Complex Numbers

Math Background

Previously, you

- Studied the real number system and its sets of numbers
- Applied the commutative, associative and distributive properties to real numbers
- Used the order of operations to simplify expressions
- Multiplied binomial expressions

In this unit you will

- Identify complex numbers.
- Perform arithmetic operations with complex numbers.

You can use the skills in this unit to

- Differentiate between a real number, an imaginary number and a complex number.
- Write numbers in standard complex form.
- Apply the properties of operations to add and subtract complex numbers.
- Apply the distributive property and multiply complex numbers.

Vocabulary

- **Associative property** – Denoting an operation where the outcome is independent of the grouping of the symbols and the numbers.
- **Commutative property** – Denoting an operation that is independent of the order of the numbers or symbols concerned.
- **Complex number** – Any number that can be written as $a+bi$, where a and b are real numbers and $i=\sqrt{-1}$.
- **Distributive property** – Denoting an operation that is independent of being carried out before or after another operation.
- **FOIL –** Technique used for multiplying two binomials. FOIL stands for "firsts, outers, inners and lasts".
- **Imaginary number –** The square root of a negative number. i is called the imaginary unit.
- **Pure imaginary number** – When $a = 0$ and the complex number is written as bi.

Essential Questions

- What is a complex number? What is the purpose for a complex number?
- How do the properties of operations apply to complex numbers? How do you add, subtract, and multiply complex numbers?

Overall Big Ideas

Complex numbers expand the number system to include square roots of negative numbers and allows applications of complex numbers to electronics. We use the properties of operations as it applies to complex numbers to simplify expressions and to build foundations to solve quadratic equations having complex solutions.

Skill

To define and use imaginary and complex numbers.

To perform arithmetic operations with complex numbers.

Related Standards

N.CN.A.1

Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a+bi$ with a and b real.

N.CN.A.2

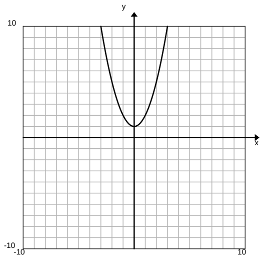
Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Notes, Examples, and Exam Questions

A Complex number is a combination of a real number and an imaginary number. Imaginary numbers are special because when squared, they give a **negative** result. Normally this doesn't happen, because when we square a positive number we get a positive result, and when we square a negative number we also get a positive result. But just imagine there is such a number, because we need it! The "unit" imaginary number (like 1 is for Real numbers) is *i*, which is the square root of -1. So, a complex number has a real part and an imaginary part, but either part can be 0, so all Real numbers and Imaginary numbers are also Complex numbers. Complex Number (in Standard Form): $a+bi$, where a is the real part of the complex number and bi is the imaginary part of the complex number

Graphically:

Imaginary Unit: $i=\sqrt{-1}$



No *x*-intercepts

$f(x)=x^2+1$

Complex Number	Real Part	Imaginary Part
3 + 2i	3	2
5	5	**0**
-6i	**0**	-6

Ex 1
Simplifying Square Roots of Negative Numbers

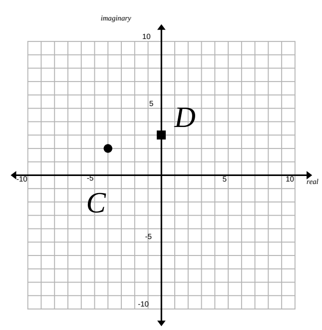
$$\sqrt{-2}=\sqrt{-1}\sqrt{2}=i\sqrt{2}$$
$$\sqrt{-4}=\sqrt{-1}\sqrt{4}=2i$$
$$\left(\sqrt{-1}\right)^2=i^2=-1$$
$$3\sqrt{-16}=3\sqrt{16}\sqrt{-1}=3\bullet 4\sqrt{-1}=12i$$
$$-\sqrt{-75}=-\sqrt{(75)(-1)}=-\sqrt{25}\sqrt{3}\sqrt{-1}=-5i\sqrt{3}$$

Ex 2
Graphing Complex Numbers in the Complex Plane

Plot the complex numbers in the complex plane: $C=-4+2i,\ D=3i$

Plot: C(-4,2) D(0,3)

Note: The horizontal axis is the real axis, and the vertical axis is the imaginary axis.



Sum and Difference of Complex Numbers: Add or subtract the real parts and the imaginary parts separately.

Ex 3

Find the sum: $(3-5i)+(-6-2i)$

$$(3+-6)+(-5i-2i)$$
$$=\boxed{-3-7i}$$

Ex 4

Find the difference: $(-4-8i)-(-3+2i)$

$$(-4-(-3))+(-8i-2i)$$
$$=\boxed{-1-10i}$$

Powers of i:

$$i^1=\sqrt{-1}$$
$$i^2=\sqrt{-1}\cdot\sqrt{-1}=-1$$
$$i^3=i\cdot i^2=-i$$
$$i^4=i^2\cdot i^2=1$$
$$i^5=i\cdot i^4=i$$
$$\ldots$$

△Note: The pattern continues every 4th power of i.

Ex 5

Evaluate i^{82}.

The exponent of 82 has a remainder of 2 when divided by 4. Therefore, i^{82} will be the same as $i^2=\boxed{-1}$.

Ex 6

What value of d makes the equation $-2+3i+12i=9i-(2-di)$ true?

If the complex numbers are equal, then the real parts must be equal and the imaginary parts must also be equal.

$$-2+3i+12i=9i-(2-di)$$
$$-2+15i=9i-2+di$$
$$-2+15i=-2+(9+d)i$$

Since (9 + d) must equal 15, d = 6.

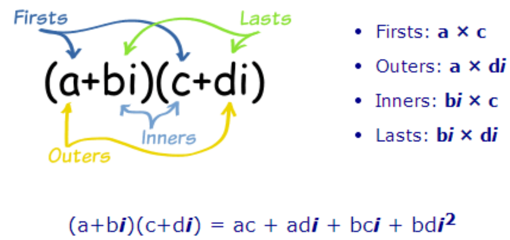
Ex 7

<u>Product of Complex Numbers</u>: Use the distributive property or FOIL method to multiply two complex numbers.

Find the product $(3-6i)(5+2i)$.

Use FOIL:

$$(3)(5)+(3)(2i)+(-6i)(5)+(-6i)(2i)$$
$$=15+6i-30i-12i^2$$
$$=15-24i-12(-1)$$
$$=\boxed{27-24i}$$



Ex 8

Multiply $6i(4-6i)$.

Use the distributive property.

$$6i(4-6i)$$
$$24i-36i^2 \rightarrow 24i-36(-1)$$
$$=24i+36$$
$$=\boxed{36+24i}$$

Ex 9

Multiply $(3-2i)^2$.

Use FOIL.

$$(3-2i)(3-2i) \rightarrow 9-6i-6i+4(i^2)$$
$$=9-12i+4(-1) \rightarrow 9-12i-4$$
$$=\boxed{5-12i}$$

Ex 10

<u>Solving Quadratic Equations with Complex Solutions</u>

Solve $x^2+4=0$.

Solve by square roots:

$$x^2=-4$$
$$\sqrt{x^2}=\sqrt{-4}$$

Write the answer(s) in complex form:

$$x=\pm\sqrt{4}\sqrt{-1}$$
$$\boxed{x=\pm 2i}$$

<u>QOD</u>: Tell whether the statement is true or false, and justify your answer. "Every complex number is an imaginary number."

SAMPLE EXAM QUESTIONS

1. **Simplify** $(3-4i)+(5-6i)$.

A. $8+10i$ **C.** $-9-38i$
B. $8-10i$ **D.** $6-10i$

Ans: B

2. **Simplify** $(1-3i)-(-3+7i)$.

A. $4-10i$ **C.** $4+4i$
B. $18+16i$ **D.** $-2+4i$

Ans: A

3. **Which is the product** $(8+i)(6+2i)$ **in standard form?**

A. $50+22i$ **C.** $48+24i$
B. $46+22i$ **D.** $48+20i$

Ans: B

4. **Express** $8\sqrt{-84}$ **in terms of** ***i*****.**

A. $-16i\sqrt{21}$ **C.** $-16\sqrt{21}$
B. $\sqrt{-5376}$ **D.** $16i\sqrt{21}$

Ans: D

5. **What is the simplified version of** $-i^8\sqrt{9}$**?**

A. 3 **C.** -3
B. $3i$ **D.** $-3i$

Ans: C

6. **Simplify** $i^5\sqrt{-24}$.

A. $-2\sqrt{6}$ **C.** $2\sqrt{6}$
B. $-2i\sqrt{6}$ **D.** $2i\sqrt{6}$

Ans: A

7. Simplify $\left(i\sqrt{5}-9\right)\left(i\sqrt{5}+9\right)$**.**

A. $5i^2-81$
B. $i\sqrt{5}-81$
C. -86
D. -76

Ans: C

8. Simplify $(9-2i)(3+i)$**.**

A. $12+4i$
B. $25+3i$
C. $27+i$
D. $29+3i$

Ans: D

9. Subtract $(5-2i)-(6+8i)$ **. Write the result in the form** $a+bi$**.**

A. $-1-10i$
B. $7-2i$
C. $-3-8i$
D. $11+6i$

Ans: A