Solving Quadratic Equations and Inequalities

Math Background

Previously, you

- Graphed quadratic functions and found the vertex and axis of symmetry in vertex and standard form.
- Solved linear equations by isolating the variable.
- Simplified radicals.
- Recognized common factors, perfect square trinomials and difference of squares patterns.
- Factored quadratic expressions.
- Defined constraints for linear functions in equations and inequalities.

In this unit you will

- Solve quadratic equations by graphing.
- Solve quadratic equations by various algebraic methods including square root method, factoring, completing the square and using the quadratic formula.
- Solve quadratic inequalities.

You can use the skills in this unit to

- Show the maximum and minimum, intercepts, and axis of symmetry of quadratic functions with and without technology.
- Determine the number and nature of quadratic solutions.
- Know multiple methods to solve a quadratic equation and know when each method is appropriate.
- Recognize when a quadratic equation does not have any real solutions.
- Interpret features of a quadratic function in terms of the real-world situation it models.

Vocabulary

- **Completing the square** – A way of simplifying or solving a quadratic equation by adding an expression to both sides to make one part of the equation a perfect square.
- **Constraints** – A limitation usually imposed upon either the domain of a function or the range of a function. In linear programming, the constraint tells you to only consider a certain range of inputs and the constraints are represented as linear inequalities.
- **Complex solutions** – The x-values that make the function equal to zero are complex numbers (have an imaginary part).
- **Discriminant** – The expression b^2-4ac in a quadratic equation $ax^2+bx+c=0$. It can be used to determine the characteristics of the solution.
- **Extracting roots** – Refers to applying the square root property as a means of solving a quadratic equation.
- **Quadratic Equation –** A polynomial equation in which the highest power of the variable is two. The general form of such equations in the variable *x* is $ax^2+bx+c=0$.

- **Quadratic Formula –** The formula for determining the roots of a quadratic equation from its coefficients.
- **Real Solutions** – Solutions of the variable that make the equation true but are either rational numbers or irrational numbers.
- **Zeros of a Function** – The x-value or x-values that make the function equal to zero. A zero may be a real number or a complex number.

Essential Questions

- What are the key features of quadratic functions?
- What are the different ways to solve quadratic equations and which ways are more efficient? Do all quadratics have real solutions?
- Why is it helpful to change the form of quadratic expressions by factoring?
- When do you use an equation versus an inequality? How can we model applications and solve specific problems?

Overall Big Ideas

A quadratic function is represented by a U-shaped curve called a parabola which intercepts one or both axes and has one maximum or minimum value. Solving quadratic functions can be done by different techniques like square rooting, factoring, completing the square, graphing and using the quadratic formula. Some techniques are more efficient than others based on the characteristics of the quadratic function. Some quadratics do not have real solutions. Changing the form of the expression by factoring reveals important attributes about the function and its graph. Variable equations or inequalities model real-life situations and generalize applications, building a foundation for solving equations and inequalities with more than one variable.

Skill

To solve quadratic equations by graphing.

To solve quadratic equations by finding the square root of both sides of the equation.

To solve quadratic equations by factoring.

To solve quadratic equations by completing the square.

To solve quadratic equations using the quadratic formula.

To solve quadratic inequalities.

Related Standards

F.IF.C.7a
Graph linear and quadratic functions and show intercepts, maxima, and minima. *(Modeling Standard)

N.CN.C.7
Solve quadratic equations with real coefficients that have complex solutions.

A.REI.B.4b
Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

A.SSE.B.3a
Factor a quadratic expression to reveal the zeros of the function it defines. *(Modeling Standard)

F.IF.C.8a
Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

A.CED.A.1-2
Create equations and inequalities in one variable and use them to solve problems. Include equations arising from all types of functions, including simple rational and radical functions. *(Modeling Standard)

A.CED.A.3-2
Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. Use all types of equations. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. *(Modeling Standard)

Notes, Examples, and Exam Questions

A quadratic equation in the form $ax^2 + bx + c = 0$ has a related function $f(x) = ax^2 + bx + c$. The zeros of the function are the x-intercepts of its graph. These x-values are the solutions or roots of the related quadratic equation. A quadratic equation can have one real solution, two real solutions, or no real solutions. Note that the three words - zeros, roots and x-intercepts - all represent the same thing – the x-values that make the function equal to zero.

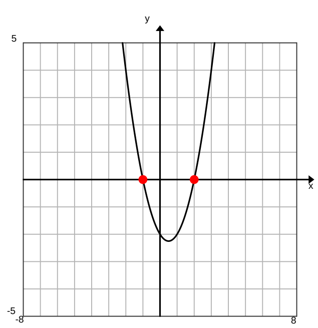
Graphing quadratic functions was discussed in Unit 3.1-3.2. By placing the function in the intercept form, we can find the points where the parabola intersects the x-axis. There are 3 cases that can occur when graphing the parabola.

x-intercept: the *x*-coordinate of the point where the curve intersects the *x*-axis

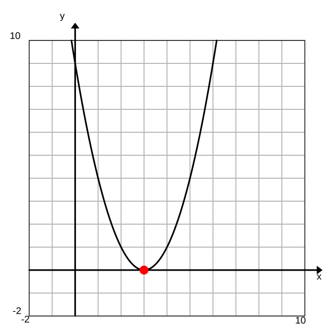
Intercept Form of a Quadratic Function: $y = a(x-p)(x-q)$ *x*-Intercepts: p, q

x-Coordinate of Vertex: $x = \dfrac{p+q}{2}$ (vertex is halfway between the *x*-intercepts)

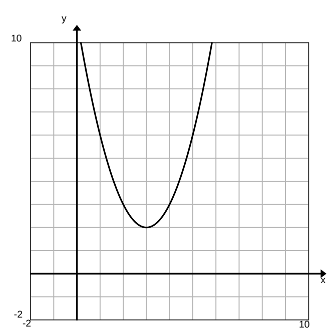
Models: A quadratic equation can have one real solution, two real solutions, or no real solutions depending on the number of times the graph crosses the x-axis.



Two real solutions



One Real Solution



No real solution

Graphing a Quadratic Function in Intercept Form

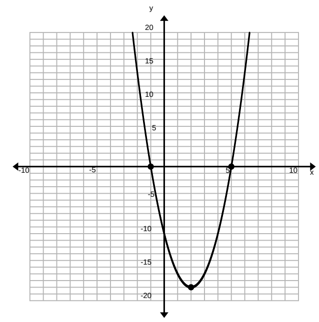
Ex 1 Graph the quadratic function $f(x) = 2(x-5)(x+1)$.

Step One: Identify the *x*-intercepts, *p* and *q*. *x*-intercepts are 5 and -1

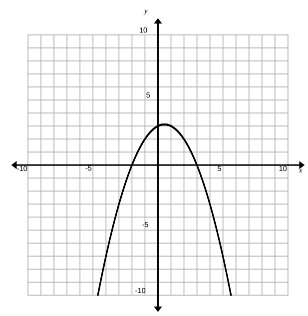
*Note: It may be helpful to write the equation as $y = 2(x-5)(x-(-1))$

Step Two: Identify the vertex. x-coordinate of vertex: $x = \frac{5+-1}{2} = 2$

y-coordinate of vertex: $y = 2\left((2)-5\right)\left((2)+1\right) = 2(-3)(3) = -18$



Step Three: Plot the x-intercepts and the vertex and draw the parabola.



Writing the Equation of a Quadratic Function in Standard Form

Ex 2 Write an equation for the parabola in standard form.

The x-intercepts (zeros) of the parabola are at -2 and 3. So the intercept form of the quadratic equation is $y = a(x+2)(x-3)$.

To solve for a, we will choose a point on the parabola and substitute it into the equation for (x, y). Choose a simple point from the graph, like (2, 2). Substituting it in to the equation above,

$$2 = a(2+2)(2-3)$$
$$2 = -4a$$
$$-\frac{1}{2} = a$$

So the intercept form of the equation is $\boxed{y = -\frac{1}{2}(x+2)(x-3)}$.

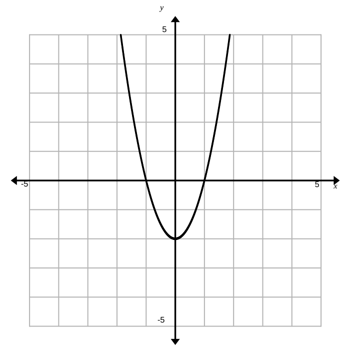
To rewrite in standard form, multiply the binomials (FOIL) and distribute the constant.

$$y = -\frac{1}{2}(x+2)(x-3) = -\frac{1}{2}\left(x^2 - x - 6\right)$$

$$\boxed{y = -\frac{1}{2}x^2 + \frac{1}{2}x + 3}$$

*****Note that when the quadratic is in standard form, the *c* represents the y-intercept. In the graph above, the parabola intersects the y-axis at (0, 3).**

Ex 3



Write the intercept form and standard form of the parabola shown in the graph.

From the graph, the intercepts are 1 and -1. The vertex is (0, -2).

$-2 = a(0+1)(0-1)$

Substituting into the intercept form: $-2 = -1a$

$2 = a$

So, the intercept form is: $\boxed{y = 2(x+1)(x-1)}$

Simplifying to find the standard form: $y = 2(x+1)(x-1) = 2(x^2-1)$

$\boxed{y = 2x^2 - 2}$

<u>QOD</u>: How can you tell from the graph of a quadratic function if the equation has one, two, or no solution?

<u>Using a Graphing Calculator to Solve Quadratic Equations</u>

Ex 4 Approximate the solution(s) of $x^2 = 1 - 4x$ using a graphing calculator.

Step One: Write the equation in the form $ax^2 + bx + c = 0$. $x^2 + 4x - 1 = 0$

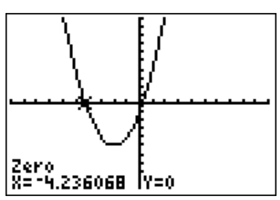
Step Two: Graph the function $y = ax^2 + bx + c$. $y = x^2 + 4x - 1$

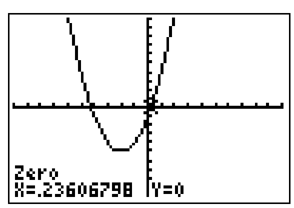
Press $\boxed{\text{Y =}}$, plug in the equation and then hit $\boxed{\text{ZOOM}}$ 6 for a nice window for the graph.

Step Three: Find the zero(s) of the function.

Press $\boxed{\text{2nd}}\boxed{\text{TRACE}}\boxed{2}$ to find the zeros. To find each zero, make sure the cursors are to the left and right of the zero.

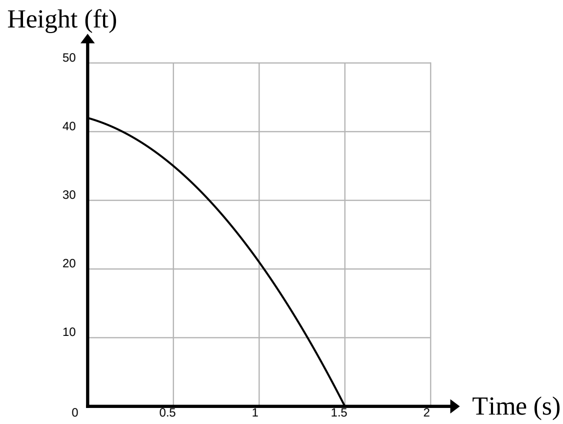
$\boxed{x \approx -4.236, 0.236}$





SAMPLE EXAM QUESTIONS

1. Use the graph provided to choose the best description of what the graph represents.



A. A ball is dropped from a height of 42 feet and lands on the ground after 3 s.

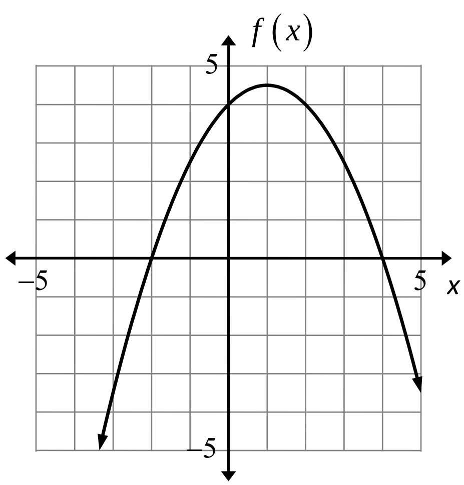
B. A ball is dropped from a height of 42 feet and lands on the ground after 1.5 s.

C. A ball is shot up in the air and reaches a height of 42 feet after 1 s.

D. A ball is shot up in the air, reaches a height of 42 feet, and lands on the ground after 1.5 s.

Ans: B

2. Look at the graph of the quadratic $f(x)$ below.



The graph of $g(x) = 3x^2 + bx - 24$ has the same *x*-intercepts.

What is the value of *b*?

A. –6 **C.** –2

B. 1 **D.** 14

Ans: A

3. Where is the axis of symmetry in the quadratic $f(x)=3(x-9)(x+5)$?

A. $x = 4$

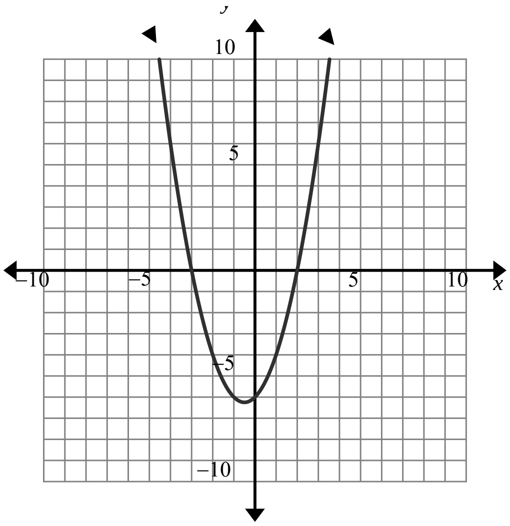
B. $x = 2$

C. $x = 6$

D. $x = -2$

Ans: B

4. Use the graph.



Which equation is represented the following graph?

A. $y = x^2 - x - 6$

B. $y = x^2 - x + 6$

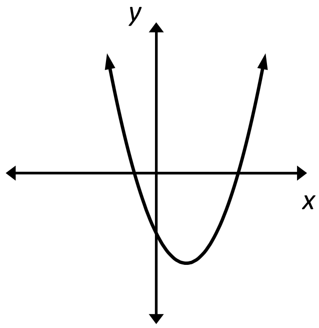
C. $y = x^2 + x - 6$

D. $y = x^2 + x + 6$

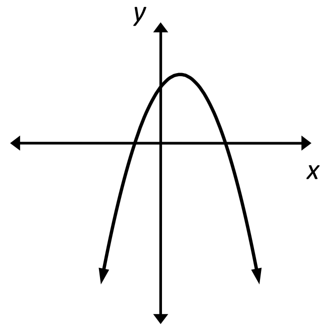
Ans: C

5. A quadratic function is given by $h(x) = ax^2 + bx + c$, where *a* and *c* are negative real numbers. Which of these could be the graph of $y = h(x)$?

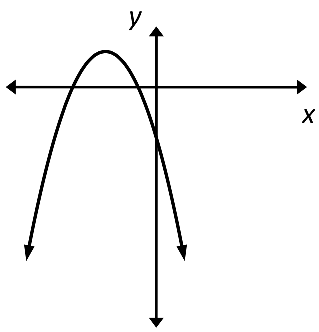
(A)



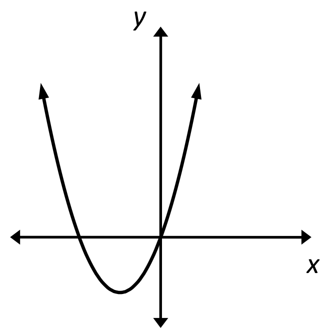
(B)



(C)



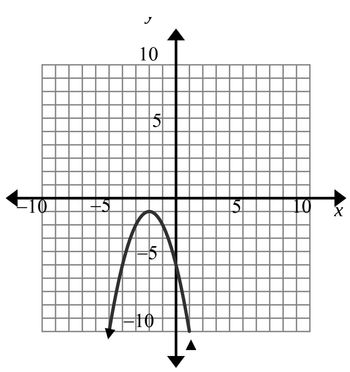
(D)



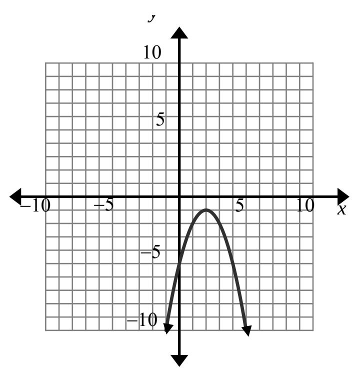
Ans: C

6. Which of the following is the graph of $y = -x^2 + 4x - 5$?

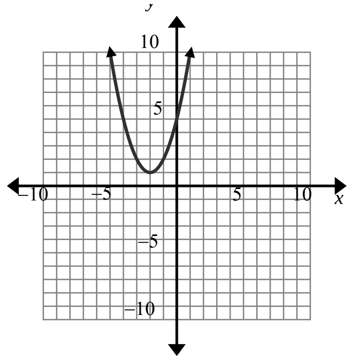
(A)



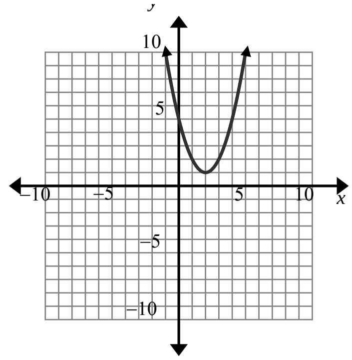
(B)



(C)



(D)



Ans: B

Extracting Roots

Another method for solving quadratic equations is by isolating the squared expression and finding the square root of both sides. This method will often involve working with radicals and simplifying.

Review: Simplifying Square Roots – A square root is *simplified* if the radicand has no perfect square factor (other than 1) and there is no radical in the denominator of a fraction.

Ex 5

Simplify the square root $\sqrt{72}$.

Method 1 – **Step One**: Find the largest perfect square that is a factor of 72. 36

Step Two: Rewrite 72 as a product using 36 as a factor. $\sqrt{36 \cdot 2}$

Step Three: Rewrite as the product of two radicals. $\sqrt{36} \cdot \sqrt{2}$

Step Four: Evaluate the square root of the perfect square. $\boxed{6\sqrt{2}}$

Method 2 – **Step One**: Rewrite 72 as a product of prime factors. $\sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$

$\sqrt{(2 \cdot 2) \cdot 2 \cdot (3 \cdot 3)}$

Step Two: Find the square root of each "pair" of factors. $= \sqrt{2^2 \cdot 3^2 \cdot 2} = \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{2}$

$= 2 \cdot 3\sqrt{2} = \boxed{6\sqrt{2}}$

Ex 6

Simplify the expression $\frac{2\sqrt{3}}{\sqrt{8}}$.

We must *rationalize* the denominator by multiplying by $1 = \frac{\sqrt{8}}{\sqrt{8}}$.

$$\frac{2\sqrt{3}}{\sqrt{8}} \cdot \frac{\sqrt{8}}{\sqrt{8}}$$
$$= \frac{2\sqrt{24}}{8}$$

Now simplify the radical and the fraction.

$$= \frac{2\sqrt{24}}{8} = \frac{2\sqrt{4 \cdot 6}}{8}$$
$$= \frac{2 \cdot 2\sqrt{6}}{8} = \frac{4\sqrt{6}}{8} = \boxed{\frac{\sqrt{6}}{2}}$$

Ex 7

<u>Solving a Quadratic Equation by Finding Square Roots</u>

Solve the equation $3x^2 - 108 = 0$.

Step One: Isolate the squared expression.

$$3x^2 = 108$$
$$x^2 = 36$$

Step Two: Find the square root of both sides.

$$\sqrt{x^2} = \sqrt{36}$$
$$|x| = 6$$

$** \sqrt{x^2} = \pm x$

Step Three: Solve for the variable.

$\boxed{x = -6 \text{ or } x = 6}$ Set Notation: $\{-6, 6\}$

Ex 8

Solve the equation $2(2n-5)^2 = 162$.

Step One: Isolate the squared expression.

$$(2n-5)^2 = 81$$

Step Two: Find the square root of both sides.

$$\sqrt{(2n-5)^2} = \sqrt{81}$$
$$2n - 5 = \pm 9$$

Step Three: Solve for the variable.

$$2n-5=9 \quad 2n-5=-9$$
$$2n=14 \quad 2n=-4$$
$$\boxed{n=7} \quad \boxed{n=-2}$$

Ex 9

Solve the equation $\frac{1}{4}(a-8)^2 = 7$,

Step One: Isolate the squared expression. $(a-8)^2 = 28$

Step Two: Find the square root of both sides.

$$\sqrt{(a-8)^2} = \sqrt{28}$$
$$a-8 = \pm 2\sqrt{7}$$

Step Three: Solve for the variable.

$$a-8 = 2\sqrt{7} \qquad a-8 = -2\sqrt{7}$$
$$a = 8+2\sqrt{7} \qquad a = 8-2\sqrt{7}$$
$$\boxed{a = 8 \pm 2\sqrt{7}}$$

**Note: The $\pm$ ("plus or minus") symbol is used to write both solutions in a shorter way. In set notation, the solutions would be written $\{8-2\sqrt{7}, 8+2\sqrt{7}\}$.

Ex 10

Real-Life Application: Free Fall

On Earth, the equation for the height (*h*) of an object for *t* seconds after it is dropped can be modeled by the function $h = -16t^2 + h_0$, where h_0 is the initial height of the object.

A ball is dropped from a height of 81 ft. How long will it take for the ball to hit the ground?

Use the free-fall function. $h = -16t^2 + h_0$ $\quad h_0 = 81, h = 0$

Initial height is 81 ft. The ball will hit the ground when its height is 0 ft.

Solve for *t*.

$$0 = -16t^2 + 81$$
$$16t^2 = 81$$
$$\sqrt{t^2} = \sqrt{\frac{81}{16}}$$
$$|t| = \frac{9}{4}$$
$$t = -\frac{9}{4}, \frac{9}{4}$$

Solution: Since time is positive, we have a constraint on this problem as time must be greater than zero. The only feasible answer is $\frac{9}{4} =$ 2.25 seconds

Ex 11

Solve the equation $7-10x^2=1$.

$-10x^2=-6 \Rightarrow x^2=\sqrt{\frac{3}{5}}$

$x=\pm\frac{\sqrt{3}}{\sqrt{5}}\cdot\frac{\sqrt{5}}{\sqrt{5}}=\frac{\sqrt{15}}{5}$

$x=\frac{\sqrt{15}}{5}$ and $-\frac{\sqrt{15}}{5}$

SAMPLE EXAM QUESTIONS

1. Solve the quadratic equation by taking the square root. $4x^2+5=-1$

(A) $x=\sqrt{\frac{-6}{2}}$

(B) $x=\frac{-6}{4}$

(C) $x=\pm\frac{i\sqrt{6}}{2}$

(D) $x=\sqrt{\frac{-6}{4}}$

Ans: C

2. Solve $x^2+25=0$ **over the set of complex numbers.**

(A) $\pm 25i$

(B) ± 5

(C) $\pm 5i$

(D) ± 25

Ans: C

3. Solve the equation for *x*: $a\left(x-h\right)^2+k=p$

A. $x=h\pm\sqrt{\frac{p}{a}-k}$

B. $x=h\pm\sqrt{\frac{p}{a}-\sqrt{k}}$

C. $x=h\pm\frac{\sqrt{p-k}}{a}$

D. $x=h\pm\sqrt{\frac{p-k}{a}}$

Ans: D

4. What is the solution set of the equation $4\left(t-3\right)^2-1=8$ **?**

A. $\left\{1\frac{1}{2},\ 4\frac{1}{2}\right\}$

B. $\left\{\frac{3}{4},\ 5\frac{1}{4}\right\}$

C. $\left\{3-\sqrt{3},\ 3+\sqrt{3}\right\}$

D. $\left\{3-\sqrt{5},\ 3+\sqrt{5}\right\}$

Ans: A

5. What is the solution set of the equation $36x^2-25=0$ **?**

A. $\left\{\frac{5}{6}\right\}$

B. $\left\{\frac{25}{36}\right\}$

C. $\left\{\frac{-5}{6},\ \frac{5}{6}\right\}$

D. $\left\{\frac{-25}{36},\ \frac{25}{36}\right\}$

Ans: C

Factoring

Review: Factoring Quadratic Trinomials into Two Binomials (Using the "ac method" or "splitting the middle term".)

1: Factoring $ax^2 + bx + c,\ a = 1$ Factor $x^2 - 7x + 12$.

Find two integers such that their product is 12 and their sum is -7. -4 and -3

Write the two binomials as a product. $\boxed{(x-4)(x-3)}$

2: Factoring $ax^2 + bx + c,\ a \neq 1$ Factor $2x^2 + 7x + 3$.

Step One: Multiply $a \cdot c$. $2 \cdot 3 = 6$

Step Two: Find two integers such that their product is $a \cdot c = 6$ and their sum is $b = 7$. 6 and 1

Step Three: Rewrite ("split") the middle term as a sum of two terms using the numbers from Step Two.

$2x^2 + 6x + 1x + 3$ (order does not matter when splitting the middle term)

Step Four: Factor by grouping. Group the first 2 terms and last 2 terms and factor out the GCF from each pair.

$$(2x^2 + 6x) + (1x + 3) = 2x(x+3) + 1(x+3)$$

Step Five: If Step Four was done correctly, there should be a common binomial factor. Factor this binomial out and write what remains from each term as the second binomial factor. $\boxed{(2x+1)(x+3)}$

3: Factoring $ax^2 + bx + c,\ a \neq 1$ Factor $5x^2 - 7x + 2$.

Step One: Multiply $a \cdot c$. $\boxed{5 \cdot 2 = 10}$

Step Two: Find two integers such that their product is $a \cdot c = 10$ and their sum is $b = -7$. $\boxed{-2 \text{ and } -5}$

Step Three: Rewrite ("split") the middle term as a sum of two terms using the numbers from Step Two.

$5x^2 - 2x - 5x + 2$

Step Four: Factor by grouping. Group the first 2 terms and last 2 terms and factor out the GCF from each pair.

$$(5x^2 - 2x) + (-5x + 2)$$
$$= x(5x-2) - 1(5x-2)$$

Step Five: If Step Four was done correctly, there should be a common binomial factor. Factor this binomial out and write what remains from each term as the second binomial factor. $\boxed{(5x-2)(x-1)}$

Special Factoring Patterns: Memorize these!

Difference of Two Squares:

$a^2 - b^2 = (a+b)(a-b)$

Perfect Square Trinomial:

$a^2 + 2ab + b^2 = (a+b)^2$

$a^2 - 2ab + b^2 = (a-b)^2$

Factor $9x^2 - 16y^4$.

This appears to be a difference of two squares, since each term is a perfect square. Rewrite each term as a monomial squared then use the pattern to factor.

$$(3x)^2 - (4y^2)^2 = \boxed{(3x+4y^2)(3x-4y^2)}$$

Factor $49m^2 - 14mn + n^2$.

This appears to be a perfect square trinomial. Rewrite the first and last terms as a monomial squared and check to see if the middle term is twice the product of these monomials. Then use the pattern to factor.

$$(7m)^2 - 14mn + (n)^2$$
$$= \boxed{(7m-n)^2}$$

(Check: $2(7m)(n) = 14mn$)

4: Factoring a GCF Monomial

Factor $72 - 50y^2$ completely.

Step One: Factor out the GCF of 2. $2(36 - 25y^2)$

Step Two: Factor the remaining polynomial. $\boxed{2(6+5y)(6-5y)}$

Factoring Method: The third method that can be used to solve quadratic equations is the factoring method. Here are the steps for using this method.

1. Write the equation in standard form: $ax^2 + bx + c = 0$
2. Factor the quadratic expression.
3. Use the Zero Product Property. This property states that if the product of two factors is zero, then one or both of the factors must equal zero. Set each factor equal to zero.
4. Solve each corresponding linear equation for the variable.
5. The solutions to a quadratic equation are roots. The roots of an equation are the value of the variable that make the equation true.

Ex 12

<u>Solving a Quadratic Equation by Factoring</u>

Solve the equation $2x^2 - 4x - 8 = -x^2 + x$.

Step One: Write the equation in standard form. $3x^2 - 5x - 8 = 0$

Step Two: Factor the quadratic using the "*ac* method".

$$a \cdot c = -24 \qquad 3x^2 - 8x + 3x - 8 = 0$$
$$b = -5 \qquad x(3x-8) + 1(3x-8) = 0$$
$$-8 \text{ and } 3 \qquad (3x-8)(x+1) = 0$$

Step Three: Set each factor equal to zero and solve.

$$3x - 8 = 0 \qquad x + 1 = 0$$
$$x = \frac{8}{3} \qquad x = -1$$

The solutions can be written in set notation: $\left\{-1, \frac{8}{3}\right\}$

**Note: We can graph quadratic functions by plotting the zeros. The vertex is halfway between the zeros.

Ex 13

Solve the equation $25 = 30y - 9y^2$.

Step One: Write the equation in standard form. $9y^2 - 30y + 25 = 0$

Step Two: Factor the quadratic. It is a perfect square trinomial.

$$(3y)^2 - 30y + (5)^2 \qquad (3y-5)^2 = 0$$

Note: $2(5)(3y) = 30y$

Step Three: Set each factor equal to zero and solve.

$$3y - 5 = 0$$
$$y = \frac{5}{3}$$

The solution can be written in set notation: $\left\{\frac{5}{3}\right\}$

***Note: There is only one solution – one root or zero. This means that the graph only crosses the x-axis in one point. This tells us the vertex is $\left(0, \frac{5}{3}\right)$.

Ex 14

Find the zero(s) of the quadratic function $y = -x^2 - 2x + 3$ and graph the parabola.

Step One: Factor the quadratic polynomial.

$$y = -\left(x^2 + 2x - 3\right)$$
$$y = -\left(x + 3\right)\left(x - 1\right)$$

Step Two: Set each factor equal to 0 and solve.

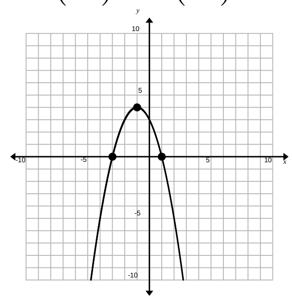
$$x + 3 = 0 \quad x - 1 = 0$$
$$x = -3 \quad x = 1$$

Step Three: Find the coordinates of the vertex.

$$x = \frac{-3 + 1}{2} = \frac{-2}{2} = -1$$

$$y = -\left(-1\right)^2 - 2\left(-1\right) + 3 = -1 + 2 + 3 = 4$$

Step Four: Plot the points and sketch the parabola.

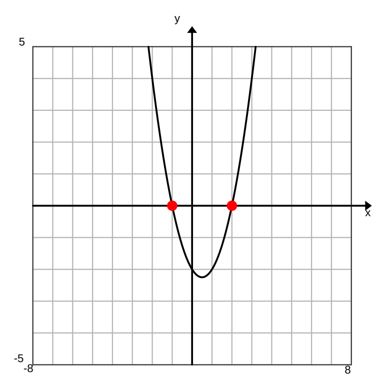


Ex 15

<u>Using Zeros to Write Functions Rules</u>

Write a quadratic function in standard form with zeros 2 and -1.

$x = 2$ or $x = -1$	Write the zeros as solutions for two equations.
$x - 2 = 0$ or $x + 1 = 0$	Rewrite each equation so that it equals 0.
$(x - 2)(x + 1) = 0$	Apply the converse of the Zero Product Property to write a product that equals 0.
$x^2 - x - 2 = 0$	Multiply the binomials.
$f(x) = x^2 - x - 2$	replace 0 with $f(x)$



Check: Graph the function $f(x) = x^2 - x - 2$ on a calculator.
The graph shows the original zeros of 2 and -1.

Application Problem: Any object that is thrown or launched into the air, such as a baseball, basketball, or soccer ball, is a *projectile* on Earth after t seconds is given below.

$$h(t) = -16t^2 + v_o t + h_o$$

Constant due to Earth's gravity in $ft/\sec^2$

Initial vertical velocity in $ft/\sec$ **(at t=0)**

Initial height in ft (at t=0)

Ex 16

A soccer ball is kicked from ground level with an initial vertical velocity of 32 ft/s. After how many seconds will the ball hit the ground?

$h(t) = -16t^2 + v_o t + h_o$

$h(t) = -16t^2 + 32t + 0$

The ball will hit the ground when its height is zero.

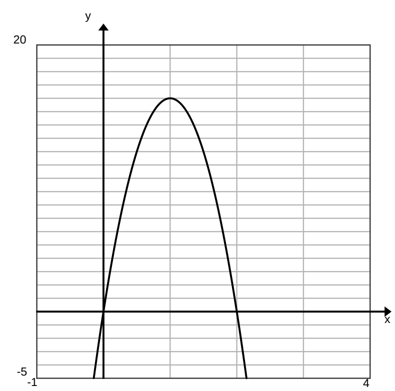
$-16t^2 + 32t = 0$

$-16t(t-2) = 0$
$-16t = 0$ or $(t-2) = 0$
$t = 0$ or $t = 2$

The ball will hit the ground in 2 seconds. Notice that the height is also zero when $t = 0$, the instant that the ball is kicked.

Check with calculator. Graph $h(t) = -16t^2 + 32t$.

Notice zeros at 0 and 2.



SAMPLE EXAM QUESTIONS

1. Factor $9x^2 + 121$.

(A) $(3x+11)(3x-11)$

(B) $(3x+11)(3x+11)$

(C) $(3x+11i)(3x+11i)$

(D) $(3x+11i)(3x-11i)$

Ans: D

2. Solve the equation $6x^2+24x=126$ by factoring.

(A) $x=7$ or $x=-3$

(B) $x=-7$ or $x=3$

(C) $x=-7$

(D) $x=3$

Ans: B

3. Which of the following is a factor of $(a-1)^2-b^2$?

(A) $a+b-1$

(B) $a-b$

(C) $a-1$

(D) $a-b+1$

Ans: A

4. Solve the quadratic $4x^2=14x+8$.

A. $x=-2$ or $x=1$

B. $x=-\frac{1}{2}$ or $x=4$

C. $x=-\frac{1}{7}$ or $x=8$

D. $x=0$ or $x=-\frac{7}{4}$

Ans: B

5. Which is a factor of $4x^2-6x-40$?

A. $2x+5$

B. $2x-5$

C. $2x+4$

D. $2x-4$

Ans: A

6. Which equation has roots of 4 and -6?

A. $(x-4)(x+6)=0$

B. $(x-4)(x-6)=0$

C. $(x+4)(x+6)=0$

D. $(x+4)(x-6)=0$

Ans: A

For questions 7–9, consider the solutions to the equation $(x+5)(x-3)=0$.

7. $x^2-15=0$ **has the same solutions as the given equation.**

A. True

B. False

Ans: B

8. $x^2+2x-15=0$ **has the same solutions as the given equation.**

A. True

B. False

Ans: A

9. $(x+1)^2-14=0$ **has the same solutions as the given equation.**

A. True

B. False

Ans: B

Completing the Square

The Square Root Property can be used to solve an equation of the form $ax^2+bx+c=d$ when ax^2+bx+c is a perfect square trinomial. If the trinomial is not a perfect square, a method called completing the square can be used to rewrite the equation so that the trinomial is a perfect square.

Review: Factoring a Perfect Square Trinomial

$$a^2+2ab+b^2=(a+b)^2$$
$$a^2-2ab+b^2=(a-b)^2$$

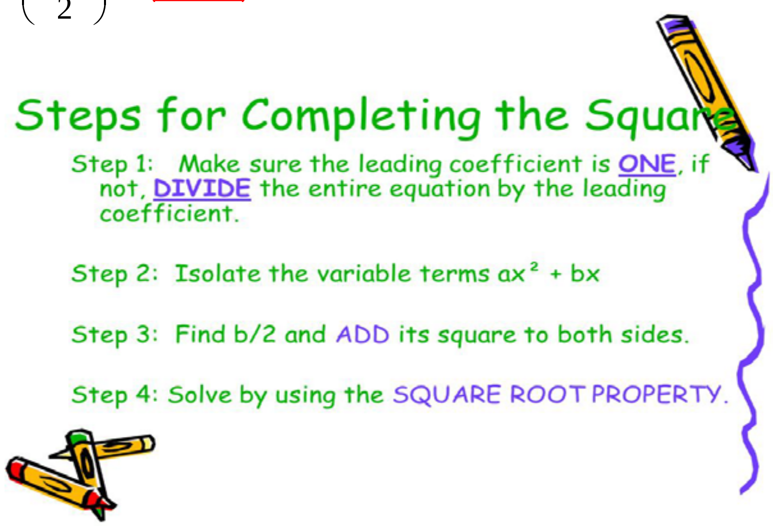
Completing the square is writing an expression of the form $x^2 + bx$ as a perfect square trinomial in order to factor it as a binomial squared. To complete the square of $x^2 + bx$, we must add $\left(\frac{b}{2}\right)^2$. Teacher Note: Algebra Tiles work well to illustrate completing the square.

Ex 17

Find the value of c such that $x^2 - 10x + c$ is a perfect square trinomial.

$b = -10$, therefore we must add $\left(\frac{-10}{2}\right)^2 = \boxed{25 = c}$ to complete the square.

Note: $x^2 - 10x + 25 = (x-5)^2$



Ex 18

Solving a Quadratic Equation by Completing the Square

Solve $2x^2 + 12x - 4 = 0$ by completing the square.

Step One: Rewrite to make the lead coefficient 1.

$$\frac{2x^2}{2} + \frac{12x}{2} - \frac{4}{2} = \frac{0}{2}$$
$$x^2 + 6x - 2 = 0$$

Step Two: Take the constant term to the other side.

$$x^2 + 6x = 2$$

Step Three: Complete the square (add $\left(\frac{b}{2}\right)^2$ to **both** sides).

$$x^2 + 6x + \boxed{\left(\frac{6}{2}\right)^2} = 2 + \boxed{\left(\frac{6}{2}\right)^2}$$
$$x^2 + 6x + 9 = 11$$

Step Four: Factor the perfect square trinomial.

$$(x+3)^2 = 11$$

Step Five: Take the square roots of both sides.

$$\sqrt{(x+3)^2} = \sqrt{11}$$
$$|x+3| = \sqrt{11}$$

Step Six: Solve for the variable.

$$x + 3 = \sqrt{11} \qquad x + 3 = -\sqrt{11}$$
$$x = -3 + \sqrt{11} \qquad x = -3 - \sqrt{11}$$

The solution set is $\boxed{\left\{-3-\sqrt{11}, -3+\sqrt{11}\right\}}$

Ex 19

Solve $3x^2 - 42x + 150 = 0$ by completing the square.

Step One: Rewrite to make the lead coefficient 1. $\frac{3x^2}{3} - \frac{42x}{3} + \frac{150}{3} = \frac{0}{3}$

$x^2 - 14x + 50 = 0$

Step Two: Take the constant term to the other side. $x^2 - 14x = -50$

Step Three: Complete the square (add $\left(\frac{b}{2}\right)^2$ to **both** sides). $x^2 - 14x + \boxed{\left(\frac{-14}{2}\right)^2} = -50 + \boxed{\left(\frac{-14}{2}\right)^2}$

$x^2 - 14x + 49 = -1$

Step Four: Factor the perfect square trinomial. $(x-7)^2 = -1$

Step Five: Take the square roots of both sides. $\sqrt{(x-7)^2} = \sqrt{-1}$

$|x-7| = i$

$x - 7 = i \quad x - 7 = -i$

Step Six: Solve for the variable. $x = 7 + i \quad x = 7 - i$

The solution set is $\boxed{\{7-i, 7+i\}}$

Ex 20

Vertex Form of a Quadratic Function: $y = a(x-h)^2 + k$ Vertex: (h,k)

Write the quadratic function in vertex form and identify the vertex of $y = 2x^2 + 4x - 7$.

Step One: Factor out the lead coefficient from the variable terms (if other than one). $y = 2(x^2 + 2x) - 7$

Step Two: Complete the square.

$$y + 2\boxed{\left(\frac{2}{2}\right)^2} = 2\left(x^2 + 2x + \boxed{\left(\frac{2}{2}\right)^2}\right) - 7$$

Note: We must add $a\boxed{\left(\frac{b}{2}\right)^2}$ to both sides by the distributive property.

$$y + 2 = 2(x^2 + 2x + 1) - 7$$

Step Three: Factor the perfect square trinomial. $y + 2 = 2(x+1)^2 - 7$

Step Four: Solve for y.

$y = 2(x+1)^2 - 7 - 2$

$\boxed{y = 2(x+1)^2 - 9}$ The equation is now in vertex form.

The vertex is $\boxed{(-1,-9)}$.

Ex 21

Write the quadratic function in vertex form and identify the vertex of $y = x^2 - 5x + 2$.

Step One: Factor out the lead coefficient (if other than one). $y = x^2 - 5x + 2$

Step Two: Complete the square.

$$y + \boxed{\left(\frac{-5}{2}\right)^2} = \left(x^2 - 5x + \boxed{\left(\frac{-5}{2}\right)^2}\right) + 2$$

$$y + \frac{25}{4} = \left(x^2 - 5x + \frac{25}{4}\right) + 2$$

Step Three: Factor the perfect square trinomial. $y + \frac{25}{4} = \left(x - \frac{5}{2}\right)^2 + 2$

Step Four: Solve for *y*.

$$y = \left(x - \frac{5}{2}\right)^2 + 2 - \frac{25}{4}$$

$$\boxed{y = \left(x - \frac{5}{2}\right)^2 - \frac{17}{4}}$$

The vertex is $\boxed{\left(\frac{5}{2}, -\frac{17}{4}\right)}$.

<u>You Try</u>:

1. A rectangle has sides x and $x + 10$. The area of the rectangle is 100. Use completing the square to find the value of *x*.

<u>QOD</u>: Why is completing the square helpful when finding the maximum or minimum value of a quadratic function?

SAMPLE EXAM QUESTIONS

1. For what values of c will $3x^2 - 2x + c = 0$ have exactly one distinct real root?

(A) $-\frac{3}{2}$

(B) $-\frac{1}{3}$

(C) $\frac{1}{3}$

(D) $\frac{2}{3}$

Ans: C

2. Which statement best describes these two functions?

$f(x) = x^2 - x + 4$

$g(x) = -3x^2 + 3x + 7$

(A) The maximum of $f(x)$ is less than the minimum of $g(x)$.

(B) The minimum of $f(x)$ is less than the maximum of $g(x)$.

(C) The maximum of $f(x)$ is greater than the minimum of $g(x)$.

(D) The minimum of $f(x)$ is greater than the maximum of $g(x)$.

Ans: B

3. Given $4x^2 + 28x + c = (2x+q)^2$, where *c* and *q* are integers, what is the value of *c*?

A. 2

B. 7

C. 14

D. 49

Ans: D

4. What value of *c* makes the expression $y^2 - 9y + c$ a perfect trinomial square?

A. –9

B. $-\frac{9}{2}$

C. 81

D. $\frac{81}{4}$

Ans: D

5. What number should be added to both sides of the equation to complete the square in $x^2 + 8x = 17$?

A. 4

B. 16

C. 29

D. 49

Ans: D

6. The height of Carl, the human cannonball, is given by $h(t) = -16t^2 + 56t + 40$ where h is in feet and t is in seconds after the launch.

a) What was his height at the launch? When t=0, h(t)=40, so the height at launch is 40 feet.

b) What is his maximum height? The maximum height occurs at the vertex. $f(x) = -16(t-1.75)^2 + 89$ $Max: 89\ feet$

c) How long before he lands in the safety net, 8 feet above the ground? 4 seconds

Quadratic Formula

The Quadratic Formula can be used to solve any equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$. To determine the roots of the equation, substitute the coefficients a and b and the constant c into the quadratic formula and then simplify the resulting expression.

Deriving the Quadratic Formula by Completing the Square on the general form of a quadratic equation:

Solve the quadratic equation $ax^2 + bx + c = 0$ by completing the square.

Step One: Rewrite so that the lead coefficient is 1.

$$\frac{ax^2}{a} + \frac{bx}{a} + \frac{c}{a} = \frac{0}{a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Step Two: Take the constant term to the other side.

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

Step Three: Complete the square (add $\left(\frac{b}{2}\right)^2$ to both sides).

$$x^2 + \frac{b}{a}x + \boxed{\left(\frac{b}{2a}\right)^2} = -\frac{c}{a} + \boxed{\left(\frac{b}{2a}\right)^2}$$

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{-4ac + b^2}{4a^2}$$

Step Four: Factor the perfect square trinomial.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

Step Five: Take the square roots of both sides.

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\left|x + \frac{b}{2a}\right| = \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

Step Six: Solve for the variable.

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a} \qquad x + \frac{b}{2a} = \frac{-\sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

for solving ... $ax^2 + bx + c = 0$

**Note: To help memorize the quadratic formula, sing it to the tune of the song "Pop Goes the Weasel".

Ex 22

Solve the quadratic equation $x^2 - 8x = -1$ using the quadratic formula.

Step One: Rewrite in standard form (if necessary). $x^2 - 8x + 1 = 0$

Step Two: Identify *a*, *b*, and *c*. $a = 1, b = -8, c = 1$

Step Three: Substitute the values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(1)}}{2(1)}$$

Step Four: Simplify.

$$x = \frac{8 \pm \sqrt{64 - 4}}{2} = \frac{8 \pm \sqrt{60}}{2} = \frac{8 \pm 2\sqrt{15}}{2} = 4 \pm \sqrt{15}$$

The solution set is $\boxed{\left\{4 - \sqrt{15}, 4 + \sqrt{15}\right\}}$

Ex 23

Solve the quadratic equation $3x - 7 = 2x^2$ using the quadratic formula.

Step One: Rewrite in standard form (if necessary). $2x^2 - 3x + 7 = 0$

Step Two: Identify *a*, *b*, and *c*. $a = 2, b = -3, c = 7$

Step Three: Substitute the values into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(7)}}{2(2)}$$

Step Four: Simplify.

$$x = \frac{3 \pm \sqrt{9 - 56}}{4} = \frac{3 \pm \sqrt{-47}}{4} = \frac{3 \pm i\sqrt{47}}{4} = \frac{3}{4} \pm \frac{\sqrt{47}}{4}i$$

The solution set is $\boxed{\left\{\frac{3}{4} - \frac{\sqrt{47}}{4}i, \frac{3}{4} + \frac{\sqrt{47}}{4}i\right\}}$

<u>Discriminant</u>: The number under the square root in the quadratic formula. $\boxed{b^2 - 4ac}$

The sign of the discriminant determines the number and type of solutions of a quadratic equation.

- If $b^2 - 4ac > 0$, then the equation has **two real solutions** (two *x*-intercepts).
- If $b^2 - 4ac = 0$, then the equation has **one real solution** (one *x*-intercept).
- If $b^2 - 4ac < 0$, then the equation has **two imaginary solutions** (no *x*-intercept).

A common mistake that students will make is they will find the values for *a*, *b*and *c* before putting the equation in standard form.

Ex 24

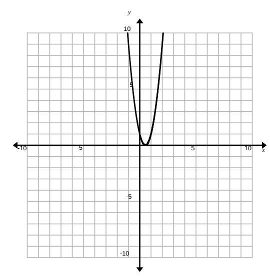
What is the discriminant of the quadratic equation $4x^2 - 4x + 1 = 0$? Give the number and type of solutions the quadratic equation has. Then, graph the quadratic function $y = 4x^2 - 4x + 1$ to verify your answer.

Discriminant: $b^2 - 4ac = (-4)^2 - 4(4)(1) = \boxed{0}$

Since the disciminant is 0, there is <u>one real solution</u>.

The *x*-coordinate of the vertex of the function $y = 4x^2 - 4x + 1$ is $x = \frac{-b}{2a} = \frac{-(-4)}{2(4)} = \frac{1}{2}$.

The *y*-coordinate of the vertex is $y = 4\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 1 = 1 - 2 + 1 = 0$.



Plot a couple of other points to graph the parabola.

Note that the graph has one *x*-intercept (the vertex).

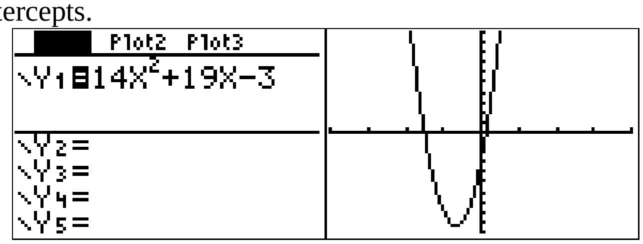
Ex 25

Determine the number and type of solutions the quadratic equation $20n^2 + 6n = 6n^2 - 13n + 3$ has. Then, solve the equation using the quadratic formula.

Put the equation in standard form: $14n^2 + 19n - 3 = 0$. Therefore, $a = 14, b = 19$ and $c = -3$.

Discriminant: $b^2 - 4ac = (19)^2 - 4(14)(-3) = \boxed{\sqrt{529} = 23}$. Since the discriminant is a positive value, there are <u>two real solutions.</u>

Check on the TI-84: The parabola has two x-intercepts.



To summarize: Five methods for solving quadratics have been discussed. When to use each one? Here's a plan of attack:

- If there is no linear term (*bx*), use the square root method or factor (if possible).
- If you have a trinomial, try to factor it first.
- If it is not factorable, then use the quadratic formula or the completing the square method.
- The graphing method can only estimate roots if they are not integers unless technology (TI-84) is handy.

SAMPLE EXAM QUESTIONS

1. **Which of the following quadratic equation has no real roots?**

(A) $2x^2-7x-9=0$

(B) $2x^2=7x$

(C) $2x^2+7x-9=0$

(D) $2x^2-7x+9=0$

Ans: D

2. **Solve the equation by using the quadratic formula.** $2x^2+5x-3=0$

(A) $x=1$ or $x=-6$ (B) $x=\frac{1}{2}$ or $x=-3$

(C) $x=\frac{1}{3}$ or $x=-1$

Ans: B

3. **How many real solutions does the equation $x^2+4=0$ have?**

A. 0 **B.** 1 **C.** 2

Ans: A

4. **What are the real solutions of $-4x^2=5x+9$?**

A. $\left\{-1, -\frac{1}{4}\right\}$ **C.** $\left\{-1, \frac{9}{4}\right\}$

B. $\left\{\frac{-5+\sqrt{119}}{4}, \frac{-5-\sqrt{119}}{4}\right\}$ **D.** There are no real solutions.

Ans: D

5. The graph of $y = x^2 - 3x - 9$ has how many *x*-intercepts?

A. 0 **B.** 1 **C.** 2

Ans: C

6. Which shows the correct use of the quadratic formula to find the solutions of $8x^2 + 2x = 1$?

A. $x = \dfrac{2 \pm \sqrt{(2)^2 - 4(8)(1)}}{2(8)}$

B. $x = \dfrac{2 \pm \sqrt{(2)^2 - 4(8)(-1)}}{2(8)}$

C. $x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(8)(1)}}{2(8)}$

D. $x = \dfrac{-2 \pm \sqrt{(2)^2 - 4(8)(-1)}}{2(8)}$

Ans: D

7. What are the solutions of $3x^2 - 6x = -2$?

A. $x = \dfrac{1 \pm \sqrt{3}}{3}$

B. $x = \dfrac{-1 \pm \sqrt{3}}{3}$

C. $x = 1 \pm \dfrac{\sqrt{3}}{3}$

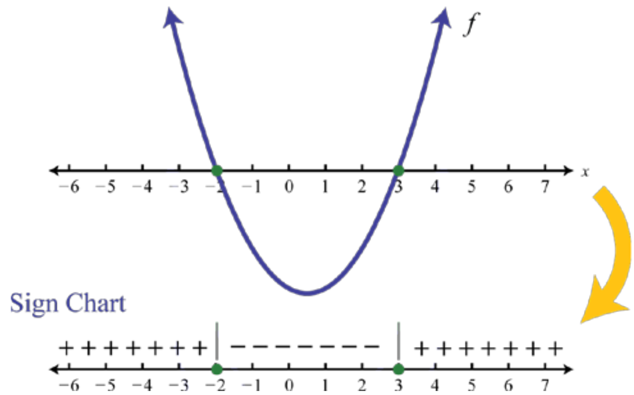
D. $x = -1 \pm \dfrac{\sqrt{3}}{3}$

Ans: C

Solving Quadratic Inequalities

Solving a Quadratic Inequality in One Variable: Solving inequalities is very similar to solving equations. We do most of the same things. When solving equations, we try to find POINTS, such as the ones marked "= 0". But when we solve inequalities, we try to find INTERVAL(S), such as one marked "< 0". So, this is what we do:

- Find the roots – the "= 0" points using the methods already learned. Start with factoring and use quadratic formula if it is not factorable.
- In between the roots are intervals that are either
  - Greater than zero (> 0), or
  - Less than zero (< 0)
- Then, pick a test value to find out which it is



**Note: In this diagram, the roots were -2 and 3. Using a number line, pick values in each interval to determine whether the function is positive or negative. The corresponding graph is shown above.

Ex 26

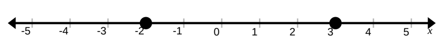
Solve $x^2 - 5x + 6 \ge 0$.

Step One: Solve the quadratic equation $x^2 - x - 6 = 0$ using any method.

We will use factoring.

$$(x-3)(x+2)=0$$
$$x-3=0 \quad x+2=0$$
$$x=3 \qquad x=-2$$

Step Two: Draw a sign chart on a number line to test which values for x satisfy the inequality.



Choose an x-value to the left of -2 and substitute into the inequality. We will try -4.

$$(-4-3)(-4+2)\ge 0$$
$$(-7)(-2)\ge 0 \text{ true}$$

Choose an x-value between -2 and 3 and substitute into the inequality. We will try 0.

$$(0-3)(0+2)\ge 0$$
$$(-3)(2)\ge 0 \text{ false}$$

Choose an x-value to the right of 3 and substitute into the inequality. We will try 4.

$$(4-3)(4+2)\ge 0$$
$$(1)(6)\ge 0 \text{ true}$$

Step Three: Write the solution as a compound inequality or in set notation.

$x \le -2$ or $x \ge 3$ $\qquad$ $(-\infty,-2]\cup[3,\infty)$

Ex 27

Solve $2x^2 + x - 4 < 0$.

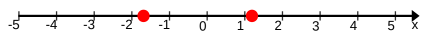
Step One: Solve the quadratic equation $2x^2 + x - 4 = 0$ using any method.

$$a = 2, b = 1, c = -4$$

We will use the quadratic formula. $x = \dfrac{-1 \pm \sqrt{1^2 - 4(2)(-4)}}{2(2)} \rightarrow x = \dfrac{-1 \pm \sqrt{33}}{4}$

$$x \approx -1.69 \text{ or } x \approx 1.19$$

Step Two: Draw a sign chart on a number line to test which values for *x* satisfy the inequality.



Choose an *x*-value to the left of -1.69 and substitute into the inequality. We will try -2.
$2(-2)^2 + (-2) - 4 < 0$
$2 < 0$ false

Choose an *x*-value between -1.69 and 1.19, substitute into the inequality. We will try 0.
$2(0)^2 + 0 - 4 < 0$
$-4 < 0$ true

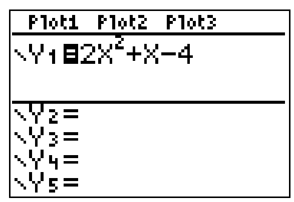
Choose an *x*-value to the right of 1.19 and substitute into the inequality. We will try 2.
$2(2)^2 + 2 - 4 < 0$
$6 < 0$ false

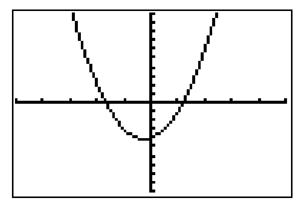
Step Three: Write the solution as a compound inequality or in set notation.

$\boxed{-1.69 < x < 1.19}$ $\boxed{(-1.69, 1.19)}$

<u>Check:</u> Using the TI-84:

The function is less than zero when the parabola is below the x-axis. As noted above, this occurs between the two zeros, -1.69 and 1.19.





Ex 28

Solve $x^2 - x + 1 < 0$.

Step One: Solve the quadratic equation $x^2 - x + 1 < 0$ using any method.

A quick check of the discriminant tells us that there are no real solutions or no x-intercepts.

$$b^2 - 4ac = (-1)^2 - 4(1)(1) = -3$$

There are no roots to put on the sign chart. But this makes things easier! Because the line does not cross the x-axis, the function must be either:

- Always > 0, or
- Always < 0

So, all we have to do is test one value (say x = 0) to see if the parabola is above or below the x-axis.

$(0)^2 - 0 + 1 = 1$
$1 > 0$

This means the function is always greater than zero. Since we are trying to find when it is less than zero, this never occurs and our answer is: $\boxed{\text{No Solution}}$

If we were trying to solve: $x^2 - x + 1 > 0$, then our solution would be: $\boxed{\text{All Real Numbers}}$

Ex 29

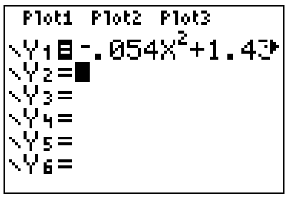
Application with Technology:

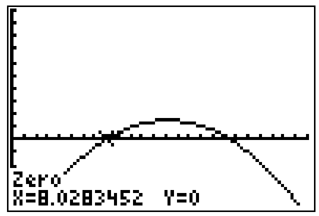
The path of a soccer ball kicked from the ground can be modeled by $y = -0.0540x^2 + 1.43x$, where x is the horizontal distance (in feet) from where the ball was kicked and y is the corresponding height (in feet).

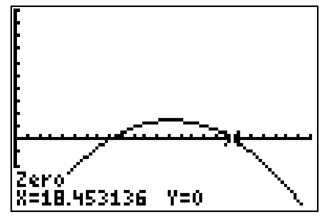
a) A soccer goal is 8 feet high. Write and solve an inequality to find at what values of x the ball is low enough to go into the goal.

$$y = -0.0540x^2 + 1.43x - 8 < 0$$

Use the TI-84 to find the roots:







From the graph, the function is below the x-axis: $x \le 8.02$ feet and $x \ge 18.4$ feet

b) A soccer player kicks the ball toward the goal from a distance of 15 feet away. No one is blocking the goal. Will the player score a goal? Explain your reasoning.

No, since 15 feet is not in our interval above, the ball will go over the goal. To find out how far above the goal, use the Table feature.

When x = 15, y = 1.3, so the ball will go over the goal by 1.3 feet.

X	Y1
12	1.384
13	1.464
14	1.436
15	1.3
16	1.056
17	.704
18	.244

Y1=1.3

SAMPLE EXAM QUESTIONS

1. What is the solution set of $y^2 - 2y \le 3y + 14$?

(A) $y \le 7$

(B) $y \le -2$ or $y \ge 7$

(C) $-7 \le y \le 2$

(D) $-2 \le y \le 7$

Ans: D

2. Solve the inequality $x^2 - 14x + 45 \le -3$.

(A) $x \le 5$ or $x \ge 9$

(B) $x \le 6$ or $x \ge 8$

(C) $6 \le x \le 8$

(D) $5 \le x \le 9$

Ans: C

3. Solve the inequality $8x^2 - 14x + 4 > -11$.

(A) $-2 < x < 0.25$

(B) $-2.5 > x > 0.75$

(C) $-2 > x > 0.25$

(D) $-2.5 < x < 0.75$

Ans: D

4. Solve the inequality $3x^2 - x > 4$.

(A) $x < -1$ or $x > \frac{4}{3}$

(B) $-1 < x < \frac{4}{3}$

(C) $x < -\frac{4}{3}$ or $x > 1$

(D) $1 < x < \frac{4}{3}$

Ans: A