OBJECTIVE #: G.CO.2

OBJECTIVE
Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle (i.e. rigid motions) to those that do not (e.g. translation vs. horizontal stretch)

BIG IDEA (Why is this included in the curriculum?)
- The study of geometry through transformations gives a thorough understanding of how geometric shapes are formed and their properties. Transformations will be used throughout this course to develop concepts such as congruency, similarity, symmetry and movement. The new terminology of mapping and transformations are connected to algebraic functions as a way to connect the previous knowledge of the student with the new geometric terminology.
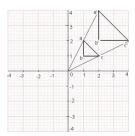
PREVIOUS KNOWLEDGE (What skills do they need to have to succeed?)
- The student must have a clear understanding of functions, domain (input), range (output) and one-to-one correspondence.
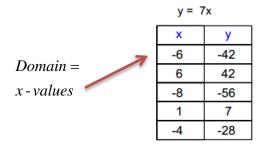
VOCABULARY USED IN THIS OBJECTIVE (What terms will be essential to understand?)
> **NEW VOCABULARY** (New Terms and definitions introduced in this objective)
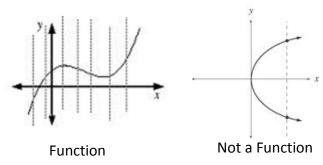>> - Dilation: A non-rigid transformation that enlarges or reduces the original figure.

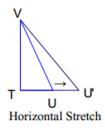>> - Domain: The collection of input values.

$$y = 7x$$

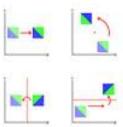| x	y
-6	-42
6	42
-8	-56
1	7
-4	-28

Domain =
x - values

- Function: A function is a rule that pairs each element in the domain (input) with exactly one element in the range (output). A function must pass the vertical line test.

Function   Not a Function
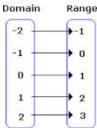
- Horizontal Stretch: A non-rigid transformation in which a plane figure is distorted horizontally.

Horizontal Stretch

- Isometry: A rigid transformation that preserves length, thus ensuring the pre-image and image are congruent.
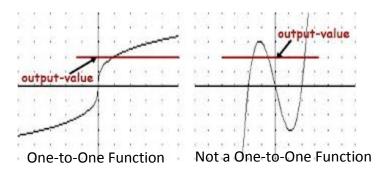
- Mapping: A correspondence between set of points, which pairs each member of the domain with an element of the range.   [G.CO.6]

Domain   Range

-2 → -1
-1 → 0
0 → 1
1 → 2
2 → 3

- Non-Rigid Transformation: A transformation that does not preserve length.

- One-to-One Function: A function in which every element in the range (output) corresponds to one and only one element in the domain (input). A one-to-one function must pass the horizontal line test. [G.CO.6]

One-to-One Function   Not a One-to-One Function

- Range: The collection of all output values.

y = 7x

x	y
-6	-42
6	42
-8	-56
1	7
-4	-28

$Range =$
y - $values$

- Reflection: A rigid transformation in which the image is a mirror image of the pre-image, thus ensuring the pre-image and the image are equidistant from the line of reflection. [G.CO.3, G.CO.4, G.CO.5, G.CO.11]

Reflection

Notation:

Reflection in the $x - axis$	$P(x, y) \rightarrow P'(x, -y)$ or $r_{x-axis}(x, y) = (x, -y)$
Reflection in the $y - axis$	$P(x, y) \rightarrow P'(-x, y)$ or $r_{y-axis}(x, y) = (-x, y)$
Reflection in $y = x$	$P(x, y) \rightarrow P'(y, x)$ or $r_{y=x}(x, y) = (y, x)$
Reflection in $y = -x$	$P(x, y) \rightarrow P'(-y, -x)$ or $r_{y=-x}(x, y) = (-y, -x)$
Reflection in the Origin	$P(x, y) \rightarrow P'(-x, -y)$ or $r_{origin}(x, y) = (-x, -y)$

- Rigid Transformation: An isometry that does not alter the shape or size of a figure (reflection, rotation, translation).

- Rotation: A rigid transformation that turns a figure about a fixed point, thus ensuring the pre-image and image are congruent. [G.CO.3, G.CO.4, G.CO.5]

Rotation

Notation:

Rotation of 90°	$R_{90°}(x, y) = (-y, x)$
Rotation of 180°	$R_{180°}(x, y) = (-x, -y)$
Rotation of 270°	$R_{270°}(x, y) = (y, -x)$

- Transformation: A one-to-one mapping of points in a plane to corresponding points in the plane.

- Translation: A rigid transformation that slides an object a fixed distance in a given direction, thus ensuring the pre-image and image are congruent. [G.CO.3, G.CO.4, G.CO.5]
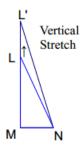

Translation

Notation:

Translation of h, k	$T_{h,k}(x, y) = (x + h, y + k)$

- Vertical Stretch: A non-rigid transformation in which a plane figure is distorted vertically.

Vertical Stretch

SKILLS (What will they be able to do after this objective?)
- The student will be able to understand the difference between transformations that are isometries (rigid) and those that are not isometries.
- The student will be able to differentiate between the various isometries, e.g., reflections, rotations, or translations.
- The student will be able to recognize the non-isometric transformations that are dilations and stretches.
- The student will be able to show a one-to-one correspondence between the original figure's coordinates (called the pre-image or input values) and the transformed figure's coordinates (called the image or output values).

SHORT NOTES (A short summary of notes so that a teacher can get the basics of what is expected.)
Transformations describe how geometric figures of the same shape are related to one another. If they are equal (isometric) they are called rigid. In previous studies students have been introduced to flip, slide and turn. Link this knowledge to new vocab: reflections, rotations and translations.
Through Common Core, this objective will be taught from a transformational standpoint instead of the traditional methodology. The students will gain an understanding of symmetry, congruency, and similarity through constructions and transformations instead of memorizing vocabulary, theorems, postulates and corollaries. This new approach promotes critical thinking which leads to a deeper understanding when discovering geometric concepts.
The students are familiar with function tables (input/output) from Algebra. That concept continues to Geometry with the new vocabulary – mapping (pre-image/image). In Algebra a one-to-one function exists if for every input (domain) there exists only one output (range). This one-to-one function in Geometry is known as a transformation.
Practice with patty paper, transparencies and the geometry software.

MISCONCEPTIONS (What are the typical errors or difficult areas? Also suggest ways to teach them.)
Make sure students are keeping track of notation and how to pronounce symbols properly. For example: P' is pronounced P prime. Consistent use of notation with correct pronunciations should help the students get needed verbal and visual cues.
Not all mapping will produce a transformation. Begin by reviewing functions from Algebra and build. Have them find which functions will and will not produce a transformation.
See attachment on Mapping, Functions and Transformations for clarity.

FUTURE CONNECTIONS (What will they use these skills for later?)
- Transformations will be used throughout this course to develop concepts such as congruency, similarity, symmetry and movement. In addition, correspondence between numerical coordinates and geometric points develop methods for algebraic/geometric applications and modeling.

ADDITIONAL EXTENSIONS OR EXPLANATIONS (What needs greater explanation?)
- Teachers need to emphasize the connection that students must make between algebra (function) and geometry (mapping).

OUTSIDE RESOURCES
www.regentsprep.org (Go to Geometry, Transformational Geometry, Review of Transformation Information, Sheet of Basic Formulas for Transformations (.pdf form))

ASSESSMENT ITEMS (What questions would evaluate these skills?)
Multiple Choice
1. The point P (-3,7) is reflected in the line y = x. What are the coordinates of P'?
A) (-3, -7)   B) (3, -7)
C) (3, 7)   D) (-7, 3)
E) None of these

Short Answer

Graph the given reflection

    2. P (2,4) in the line x = 1

    3. Explain how a rotation and a reflection are alike.   They both preserve congruence

       How are they different?   In a rotation, every point moves along a circular path around a fixed point.

    4. Name three isometric transformations. Reflection, Rotation, Translation

Multi-Step Problems

    5. The coordinates of Δ ABC are A (1, 1), B (-2, 4), and C(-2, -1). The coordinates ofΔ A'B'C' are A'(2, -3), B'(-1,0), and C'(-1, -5).

       a.   Name the transformation. ΔABC is translated right 1 unit and down 4 units

       b.   Is this an isometry? Yes

       c.   If m ≤B = 45°, find m ≤B'. 45°

From CCSD Geometry Honors Semester 1 Practice Exam 2012-2013

    1. What term describes a transformation that does not change a figure's size or shape?

       (A) similarity

       (B) isometry

       (C) collinearity

       (D) symmetry

For questions 2–5, determine if the described transformation(s) is/are an isometry.

    2. A reflection is an isometry.

       (A) True

       (B) False

    3. A composition of two reflections is an isometry.

       (A) True

       (B) False

    4. A dilation is an isometry.

       (A) True

       (B) False

    5. A composition of a rotation and a dilation is an isometry.

       (A) True

       (B) False