OBJECTIVE #: G.CO.4

OBJECTIVE
Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular line, parallel lines, and line segment.

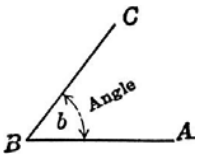
BIG IDEA (Why is this included in the curriculum?)
- All two dimensional geometric figure can be created by transformations. Congruency and similarity may be proven by one or more transformation(s) on the pre-image.

PREVIOUS KNOWLEDGE (What skills do they need to have to succeed?)
- The student must have a thorough knowledge of all types of angles.
- The student must also understand a complete rotation is 360°, as it relates to circles.
- The student must understand the properties of parallel and perpendicular lines.
- The student must know how to find the length of a line segment using the distance formula.
- The student must know how to find the slope of a line.

VOCABULARY USED IN THIS OBJECTIVE (What terms will be essential to understand?)

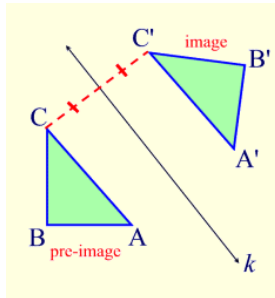
PREVIOUS VOCABULARY (Terms used but defined earlier)
- Angle of Rotation: The angle formed when rays are drawn from the center of rotation to a point and to its image.
- Center of Rotation: A fixed point around which a figure is rotated.
- Circle: The set of all points in a plane that are equidistant from a given point, called the center.
- Initial Point: The starting point of a ray or vector.
- Line Segment: A portion of a line that consists of two endpoints and all points in between the two endpoints.
- Negative Rotation: A clockwise rotation.
- Parallel Lines: Two lines that are coplanar and do not intersect.
- Perpendicular: Two lines/segments/rays that intersect to form right angles.
- Positive Rotation: A counterclockwise rotation.
- Reflection: A rigid transformation in which the image is a mirror image of the pre-image, thus ensuring the pre-image and the image are equidistant from the line of reflection.
- Rotation: A rigid transformation that turns a figure about a fixed point, thus ensuring the pre-image and image are congruent.
- Translation: A rigid transformation that slides an object a fixed distance in a given direction, thus ensuring the pre-image and image are congruent.

NEW VOCABULARY (New Terms and definitions introduced in this objective)
- Angle: A geometric figure formed by rotating a ray about its initial point. [G.CO.7, G.CO.8, G.CO.9]



Notation: $\angle B$, $\angle ABC$, $\angle CBA$

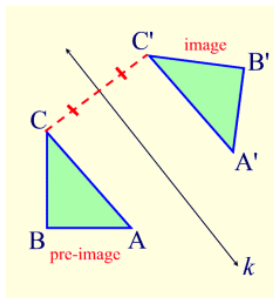
- Image: The new figure that results from any transformation of a figure in the plane. [G.CO.5, G.CO.6]



Notation: $\Delta A'B'C'$

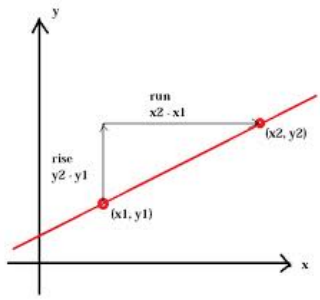
- Orientation: The arrangement of points, relative to one another, after a transformation has occurred.

- Pre-Image: The original figure in the transformation of a figure in the plane. [G.CO.5, G.CO.6]



Notation: ΔABC

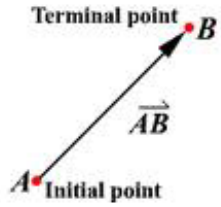
- Slope of a Line: The steepness of a line, which is represented by *m*.



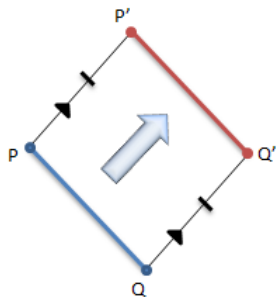
Notation: m

Formula: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

- Terminal Point: The ending point of a vector.



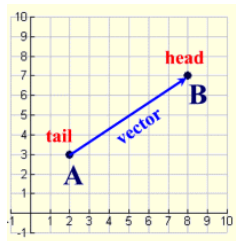
- Translation: A type of transformation that maps every two points P and Q in the plane to points P' and Q', so that the following two properties are true. (1) $PP' = QQ'$. (2) $\overline{PP'} \parallel \overline{QQ'}$ or $\overline{PP'}$ and $\overline{QQ'}$ are collinear.



Notation: $T_{h,\,k}(x, y) = (x + h,\ y + k)$

Vector Notation: $T_{\langle h,\,k\rangle} = (x + h,\ y + k)$

- Vector: A quantity that has both direction and magnitude, and is represented by an arrow drawn between two points.



Notation: $\overrightarrow{AB}$

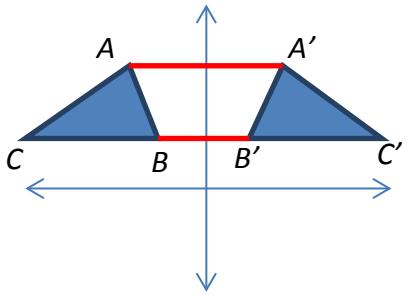
SKILLS (What will they be able to do after this objective?)

- Students will be able to develop and utilize the definitions of rotations, reflections, and translations
- Students will be able to describe translations and rotations in terms of reflections
- Students will be able to determine if the orientation of the pre-image is maintained after the transformation.
- Students will be able to identify and perform positive and negative rotations on a given pre-image.

SHORT NOTES (A short summary of notes so that a teacher can get the basics of what is expected.)

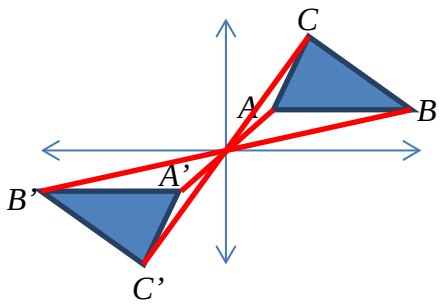
Transformation	Distance between pre-image and image	Orientation of pre-image and image
Reflection	The distances are different	The orientation changes
Rotation	The distances are different	The orientation stays the same
Translation	The distances are the same	The orientation stays the same

Reflections:



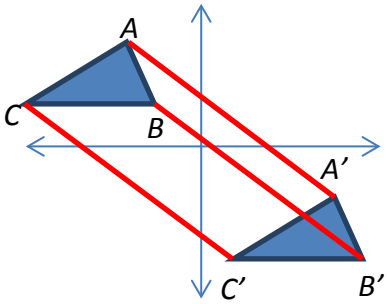
- Isometric – Within the shape distances, angle measures, parallelism, collinearity are all preserved.
- Orientation is reversed.

Rotations:



- Isometric – Within the shape distances, angle measures, parallelism, collinearity are all preserved.
- Orientation is preserved.

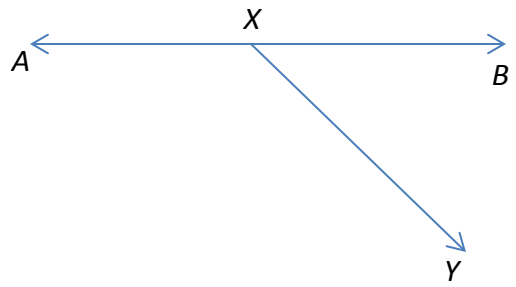
Translations:



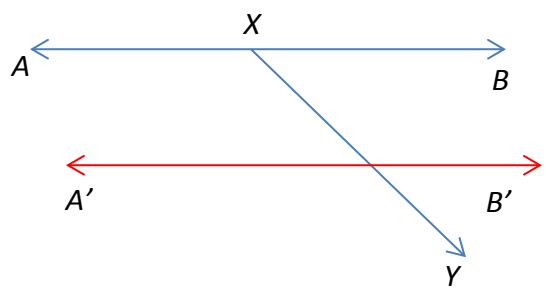
- Isometric – Within the shape distances, angle measures, parallelism, collinearity are all preserved.
- Orientation is preserved.

- In order to help student understand the rules for different transformations, it is useful to provide each student with a coordinate grid. Have students pick points on the coordinate plane to create a polygon. Allow students to work in groups to determine the rules for the following transformations
  - Reflection over the *y*-axis $R_{x-axis}(x,\ y) = (__, __)$ $(x, -y)$
  - Reflection over the *x*-axis $R_{y-axis}(x,\ y) = (__, __)$ $(-x, y)$
  - Rotation by 90° about the origin $R_{O,90°}(x,\ y) = (__, __)$ $(-y, x)$
  - Rotation by 180° about the origin $R_{O,180°}(x,\ y) = (__, __)$ $(-x, -y)$
  - Rotation by 270° (-90°) about the origin $R_{O,270°}(x,\ y) = (__, __)$ $(y, -x)$
  - Reflection over a vertical line $x = c$ $R_{x=c}(x,\ y) = (__, __)$ $(-x + 2c, y)$
  - Reflection over a horizontal line $y = b$ $R_{y=b}(x,\ y) = (__, __)$ $(x, -y + 2b)$
  - Reflection over the $y = x$ line $R_{y=x}(x,\ y) = (__, __)$ (y, x)
  - Reflection over the $y = -x$ line $R_{y=-x}(x,\ y) = (__, __)$ $(-y, -x)$

- Be sure students understand when a translation is preformed along a vector, it creates parallel lines.
  - $\overleftrightarrow{AB}$ is translated along $\overrightarrow{XY}$



  - This results in $\overleftrightarrow{AB}$ and $\overleftrightarrow{A'B'}$ being parallel lines.



MISCONCEPTIONS (What are the typical errors or difficult areas? Also suggest ways to teach them.)

- Rotations are counterclockwise – relate this to the numbering of the 4 quadrants.
- Students often confuse reflections over the *x*-axis. Make sure students realize a reflection across the *x*-axis is a vertical movement.
- Students confuse the equations of vertical and horizontal lines. For instance, $y = 3$ would be parallel to the *x*-axis, not the *y*-axis.

FUTURE CONNECTIONS (What will they use these skills for later?)

- These transformations will be used throughout the year to help build understanding of similarity and congruency.

ADDITIONAL EXTENSIONS OR EXPLANATIONS (What needs greater explanation?)

- Lessons should stress the correct notation for the different transformations.
- Circle: A geometric figure constructed by rotating a point about a given center across 360°.
- Parallel Lines: Lines that are formed by translating a line in a plane.
- Perpendicular Lines: Lines that are formed by rotating a line 90°.

ASSESSMENT ITEMS (What questions would evaluate these skills?)

1) A double reflection over parallel lines can be described as a single transformation. What is the transformation? Translation

2) A double reflection over intersecting lines can be described as a single transformation. What is the transformation? Rotation

3) If $C(3, 5)$ was reflected to $C'(-3, 5)$, which axis was used? *y*-axis

4) If $B(2, -1)$ was reflected by a line to $B'(-1, 2)$, which line was used? $y = x$

5) If $B(5, 3)$ was reflected by a line to $B'(1, 3)$, which line was used? $x = 3$

6) If $B(2, -1)$ was reflected by a line to $B'(2, 5)$, which line was used? $y = 2$

7) $R_{(O,90°)}\ (-1,\ -5) =$ $(5, -1)$

8) $R_{(O,180°)}\ (__, __) = (5, -4) =$ $(-5, 4)$

9) $R_{(y=x)}\ (__, __) = (2,\ -1) =$ $(-1, 2)$

10) $R_{(O,270°)}\ (-3,\ 1) =$ $(1, 3)$

11) $R_{(y=2)}\ (2,\ 3) =$ $(2, 1)$

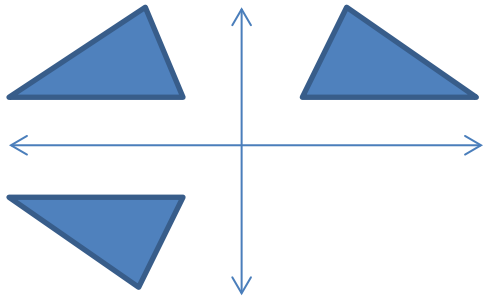
12) $R_{(y-axis)}\ (-6,\ -3) =$ $(6, -3)$

13) You start at the point (5, 2). Follow the given transformations to find the coordinates of the final point. Your pre-image for each step is the answer from the previous problem.

a. Rotate 270° about the origin (2, -5)
b. Reflect over the *x*-axis (2, 5)
c. Reflect over the *y*-axis (-2, 5)
d. Translate $(x, y) \rightarrow (x+1,\ y-2)$ (-1, 3)
e. Reflect over y=x (3, -1)

14) Jack said that a rotation about the origin of 180° $(R_{(O,180°)})$ was the same as a reflection over the *y*-axis (R_{y-axis}) and then a reflection over the *x*-axis (R_{x-axis}). Do you agree or disagree? Draw a sketch to support your answer.

Agree - Sketches may vary.



<u>From CCSD Geometry Honors Practice Semester 1 Exam 2012 – 2013</u>

1. Which of these is equivalent to a translation?

(A) a reflection across one line
(B) a composition of two reflections across intersecting lines
(C) a composition of two reflections across parallel lines