OBJECTIVE #:   G.CO.3

OBJECTIVE
Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.

BIG IDEA (Why is this included in the curriculum?)
- The study of symmetries (rotational and reflection) relates directly to the transformations previously studied. In addition, through understanding symmetries and transformations, students will gain a more thorough understanding of the unique characteristics of rectangles, parallelograms, trapezoids and regular polygons.

PREVIOUS KNOWLEDGE (What skills do they need to have to succeed?)
- The student must have a basic understanding of quadrilaterals and their basic properties.

VOCABULARY USED IN THIS OBJECTIVE (What terms will be essential to understand?)
    PREVIOUS VOCABULARY (Terms used but defined earlier)
- Parallelogram: A quadrilateral with both pairs of opposite sides parallel.
  - Rectangle: A parallelogram with four right angles.
- Reflection: A rigid transformation in which the image is a mirror image of the pre-image, thus ensuring the pre-image and the image are equidistant from the line of reflection.
- Regular Polygon: A convex polygon with all angles congruent and all sides congruent.
- Rotation: A rigid transformation that turns a figure about a fixed point, thus ensuring the pre-image and image are congruent.
- Translation: A rigid transformation that slides an object a fixed distance in a given direction, thus ensuring the pre-image and image are congruent.
- Trapezoid: A quadrilateral with exactly one pair of parallel sides.

    NEW VOCABULARY (New Terms and definitions introduced in this objective)
- Angle of Rotation: The angle formed when rays are drawn from the center of rotation to a point and to its image. [G.CO.4, G.CO.5]
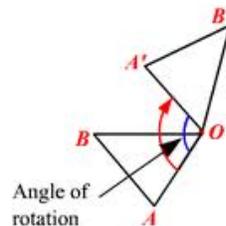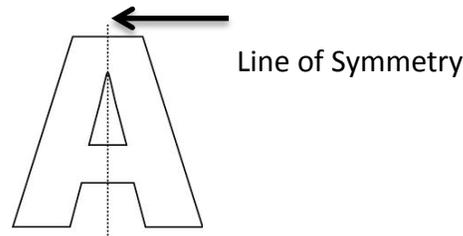

- Center of Rotation: A fixed point around which a figure is rotated. [G.CO.4, G.CO.5]
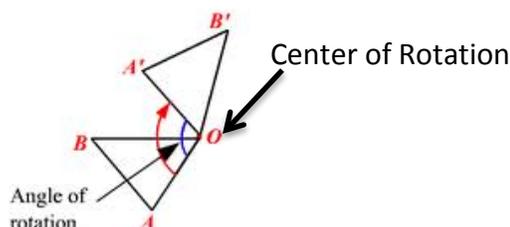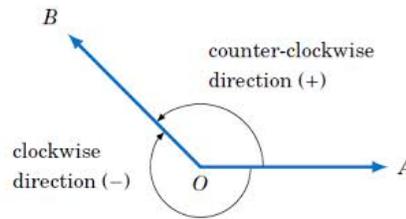
- Line of Symmetry: Line that divides a figure into two congruent parts, each of which is the mirror image of the other.
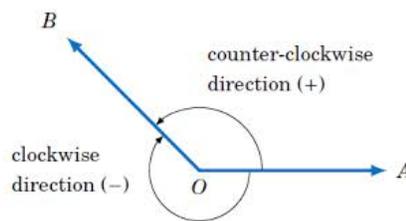
Line of Symmetry

- Negative Rotation: A clockwise rotation.

- Order: The number of positions in which the object looks exactly the same.
    - Order 1 implies no true rotational symmetry since a full $360°$ rotation is needed.

- Point Symmetry: Type of symmetry in which every part of the pre-image has a matching part of the image the same distance from the center point, but in the opposite direction.
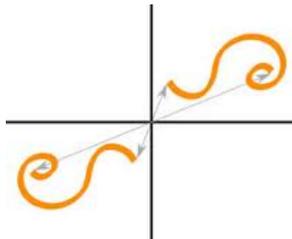
- Positive Rotation: A counterclockwise rotation.

- Reflection Symmetry: Type of symmetry in which one half of an object is a mirror image of the other half.
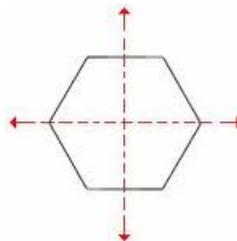
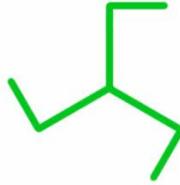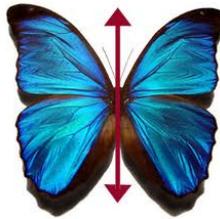- Rotational Symmetry: Type of symmetry in which a figure is an image of itself under a rotation about a point through an angle whose measure is strictly between 0 and 360 degrees.

- Symmetry: Property when a figure can be mapped onto itself through a rigid transformation.

SKILLS (What will they be able to do after this objective?)
- The student will be able to describe the symmetries (rotational and reflection) of a rectangle, parallelogram, trapezoid, and regular polygon onto itself through a thorough understanding of transformations. Students will also be able to identify the unique characteristics of a rectangle, parallelogram, trapezoid and regular polygon.

SHORT NOTES (A short summary of notes so that a teacher can get the basics of what is expected.)
When a figure is reflected, rotated, or translated, the resulting figure is congruent to the original. If a point not on a line is reflected in the line, then the line is the perpendicular bisector of the segment joining the point and its image. These facts are not new, but will now be used to solidify relationships later.
Reflectional symmetry is when a figure in the plane is mapped onto itself by a line of reflection.
Rotational symmetry is when a figure in the plane is mapped onto itself by a rotation between 0° and 360°.
Review characteristics of quadrilaterals and polygons. Include in the discussion the similarities and differences. Teaching these through transformations can cement the characteristics of these figures in a way that memorizing vocabulary does not. The students will then be able to recreate these properties through transformation later on in objective G.CO.11.
The rotations and reflections that carry a figure onto itself determine the kinds of symmetry, if any, that the figure has. Reflections determine line symmetry, and rotations determine rotational symmetry.

Example: Line Symmetry and Rotational Symmetry

Rectangle:	**Parallelogram:**	**Isosceles Trapezoid:**	**Regular Pentagon:**
2 lines of symmetry	no lines of symmetry	1 line of symmetry	1 line of symmetry
180° rotational symmetry	180° rotational symmetry	no rotational symmetry	72°, 144°, 216°, 288° rotational symmetry

Also discuss direction when rotating: clockwise and counterclockwise. Review the 4 quadrants and how the angle of rotation moves counterclockwise to create positive angle values. Remind them that 0° in the coordinate plane refers to the horizontal line from the origin to the right.
When checking for symmetry on lines other than vertical or horizontal, suggest that the students use patty paper to check for symmetry. A rectangle may appear to have 4 lines of symmetry at first glance but if patty paper is used they will be able to determine that there is only symmetry horizontally and vertically.

MISCONCEPTIONS (What are the typical errors or difficult areas? Also suggest ways to teach them.)
You can model it on the board. Draw a quadrilateral and a line of reflection and ask your students to predict the relationship of the image and pre-image to the line of reflection. After working with patty paper or transparencies assess their predications.

The students will get confused between clockwise and counterclockwise rotations because the positive angle actually moves counterclockwise while they tend to assume that positive would relate to a clockwise rotation. The language of clockwise and counterclockwise leads them to associate clockwise as a positive angle and counterclockwise as a negative angle. This is backwards. Refer to the numbering of the quadrants on the coordinate plane.

FUTURE CONNECTIONS (What will they use these skills for later?)
- Students will continue to define geometric concepts and develop proofs of theorems/concepts using transformations.

ADDITIONAL EXTENSIONS OR EXPLANATIONS (What needs greater explanation?)
- Parallelogram: A quadrilateral with both pairs of opposite sides parallel that can be mapped onto itself using a 180 degree rotation.
- Rectangle: A quadrilateral with four right angles that can be mapped onto itself using a 180 degree rotation. A rectangle displays reflection symmetry if the line of symmetry contains the midpoints of either pair of opposite sides.
- Regular Polygon: A polygon with all equal sides and all equal angles that can be mapped onto itself based on the degree of its central angle and multiples thereof, between 0 and 360 degrees. Regular polygons also display reflection symmetry; if a regular polygon has an odd number of sides, then the lines of symmetry connect the vertex to the midpoint of the opposite side; if a regular polygon has an even number of sides, then ½ of the lines of symmetry connect two opposite vertices and ½ of the lines of symmetry connect the midpoints of two opposite sides.
- Trapezoid: A quadrilateral with exactly one pair of parallel sides that can be mapped onto itself using a 360 degree rotation, therefore not displaying rotational symmetry. An isosceles trapezoid would display reflection symmetry if the line of symmetry contained the midpoints of the bases.

ASSESSMENT ITEMS (What questions would evaluate these skills?)

Multiple Choice
1. How many lines of symmetry does the infield of a baseball field have? (ignoring the pitcher's mound)
   A. 0
   B. 1
   C. 2
   D. 3
   E. 4

Short Answer
2. What is the minimum number of lines of symmetry that an octagon can have?   zero
   What is the maximum number? 8
3. Find 4 capital letters that have both vertical and horizontal symmetry? H, I, O, X
4. Write a word that has rotational symmetry. Many possibilities - SOS, SIS

Multi-Step Problems

5. Explain what happens to the orientation of a quadrilateral when it is reflected on a line? The orientation changes. For example, if the vertices of ▱ *ABCD* are labeled in a counterclockwise direction, the vertices of its image ▱ *A'B'C'D'* will be labeled in a clockwise position.

6. Draw a parallelogram in the plane.   Sample drawings
   Does it have rotational symmetry? Yes
   If so, describe the rotations that map the figure onto itself. It can be mapped onto itself by clock/counterclockwise rotation of 180°

From CCSD Geometry Honors Practice Semester 1 Exam 2012 – 2013

For questions 1–3, use the diagram showing parallelogram *ABCD*.
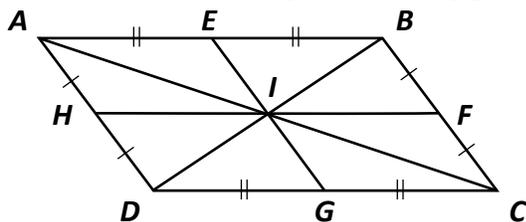
1. A reflection across $\overleftrightarrow{EG}$ carries parallelogram *ABCD* onto itself.
   (A) True   (B) False

2. A rotation of 90° about *I* carries parallelogram *ABCD* onto itself.
   (A) True   (B) False

3. A rotation of 180° about *I* carries parallelogram *ABCD* onto itself.
   (A) True   (B) False

4. A regular polygon with *n* sides is carried onto itself by a positive rotation about its center that is a multiple of 60°, but less than 360°. Which could NOT be the value of *n*?
   (A) 3
   (B) 4
   (C) 5
   (D) 6

5. Use the Venn diagram.
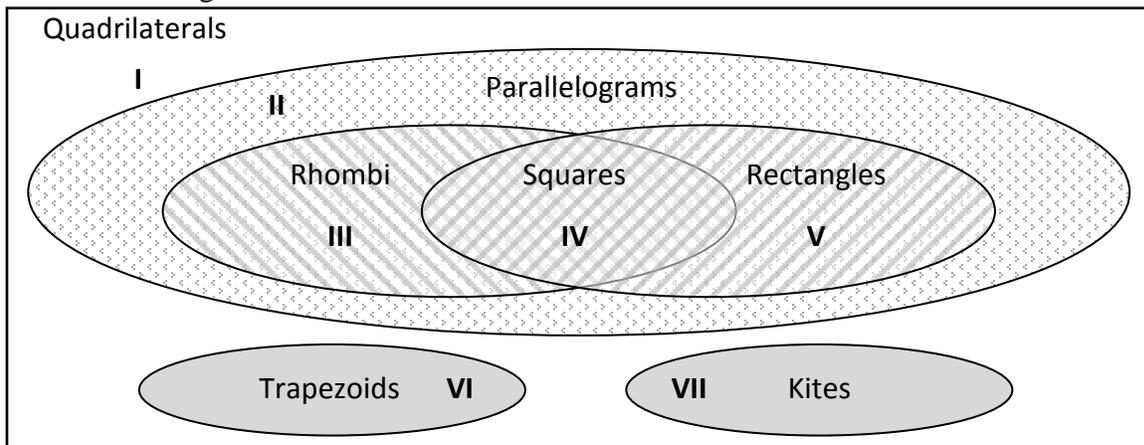
   A quadrilateral *ABCD* has 4 lines of symmetry. Identify the area of the diagram in which *ABCD* resides.
   (A) III
   (B) IV
   (C) V
   (D) VII