined earlier)

- Angle: A geometric figure that consists of two different rays that have the same initial point.
 - Acute Angle: An angle whose measure is less than 90° .
 - Right Angle: An angle whose measure is exactly 90° .
 - Obtuse Angle: An angle whose measure is greater than 90° but less than 180° .
 - Straight Angle: An angle whose measure is exactly 180° .
- Angle Bisector: A line/segment/ray that divides an angle into two congruent parts. Arc: A portion of circle.
- Bisect: To divide into two congruent parts.
- Circle: The set of all points in a plane that are equidistant from a given point, called the center.
- Distance: The length of a line segment between two points.
- Endpoint: A point marking either end of a line segment.
- Equidistant: Two points that are the same distance from a given point.
- Hexagon: A polygon with six sides.
- Line: An undefined term that extends in one dimension, which is usually represented by a straight line with two arrowheads.
- Line Segment: A portion of a line that consists of two endpoints and all points in between the two endpoints.
- Parallel Lines: Two lines that are coplanar and do not intersect.
- Perpendicular Bisector: A perpendicular line/segment/ray that intersects a segment at its midpoint.
- Perpendicular: Two lines/segments/rays that intersect to form right angles.
- Plane: An undefined term that extends in two dimensions, which is usually represented by a quadrilateral.
- Point: An undefined term that has no dimension which is usually represented by a dot indicating a specific location.
- Polygon: A closed figure created by line segments.
- Radius: The distance from the center of a circle to a point on the circle.

- Ray: A portion of a line that consists of a point called an initial point, and all points on the line that extend in one direction.
- Regular Polygon: A convex polygon with all angles congruent and all sides congruent.
- Sides of an Angle: Two rays that share a common endpoint and form the angle.
- Triangle: A figure formed by three segments joining three non-collinear points, called vertices.
 - Equiangular Triangle: A triangle with three congruent angles.
 - Equilateral Triangle: A triangle with three congruent sides.
- Vertex of an Angle: The initial point formed by the two rays (sides of an angle).

NEW VOCABULARY (New Terms and definitions introduced in this objective)

- NONE

SKILLS (What will they be able to do after this objective?)

- Using basic construction techniques, construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

SHORT NOTES (A short summary of notes so that a teacher can get the basics of what is expected.)

- The equilateral triangle construction works because the three inscribed angles measure 60° therefore the corresponding intercepted arcs measure 120° . Three 120° arcs will form one 360° circle.
- The construction of an inscribed square is based on the perpendicular bisector of the diameter.
- The inscribed hexagon is based on the equilateral triangle and its' 180° rotation.

MISCONCEPTIONS (What are the typical errors or difficult areas? Also suggest ways to teach them.)

FUTURE CONNECTIONS (What will they use these skills for later?)

- Will be used to prove that an inscribed quadrilateral has supplementary opposite angles.

ADDITIONAL EXTENTIONS OR EXPLANATIONS (What needs greater explanation?)

- It will be beneficial for students to see demonstrations of how to construct each shape, particularly noting that the side length of the hexagon-in-a-circle is equal to the radius of the circle. (That might be a fun in-class exploration, actually. Can they discover that relationship on their own?)

ASSESSMENTS (Questions that get to the heart of the objective – multiple choice, short answer, multi-step)

Short Answer

- 1) Given a line segment AB, what is the first step in constructing an equilateral triangle ABC?
Using a compass, mark the length of the segment AB. Using A as the center, draw an arc above the line segment.
- 2) What would be the next step in constructing the equilateral triangle ABC?
Without altering the compass measurement, draw an arc above the line segment using point B as the center. The two arcs should intersect one another at point C.
- 3) What is the final step of constructing the equilateral triangle ABC?
Using a straightedge, draw line segments AC and BC.

Multi-Step Problems

- 1) Use the segment below to complete each of the following constructions.



- Using segment AB , construct equilateral triangle ABC . Use mathematics to explain the process you used to construct the triangle. Use words, symbols, or both in your explanation.
See students' work
 - Construct circle O so that $\triangle ABC$ is inscribed in circle O . Use mathematics to explain your process. Use words, symbols, or both in your explanation.
See students' work
- 2) Mary plans to build a picnic table. She needs to create a drawing of the picnic table surface.
- Draw a regular hexagon with side lengths of 4 centimeters each. Label the vertices of your hexagon $A, B, C, D, E,$ and F . Label the center of your hexagon O . Explain the steps you used in your drawing.
See students' work
 - Classify $\triangle AOF$ according to its sides and/or angles. Use mathematics to justify your answers.
Equilateral, equiangular.
 - Classify $\triangle ABD$ according to its sides and/or angles. Use mathematics to justify your answer.
Scalene, right (30, 60, 90).
 - Classify $\triangle ABC$ according to its sides and/or angles. Use mathematics to justify your answer.
Isosceles, Obtuse.
- 3) How would you construct an equilateral triangle inscribed in a circle?
Draw a circle with a diameter. Adjust the compass so that its length is equal to the length of the radius. Using one of the diameter's points on the circle as the center, draw a semicircle such that two intersections are formed. With a straightedge, draw line segments that connect the two intersections and the opposite endpoint of the diameter. These three segments make an equilateral triangle.
- 4) Given two perpendicular and congruent sides, how would you construct a square?
Using the compass, measure the length of the sides. Draw an arc of this length from both the endpoints (not the vertex of the perpendicular angle) of the segments. The two arcs should intersect. With a straightedge, connect this point to the two endpoints of the segments.
- 5) Given only a line segment, how would you construct a square?
Construct a segment perpendicular to the given segment at one of the given endpoints. Using a compass, mark the length of the newly drawn segment at the same length as the given segment. Complete the construction using the answer to question 4.
- 6) How would you construct a square inscribed in a circle?
Construct a circle and draw its diameter with a straightedge. Construct a segment perpendicular to the diameter at the center of the circle. These two segments should cross the circle at four points. With a straightedge, connect these four points.
- 7) How would you construct a square given a line segment that must be the diagonal of the square?
First, construct the perpendicular bisector of the given segment. Using a compass, mark the distance from the midpoint to one of the endpoints and draw a complete circle. This circle should intersect the given line segment and its perpendicular bisector at four points. Connect these four points with a straightedge.
- 8) How would you construct a regular hexagon inscribed within a circle?
Using a compass, mark the distance of the radius of the circle. Choosing an arbitrary point on the circle as a starting place, draw a small arc that intersects with the circle keeping the compass at the same length. Use this new intersection as the new point, repeat this process until six you have six intersections between the circle and these arcs. These will be the six vertices of the hexagon. Connect these vertices with a straightedge.

9) How would you construct a regular hexagon given one of the side lengths?

With a compass, mark the distance of the segment. Draw two small intersecting arcs above the segment using the two endpoints as vertices. The intersection of these two arcs will form the center of the circle with a radius that is the same length as the given side. Draw this circle and continue using the answer to question 8.