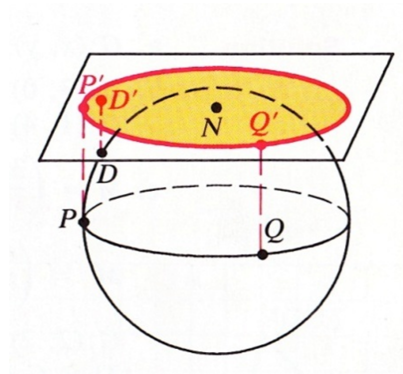
MAPPING, FUNCTIONS AND TRANSFORMATIONS



Have you ever wondered how maps of the round Earth can be made on flat paper? The diagram illustrates the idea behind a polar map of the southern hemisphere. A plane is placed tangent to a globe of the Earth at its North Pole, *N*. Every point *P* of the globe is *projected* on to the paper at exactly one point, called *P'* (said *P* prime). *P'* is called the **image** of *P*, and *P* is called the **pre-image** of *P'*. Because the North Pole projects to itself, then *N* = *N'*.

This correspondence between points of the globe's northern hemisphere and points in the plane is an example of a **mapping**. If we call this mapping *M*, then we could indicate that *M* maps *P* to *P'* by writing *M*: *P* → *P'*. Notice it is also true that *M*: *Q* → *Q'*

The word **mapping** is used in geometry as the word **function** is used in algebra. While a mapping is a correspondence between sets of points, a function is a correspondence between sets of numbers. Each number in the first set corresponds to exactly one number in the second set. For example, the squaring function maps each real number *x* to its square x^2. We can write $f: x \rightarrow x^2$. Another way to indicate that the value of the function at *x* is x^2 is to write it in its typical function notation, $f(x) = x^2$ (read *f* of *x* equals x^2). Similarly, for the mapping *M*, above, we can write *M*(*P*) = *P'* to indicate that the image of *P* is *P'*.

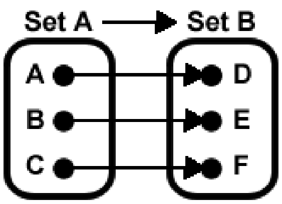
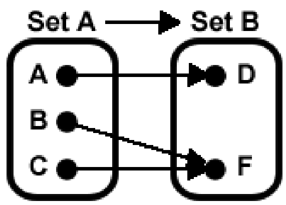
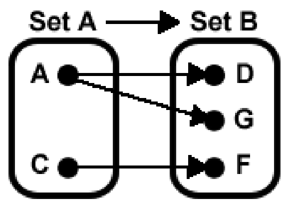
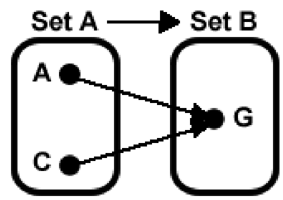
<table>
<tr><th>MAPPING NOTATION (Geometry)</th><th>FUNCTION NOTATION (Algebra)</th></tr>
<tr><td>M: P → P'</td><td>$f(x) = x^2$</td></tr>
<tr><td>The mapping that takes P to P'.</td><td>The function that takes x to x^2</td></tr>
<tr><td></td><td></td></tr>
<tr><td colspan="2">It should not surprise you that mathematicians
often use function and mapping interchangeably.

They both take a have a correspondence between an INPUT VALUE and an OUTPUT VALUE.</td></tr>
</table>

MAPPING NOTATION (Geometry)	FUNCTION NOTATION (Algebra)
$M: (x, y) \rightarrow (x + 3, y - 2)$ $M(x, y) = (x + 3, y - 2)$	$f(x) = \|x\|$
This mapping that moves all points three right and two down.	The function takes the absolute value of each *x* value.
M: (1, 5) → (4, 3)	$f(-4) = \|-4\| = 4$
P(1, 5) and *P'* (4, 3)	$x = -4$ and $f(x) = 4$
Pre-Image and Image	Input and Output

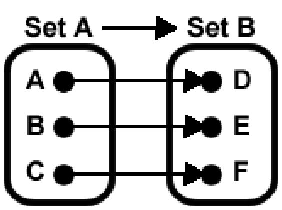
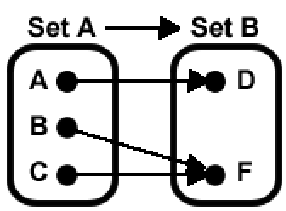
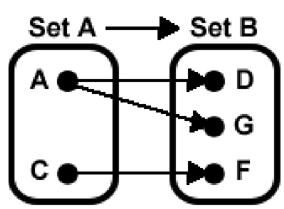
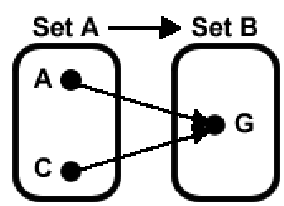
Mappings and Functions

A correspondence between two sets *A* and *B* is a **mapping** of *A* to *B* <u>IF AND ONLY IF</u> each member of *A* corresponds to one and only one member of *B* (This is also the definition for functions).

			
Mapping/Function	**Mapping/Function**	**<u>NOT</u>**	**Mapping/Function**
Each value in set *A* has exactly one value in set *B*.	Each value in set *A* has exactly one value in set *B*.	The *A* value has TWO values in set *B*.	Each value in set *A* has exactly one value in set *B*.
Example	**Example**	**Example**	**Example**
$f(x) = x + 3$	$f(x) = x^2$	$f(x) = \pm x$	$f(x) = x^4 + 1$
$f(2) = 2 + 3 = 5$ (2, 5)	$f(3) = 3^2 = 9$ (3, **9**) $f(-3) = (-3)^2 = 9$ (-3, **9**)	$f(5) = \pm 5$ (**5**, 5) & (**5**, -5)	$f(2) = 2^4 + 1$ (2, 17) $f(-2) = (-2)^4 + 1$ (-2, 17)

Transformations and One to One Correspondence Functions

A mapping is a **transformation** <u>if and only if</u> it is a one to one mapping of the plane onto itself.

			
TRANSFORMATION	**<u>NOT</u>**	**<u>NOT</u>**	**<u>NOT</u>**
One to One Correspondence.	F has two pre-images.	A has two images.	G has two pre-images.

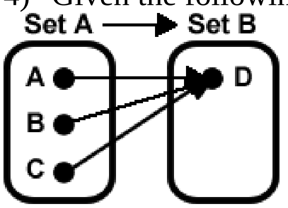
<u>A transformation is a one to one correspondence between points of the plane such that every point in the plane is the image of a point of the plane and no two points have the same image.</u>

<u>QUESTIONS</u>

1) *A* = {…, -3, -1, 0, 1, 2, 3, …}, all the integers, and *B* = = {0, 1, 4, 9, 16, …}, all the perfect squares. Let *C* be a correspondence between each integer and its square.
   a) Find *C*(-5), *C*(3), *C*(0) and *C*(5). b) Is *C* a mapping? Explain.
   c) Find the pre-image(s) of 0; and of 16. d) Is *C* a one to one mapping? Explain.
2) Suppose *T*(*x*, *y*) = (*x*, *y* – 2) is a transformation. Given *A*(-1, 5), *B*(2, 2) and *C*(0, 9).
   a) Find the images of *A*, *B* & *C*. b) Find *E* the pre-image of *E'*(3, 4).

3) Suppose $T(x, y) = (2x, 3y)$ is a transformation. Given $A(-1, 5)$, $B(2, 2)$ and $C(0, 9)$.
   a) Find the images of A, B & C. b) Find E the pre-image of $E'(4, 12)$.
   c) Find F the pre-image of $F'(-12, 6)$. d) Find G the pre-image of $G'(5, 10)$.

4) Given the following correspondences, determine the following.

#1


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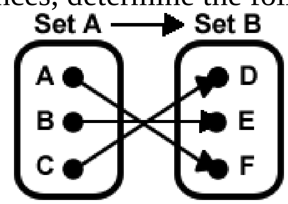

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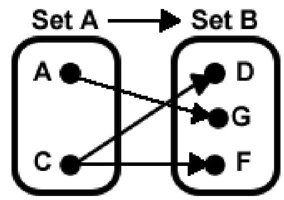

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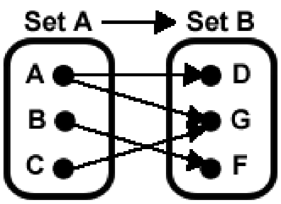

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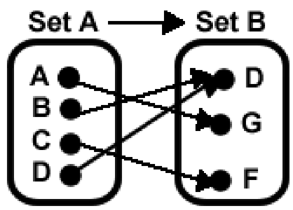

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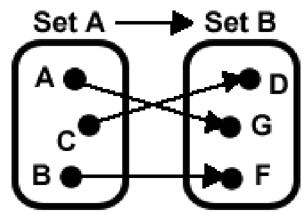


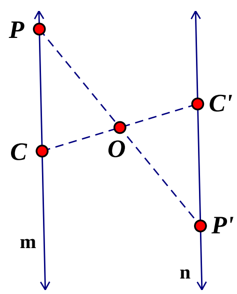
   a) Which of the provided correspondences are mappings (functions)?
   b) Which of the provided correspondences are transformations (one to one functions)?

5) a) If $f(x) = |x|$, find $f(-3)$, $f(4)$, $f(3)$ and $f(8)$. b) Is f a one to one function? Explain.

6) a) If mapping M: $(x, y) \rightarrow (2x, y + 3)$, find the images of $P(-1, 4)$ and $H(3, 5)$.
   b) Is M a transformation? Explain.

7) a) If $g(x) = 2x - 1$, find $g(8)$ and $g(-8)$. b) Find the image of 5.
   c) Find the pre-image of -7. d) Find the pre-image of 18.

8) O is a point equidistant from parallel lines m and n. A mapping M maps each point P of line m to the point P' where $\overrightarrow{PO}$ intersects line n.



Is the mapping a one to one mapping from line m to line n?