**OBJECTIVE #:**  G.CO.9

**OBJECTIVE**
- Prove theorems about lines and angles.  Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

**BIG IDEA** (Why is this included in the curriculum?)
- To establish angle and length relationships as they relate to parallel and perpendicular lines.

**PREVIOUS KNOWLEDGE** (What skills do they need to have to succeed?)
- The student must understand parallel lines.
- The student must understand the types of transformations.

**VOCABULARY USED IN THIS OBJECTIVE** (What terms will be essential to understand?)
   **PREVIOUS VOCABULARY** (Terms used but defined earlier)
   - Angle:  A geometric figure formed by rotating a ray about its initial point.
   - Corresponding Angles:  Two angles that are formed by two lines and a transversal which lie in the same position with respects to the lines and the transversal.
   - Equidistant:  Two points that are the same distance from a given point.
   - Parallel Lines:  Two lines that are coplanar and do not intersect.
   - Perpendicular:  Two lines/segments/rays that intersect to form right angles.
   - Perpendicular Bisector:  A perpendicular line/segment/ray that intersects a segment at its midpoint.

   **NEW VOCABULARY** (New Terms and definitions introduced in this objective)
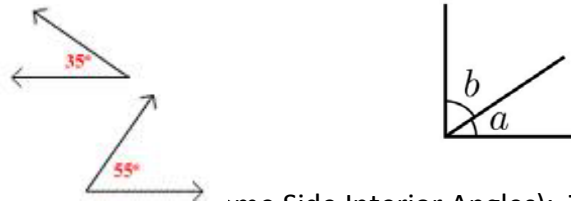   - Alternate Interior Angles:  Two angles that are formed by two lines and a transversal and that lie between the two lines on opposite sides of the transversal.

Alternate interior angles
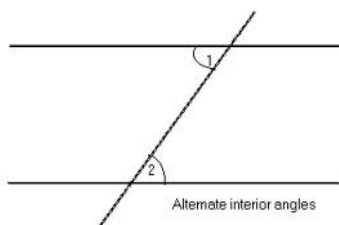
   - Alternate Exterior Angles:  Two angles that are formed by two lines and a transversal and that lie outside the two lines on opposite sides of the transversal.

Alternate exterior angles
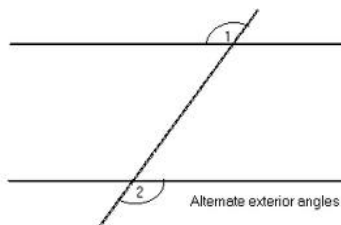
- Complementary Angles:  Two angles whose measures have the sum 90°. [G.CO.10]

- Consecutive Interior Angles (Same Side Interior Angles):  Two angles that are formed by two lines and a transversal and that lie between the two lines on the same side of the transversal.
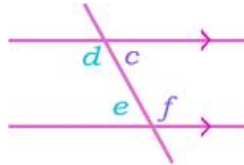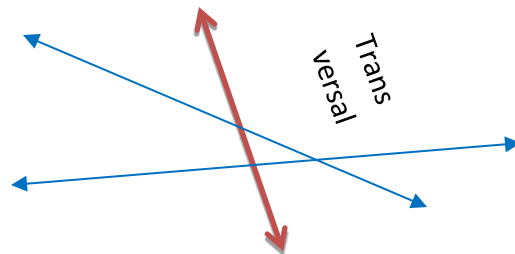
- Linear Pair:  Two adjacent angles whose non-common sides are opposite rays, thus making them supplementary. [G.CO.10]
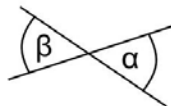
- Supplementary Angles:  Two angles whose measures have the sum 180°. [G.CO.10]
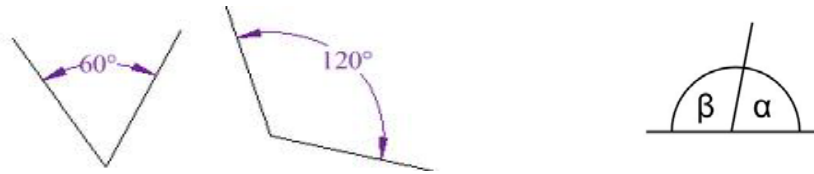
- Transversal Line:  A line that intersects two or more coplanar lines at different points

- Vertical Angles:  The two non-adjacent angles formed by intersecting lines.

**SKILLS** (What will they be able to do after this objective?)
- Students will be able to identify all of the types of angle relationships formed when two parallel lines are cut by a transversal.
- Students will be able to know all of the relationships between pairs of angles.
- Students will be able to prove that all points on a perpendicular bisector of a segment are equidistant from the segment endpoints.

**SHORT NOTES** (A short summary of notes so that a teacher can get the basics of what is expected.)
- This objective is introduced in geometry using the transformations described in G.CO.4 by translating along vectors.

- For example, corresponding angles are formed by translating one angle along the transversal onto its image.
    - See attached files for summarization that may be used for this objective.
  - $\angle AXY$ is translated along $\overrightarrow{XY}$
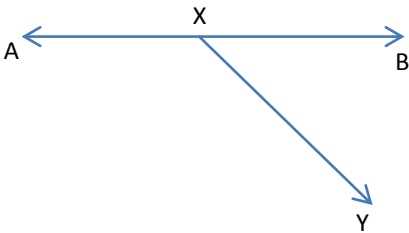


  - This results in $\overleftrightarrow{AB}$ and $\overleftrightarrow{A'B'}$ being parallel lines, therefore creating congruent corresponding angles $\angle AXY$ and $\angle A'X'Y$.
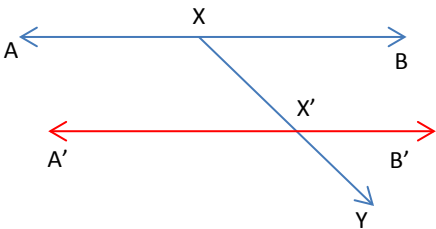


  - Students should be encouraged to explore the different types of transformations or composite transformations that form these pairs of angles.
- Have the students construct the perpendicular bisector of a segment. Allow students to choose points along the bisector and measure the distance from the endpoints of the segment.
  - Example: Student drew $\overline{BC}$ and constructed the perpendicular bisector. Student then chose point A and measured the distance from A to C and from A to B. Students can compare their findings (that $\overline{AC} \cong \overline{AB}$) to determine this holds true for any point along the perpendicular bisector.
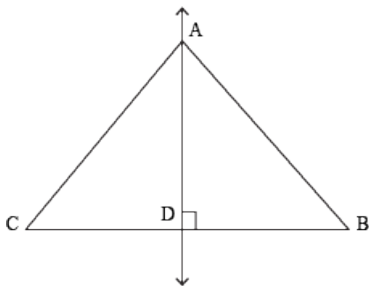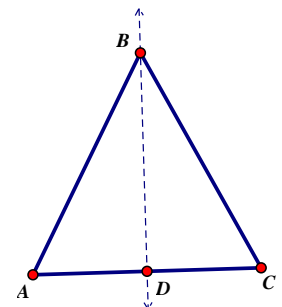


Below is the formal two-column proof.

**Given:** $\triangle ABC$ with perpendicular bisector $\overleftrightarrow{BD}$
**Prove:** $\overline{AB} \cong \overline{BC}$



| STATEMENT | REASON |
|---|---|
| $\overline{BD}$ is the $\perp$ bisector of $\angle B$ | Given |
| $\overline{AD} \cong \overline{CD}$ | Definition of a $\perp$ Bisector (D is the midpoint of $\overline{AC}$) |
| $\angle ADB$ & $\angle CDB$ are right $\angle$'s | Definition of a $\perp$ Bisector ($\overline{BD}$ is $\perp$ to $\overline{AC}$) |
| $\angle ADB \cong \angle CDB$ | All Right Angles are $\cong$ |
| $\overline{BD} \cong \overline{BD}$ | Reflexive Property (Common Side) |

| $\triangle ABD \cong \triangle CBD$ | SAS |
| --- | --- |
| $\overline{AB} \cong \overline{BC}$ | CPCTC (Corresponding Parts of Congruent Triangles are Congruent) |

**MISCONCEPTIONS** (What are the typical errors or difficult areas?  Also suggest ways to teach them.)
- Although these pairs of angles are formed without the lines being parallel, the congruence and supplementary aspects are not applicable if the original lines are not parallel.

**FUTURE CONNECTIONS** (What will they use these skills for later?)
- These angle relationships of parallel lines and perpendicular bisector will be utilized to discover the characteristics of different parallelograms.

**ADDITIONAL EXTENSIONS OR EXPLANATIONS** (What needs greater explanation?)
- Students will utilize congruent triangles to prove all points on a perpendicular bisector of a segment are equidistant from the segment endpoints.
- Corresponding Angles:  Two angles formed by translating a given angle in a plane.

**ASSESSMENTS** (Questions that get to the heart of the objective – multiple choice, short answer, multi-step)

1) Correctly match the following and include the transformation that maps one angle onto its pair.

_____ 1. Alternate Interior Angles  E              A.    ∠4 and ∠6

Translate ∠3 along the transversal onto ∠7.  Then rotate 180° onto ∠6

_____ 2. Alternate Exterior Angles  C              B.    ∠1 and ∠ 5

Rotate ∠2 180° and then translate along the transversal onto ∠7

_____ 3. Corresponding Angles  B              C.    ∠2 and ∠7

Translate ∠1 along the transversal onto ∠5

_____ 4. Consecutive Interior Angles A              D.    ∠1 and ∠4

Not a transformation

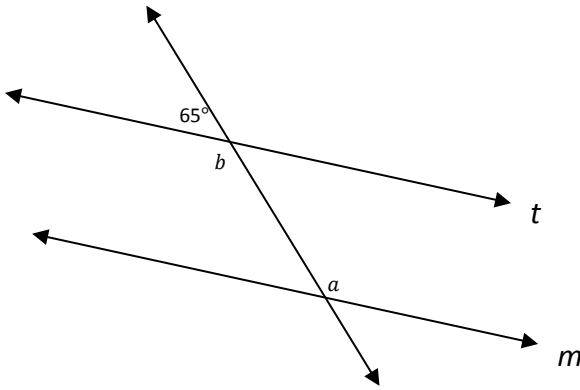_____ 5. Vertical Angles  D              E.    ∠3 and ∠6

Rotate ∠1 180° onto ∠4
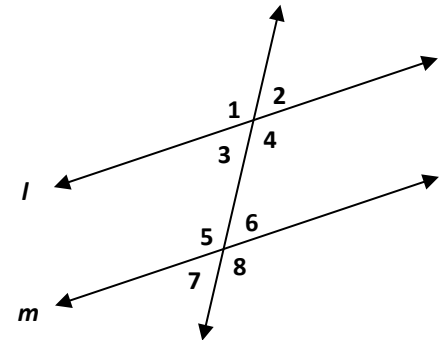
2)   Find the measure of angle *a* and *b* if $t \parallel m$.  Justify your reasoning using transformations.
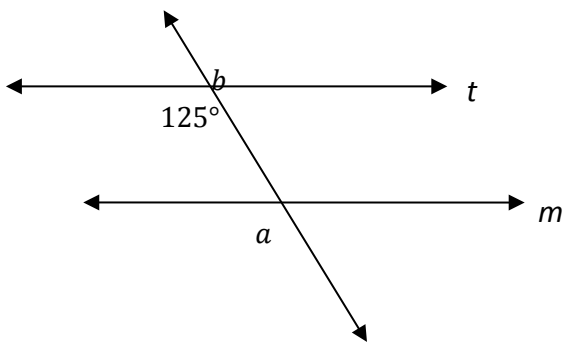
*a* = 125° by translating the given angle along the transversal
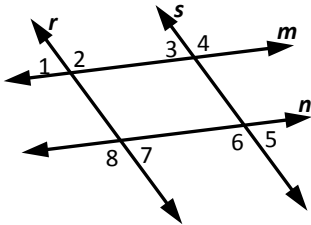
*b* = 125° by rotating the given angle 180°

3) Find the measure of angle *a* and *b* if $t \parallel m$. Justify your reasoning using transformations.



The given angle and *b* form a linear pair, therefore *b* = 115°.
Rotate *b* 180° and translate along the transversal onto *a*.
Therefore *a* = 115°

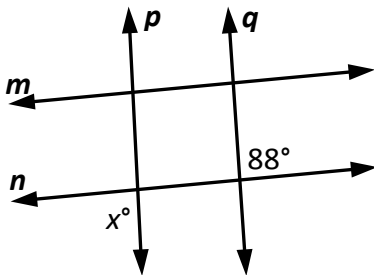**From CCSD Geometry Honors Semester 1 Practice Exam 2012 – 2013**

1. Use the diagram.



Which statement would be used to prove lines *r* and *s* are parallel?
(A) ∠1 and ∠3 are congruent
(B) ∠2 and ∠7 are complementary
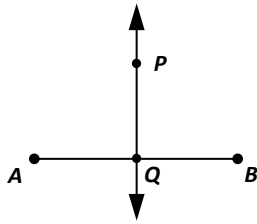(C) ∠4 and ∠1 are congruent
(D) ∠8 and ∠6 are supplementary

2. In the diagram, $m \parallel n$ and $p \parallel q$.



What is the value of *x*?
(A) 44
(B) 88
(C) 92
(D) 176

**For questions 3 – 6, use the diagram where $B$ is the reflection of $A$ across $\overleftrightarrow{PQ}$.**



3.  $PA = PB$
    (A) True
    (B) False

4.  $\overleftrightarrow{PQ} \perp \overline{AB}$
    (A) True
    (B) False

5.  $AQ = QB$
    (A) True
    (B) False

6.  $PQ = \dfrac{1}{2}AB$
    (A) True
    (B) False