

GEOMETRY

Basics of Geometry – Precise Definitions



OBJECTIVE #: G.CO.1

OBJECTIVE

Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

BIG IDEA (Why is this included in the curriculum?)

- The study of geometry is based on three undefined terms: a point, a line and a plane. These three notions are used to define all other terms in geometry. In addition, one primary goal in geometry is to expand the geometric concepts taught in previous grades by developing more precise definitions, which will lead to a better understanding of the geometric theorems.

This objective will be implemented throughout the year.

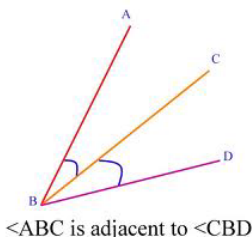
PREVIOUS KNOWLEDGE (What skills do they need to have to succeed?)

- The student must have a general knowledge of basic geometry vocabulary and notation.

VOCABULARY USED IN THIS OBJECTIVE (What terms will be essential to understand?)

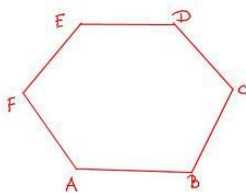
VOCABULARY (Terms used but defined earlier)

- Adjacent Angles: Two angles with a common vertex and side but no common interior points.

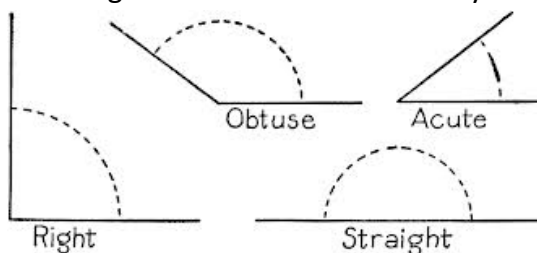


- Adjacent Sides: Two sides of a polygon with a common vertex.

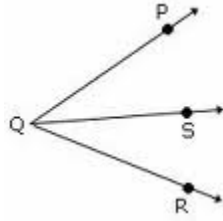
\overline{BA} is adjacent to \overline{AF}



- Angle: A geometric figure that consists of two different rays that have the same initial point.
 - Acute Angle: An angle whose measure is less than 90° . [G.CO.10, G.CO.12]
 - Right Angle: An angle whose measure is exactly 90° . [G.CO.10, G.CO.12]
 - Obtuse Angle: An angle whose measure is greater than 90° but less than 180° . [G.CO.10, G.CO.12]
 - Straight Angle: An angle whose measure is exactly 180° .

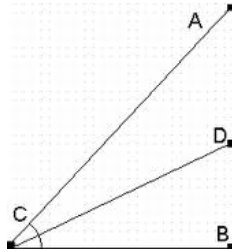


- Angle Addition Postulate: If S is in the interior of $\angle PQR$, then $m\angle PQS + m\angle SQR = m\angle PQR$.

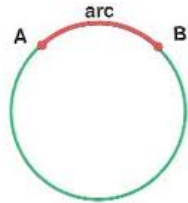


- Angle Bisector: A line/segment/ray that divides an angle into two congruent parts. [G.CO.10, G.CO.12]

\overline{CD} bisects $\angle ACB$

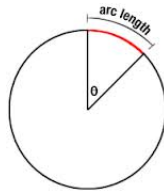


- Arc: A portion of circle. [G.CO.12]



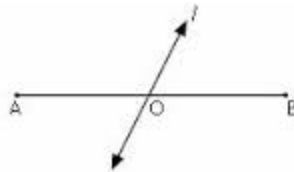
Notation: \widehat{AB}

- Arc length: A portion of the circumference of a circle.

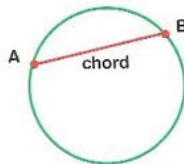


- Bisect: To divide into two congruent parts. [G.CO.11, G.CO.12]

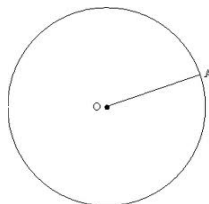
line l bisects \overline{AB}



- Chord: A segment whose endpoints are points on a circle.

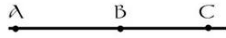


- Circle: The set of all points in a plane that are equidistant from a given point, called the center. [G.CO.4, G.CO.12]



Notation: $\odot O$

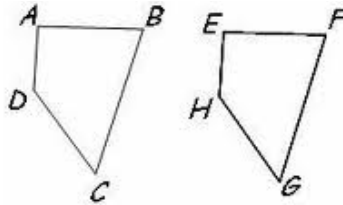
- Collinear Points: Points that lie on the same line.



- Concave: A polygon such that a line containing a side of a polygon contains a point in the interior of the polygon. A polygon that is not convex.



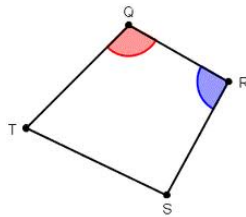
- Congruent Figures: Two geometric figures that have exactly the same size and shape.



Notation: $ABCD \cong EFGH$

- Consecutive Angles: Two angles that share a side of a polygon. [\[G.CO.11\]](#)

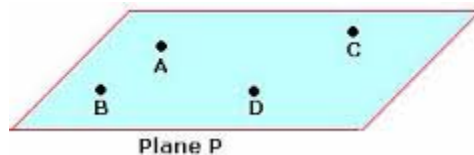
$\angle Q$ and $\angle R$
are consecutive
angles



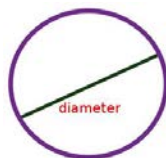
- Convex: A polygon such that no line containing a side of the polygon contains a point in the interior of the polygon.



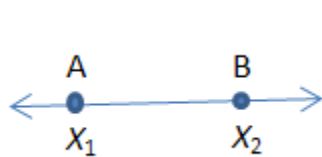
- Coplanar Points: Points that lie on the same plane.



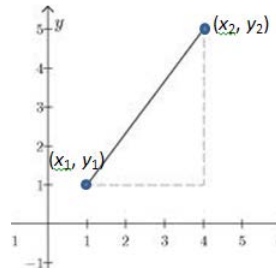
- Diameter: A chord that passes through the center of the circle.



- Distance: The length of a line segment between two points. [G.CO.12]



$$AB = |X_2 - X_1|$$



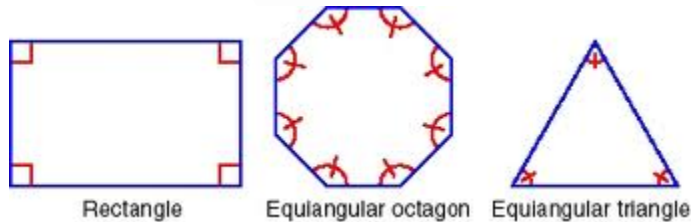
$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Notation: AB

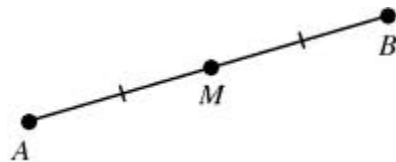
- Endpoint: A point marking either end of a line segment. [G.CO.12]



- Equiangular Polygon: A polygon with all angles congruent. [G.CO.11]

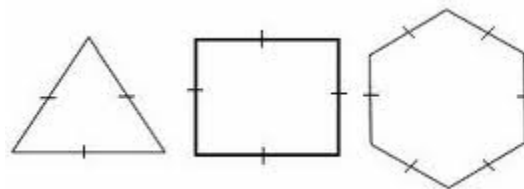


- Equidistant: Two points that are the same distance from a given point. [G.CO.9,G.CO.12]



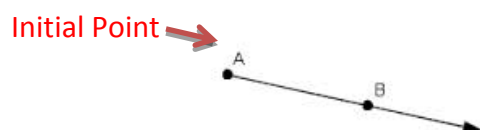
Points A and B are equidistant from Point M

- Equilateral Polygon: A polygon with all sides congruent. [G.CO.11]

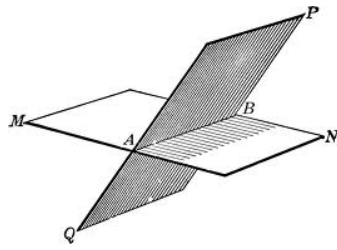


- Exterior of an Angle: All points not on an angle or in the interior of an angle.

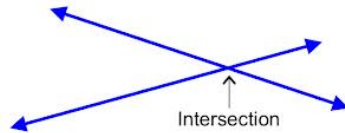
- Initial Point: The starting point of a ray or vector. [G.CO.4]



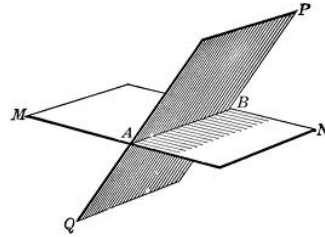
- Interior of an Angle: All points between the points that lie on each side of an angle.
- Intersect: To have one or more points in common.



- Intersection: The set of all points that two or more geometric figures have in common.



Intersect at 1 point



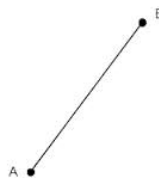
\overline{AB} is the intersection

- Length: The distance between the endpoints of a segment. **[SEE DISTANCE]**
- Line: An undefined term that extends in one dimension, which is usually represented by a straight line with two arrowheads. **[G.CO.12]**



Notation: \overline{AB} or Line t

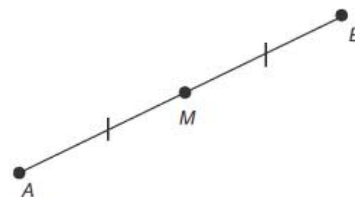
- Line Segment: A portion of a line that consists of two endpoints and all points in between the two endpoints. **[G.CO.4. G.CO.12]**



Notation: \overline{AB}

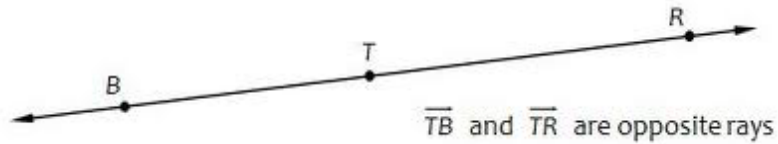
- Midpoint: The point that divides, or bisects, a segment into two congruent segments.

Midpoint: $(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

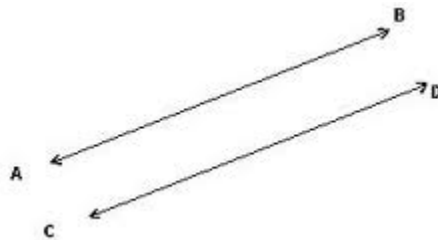


M is the midpoint of \overline{AB}

- Opposite Rays: Two rays that share a common endpoint, but extend in opposite directions, thus forming a straight line.

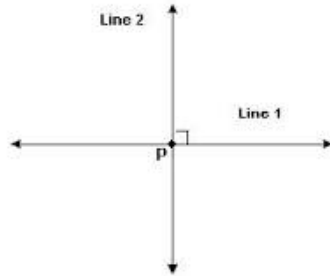


- Parallel Lines: Two lines that are coplanar and do not intersect. [G.CO.4, G.CO.9, G.CO.12]



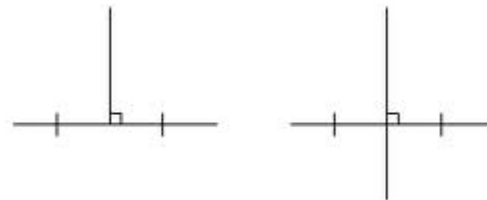
Notation: $\overline{AB} \parallel \overline{CD}$

- Perpendicular: Two lines/segments/rays that intersect to form right angles. [G.CO.4, G.CO.9, G.CO.11, G.CO.12]

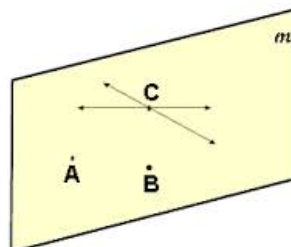


Notation: $line\ 1 \perp line\ 2$

- Perpendicular Bisector: A perpendicular line/segment/ray that intersects a segment at its midpoint. [G.CO.9, G.CO.10, G.CO.12]



- Plane: An undefined term that extends in two dimensions, which is usually represented by a parallelogram. [G.CO.12]



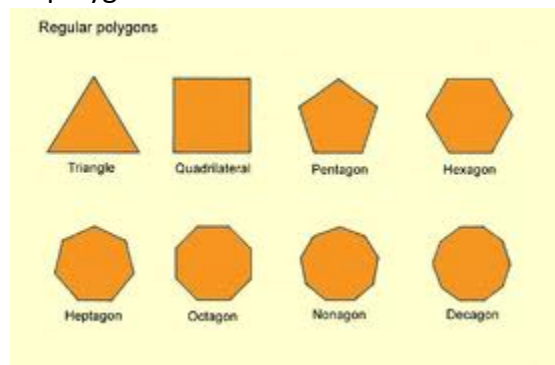
Notation: Plane ABC or Plane \mathcal{M}

- Point: An undefined term that has no dimension which is usually represented by a dot indicating a specific location. [G.CO.12]

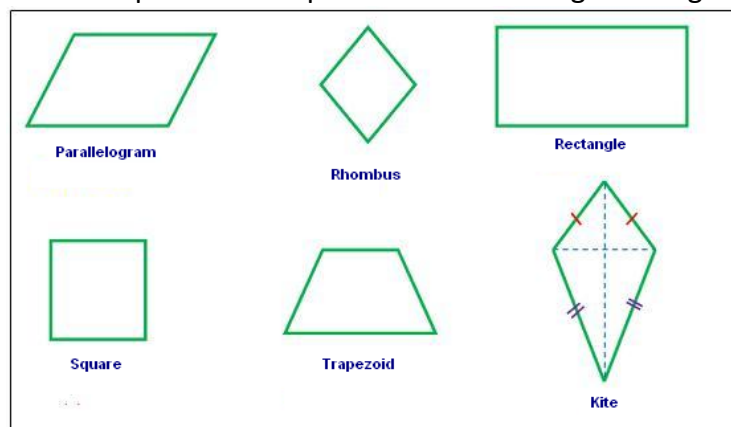


Notation: Point P

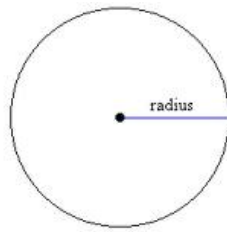
- Polygon: A closed figure created by line segments. [G.CO.12]
 - Triangle: A three sided polygon.
 - Quadrilateral: A four sided polygon.
 - Pentagon: A five sided polygon.
 - Hexagon: A six sided polygon.
 - Heptagon: A seven sided polygon.
 - Octagon: An eight sided polygon.
 - Nonagon: A nine sided polygon.
 - Decagon: A ten sided polygon.
 - Dodecagon: A twelve sided polygon.
 - n -gon: An n sided polygon.



- Quadrilateral: A four sided polygon.
 - Kite: A quadrilateral that has two pairs of consecutive congruent sides, but in which opposite sides are not congruent.
 - Parallelogram: A quadrilateral with both pairs of opposite sides parallel. [G.CO.3, G.CO.11]
 - Rectangle: A parallelogram with four right angles. [G.CO.3, G.CO.11]
 - Rhombus: A parallelogram with four congruent sides. [G.CO.11]
 - Square: A parallelogram with four right angles and four congruent sides. [G.CO.11]
 - Trapezoid: A quadrilateral with exactly one pair of parallel sides. [G.CO.3p]
 - Isosceles Trapezoid: A trapezoid with two congruent legs.



- Radius: The distance from the center of a circle to a point on the circle. [G.CO.12]

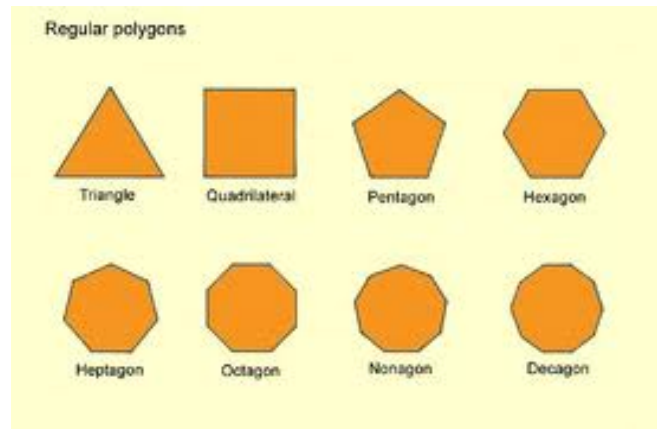


- Ray: A portion of a line that consists of a point called an initial point, and all points on the line that extend in one direction. [G.CO.12]

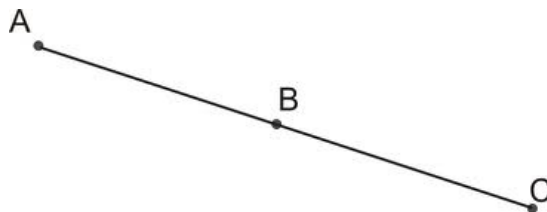


Notation: \overrightarrow{AB}

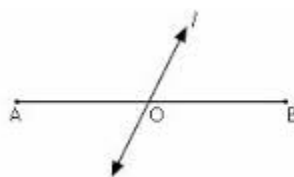
- Regular Polygon: A convex polygon with all angles congruent and all sides congruent. [G.CO.3, G.CO.12]



- Segment Addition Postulate: If B is between A and C , then $AB + BC = AC$. [G.CO.10]

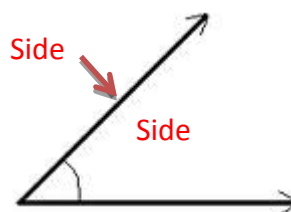


- Segment Bisector: A segment/line/ray/plane that intersects a segment at its midpoint.

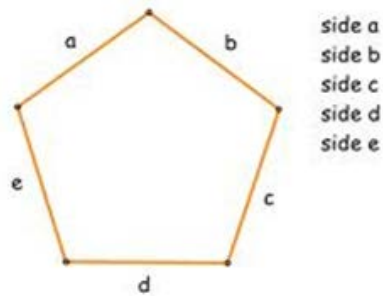


Line l bisects \overline{AB}

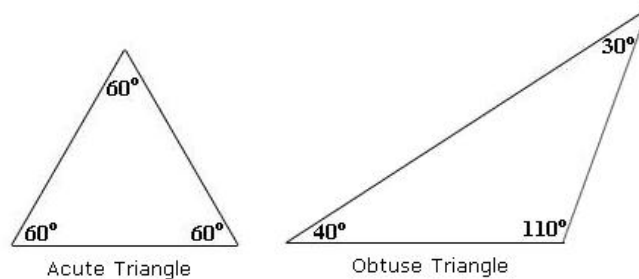
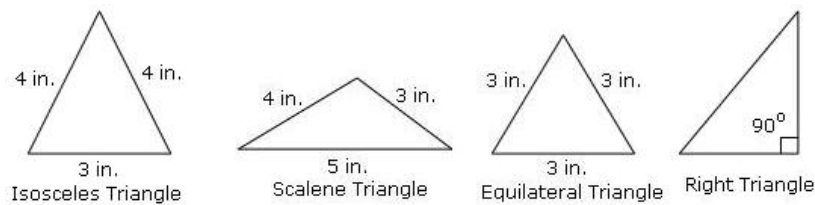
- Sides of an Angle: Two rays that share a common endpoint and form the angle. [G.CO.12]



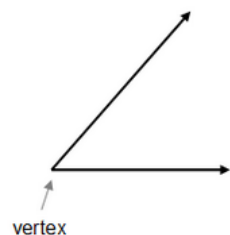
- Side of a Polygon: The segment formed by connecting two adjacent vertices of a polygon. [G.CO.7, G.CO.8]



- Triangle: A figure formed by three segments joining three non-collinear points, called vertices. [G.CO.7, G.CO.8, G.CO.12]
 - Acute Triangle: A triangle with three acute angles. [G.CO.10]
 - Right Triangle: A triangle with exactly one right angle. [G.CO.10]
 - Obtuse Triangle: A triangle with exactly one obtuse angle. [G.CO.10]
 - Equiangular Triangle: A triangle with three congruent angles. [G.CO.10, G.CO.12]
 - Scalene Triangle: A triangle with no congruent sides. [G.CO.10]
 - Isosceles Triangle: A triangle with at least two congruent sides. [G.CO.10]
 - Equilateral Triangle: A triangle with three congruent sides. [G.CO.10, G.CO.12]



- Vertex of an Angle: The initial point formed by the two rays (sides of an angle). [G.CO.12]



Common point of two rays

SKILLS (What will they be able to do after this objective?)

- The student will be able to define any geometry term using the three basic undefined terms.

SHORT NOTES (A short summary of notes so that a teacher can get the basics of what is expected.)

The definitions/diagrams/ and notations noted above are the notes for G.CO.1. Students must familiarize themselves with the definitions and notation, so they can continue to apply these concepts throughout the course.

In addition, you will need to make sure the students have an understanding of the following postulates:

Point, Line and Plane Postulates	Through any two points there exists exactly one line.
	A line contains at least two points.
	If two lines intersect, then their intersection is exactly one point.
	Through any three noncollinear points there exists exactly one plane.
	A plane contains at least three noncollinear points.
	If two points lie in a plane, then the line containing them lies in the plane.
	If two planes intersect, then their intersection is a line.

Please see the **vocabulary template** that students can use to help them practice with vocabulary throughout the course.

MISCONCEPTIONS (What are the typical errors or difficult areas? Also suggest ways to teach them.)

Vocabulary is critical for students' success in geometry, other higher level math classes, and on the proficiency exam. It is recommended that you include vocabulary on every test/quiz that is given in class.

FUTURE CONNECTIONS (What will they use these skills for later?)

- Students will continue to define geometric concepts and develop proofs of theorems/concepts using the three undefined terms.

ADDITIONAL EXTENSIONS OR EXPLANATIONS (What needs greater explanation?)

- This standard will repeat itself throughout the year when new geometric relationships and objects are introduced. Defining and notating things correctly is an essential skill of geometry.

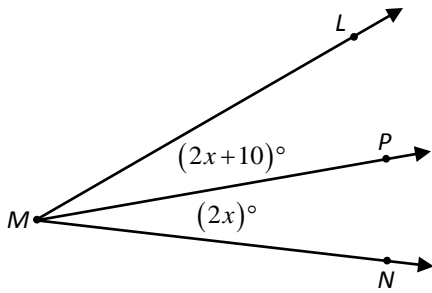
ASSESSMENTS (Questions that get to the heart of the objective – multiple choice, short answer, multi-step)

Multiple Choice

Questions #1-15 were taken from the RPDP released Semester 1 Exams from 08-11.

- 1) What is the distance between points $A(3, 12)$ and $B(-3, 4)$?
(A) $\sqrt{14}$
(B) $\sqrt{28}$
(C) 10
(D) 100
- 2) What are the coordinates of the midpoint of \overline{AB} with endpoints $A(7, -2)$ and $B(5, 6)$?
(A) $(1, -4)$
(B) $\left(\frac{5}{2}, \frac{11}{2}\right)$
(C) $(6, 2)$
(D) $(12, 4)$

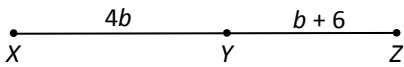
3) In the diagram, $m\angle LMN = 54^\circ$.



What is the value of x ?

- (A) 27
- (B) 22
- (C) 16
- (D) 11

4) In the diagram, Y is between X and Z , and $XZ = 36$ centimeters.



What is the length of \overline{XY} ?

- (A) 24 cm
- (B) 12 cm
- (C) 10 cm
- (D) 8 cm

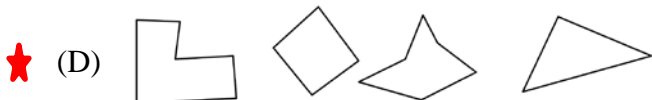
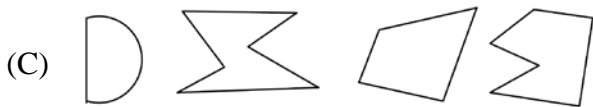
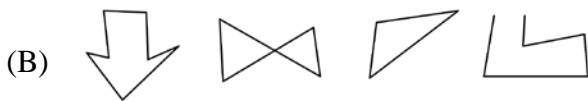
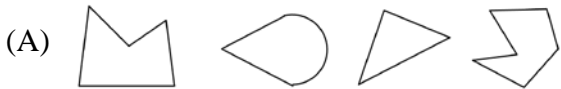
5) Given $A(-2, 5)$ and $B(4, -2)$. What is the distance from A to B ?

- (A) $\sqrt{13}$
- (B) $\sqrt{45}$
- (C) $\sqrt{53}$
- (D) $\sqrt{85}$

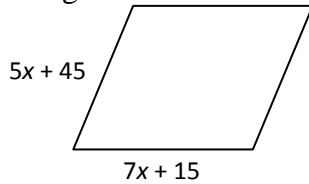
6) How many sides does a decagon have?

- (A) 6
- (B) 8
- (C) 10
- (D) 12

7) Which group of figures are all polygons?



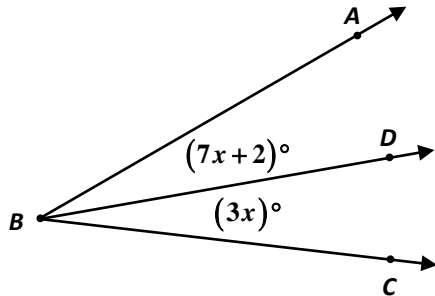
8) The figure below is a rhombus.



What is the value of x ?

- (A) 2.5
- (B) 10
- (C) 15
- (D) 30

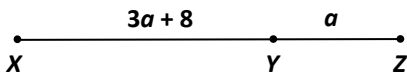
9) In the diagram below, $m\angle ABC = 42^\circ$.



What is the value of x ?

- (A) 2
- (B) $3\frac{1}{2}$
- (C) 4
- (D) $4\frac{2}{5}$

10) In the figure below, Y is between X and Z and $XZ = 40$ cm.



What is the value of a ?

- (A) 4
- (B) 8
- (C) 12
- (D) 16

11) What is the distance between points $A(-2, -6)$ and $B(-2, -3)$?

- (A) 3
- (B) $\sqrt{41}$
- (C) 9
- (D) $\sqrt{89}$

12) What are the coordinates of the midpoint of the segment joining the points $A(-3, -4)$ and $B(4, 2)$?

(A) $\left(-3\frac{1}{2}, 3\right)$

(B) $\left(-\frac{1}{2}, -1\right)$

(C) $\left(\frac{1}{2}, -1\right)$

(D) $\left(\frac{1}{2}, -3\right)$

13) Which is a valid classification for a triangle?

(A) Acute right

(B) Isosceles scalene

(C) Isosceles right

(D) Obtuse equiangular

14) How many sides does a nonagon have?

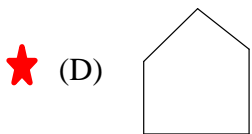
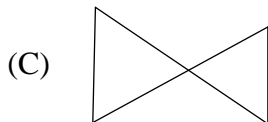
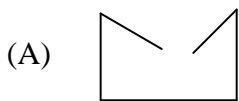
(A) 7

(B) 9

(C) 11

(D) 19

15) Which figure is a polygon?



Short Answer

Fill in the blank.

1) A(n) _____ angle is an angle whose measure is less than 90 degrees. **acute**

2) A(n) _____ triangle is a triangle with three acute angles. **acute**

3) _____ angles are two angles with a common vertex and side but no common interior points.
Adjacent

4) _____ sides are two sides of a polygon with a common vertex. **Adjacent**

5) A(n) _____ is a geometric figure that consists of two different rays that have the same initial point. **angle**

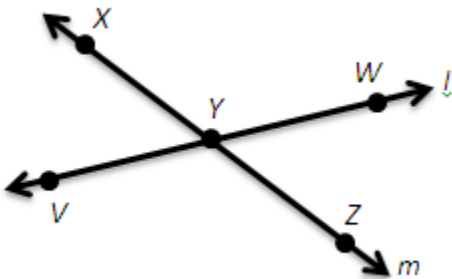
6) An angle _____ is a line/segment/ray that divides an angle into two congruent parts. **bisector**

7) A(n) _____ is a portion of a circle. **arc**

- 8) _____ is a portion of the circumference of a circle. **Arc length**
- 9) _____ is to divide something into two congruent parts. **Bisect**
- 10) A(n) _____ is a segment whose endpoints are points on a circle. **chord**
- 11) A(n) _____ is the set of all points in a plane that are equidistant from a given point, called the center. **circle**
- 12) _____ points are points that lie on the same line. **Collinear**
- 13) A _____ polygon is a polygon such that a line containing a side of a polygon contains a point in the interior of the polygon. **concave**
- 14) _____ figures are two geometric figures that have exactly the same size and shape. **Congruent**
- 15) _____ angles are two angles that share a side of a polygon. **Consecutive**
- 16) A _____ polygon is a polygon such that no line containing a side of a polygon contains a point in the interior of the polygon. **convex**
- 17) _____ points are points that lie on the same plane. **Coplanar**
- 18) a) A _____ is a three sided polygon. **triangle**
 b) A _____ is a four sided polygon. **quadrilateral**
 c) A _____ is a five sided polygon. **pentagon**
 d) A _____ is a six sided polygon. **hexagon**
 e) A _____ is a seven sided polygon. **heptagon**
 f) A _____ is an eight sided polygon. **octagon**
 g) A _____ is a nine sided polygon. **nonagon**
 h) A _____ is a ten sided polygon. **decagon**
 i) A _____ is a twelve sided polygon. **dodecagon**
- 19) A _____ is a chord that passes through the center of the circle. **diameter**
- 20) _____ is the length of a line segment between two points. **Distance**
- 21) A(n) _____ is a point marking either end of a line segment. **endpoint**
- 22) A(n) _____ polygon is a polygon with all angles congruent. **equiangular**
- 23) A(n) _____ triangle is a triangle with three congruent angles. **equiangular**
- 24) If two points are _____ from a point, then they are the same distance from the given point. **equidistant**
- 25) A polygon with all sides congruent is a(n) _____ polygon. **equilateral**
- 26) A triangle with all three sides congruent is a(n) _____ triangle. **equilateral**
- 27) The _____ of an angle are all points not on an angle or in the interior of an angle. **exterior**
- 28) The _____ point is the starting point of a ray or vector. **Initial point**
- 29) The _____ of an angle are all points between the points that lie on each side of an angle. **interior**
- 30) To _____ is to have one or more points in common. **intersect**
- 31) The _____ is the set of all points that two or more geometric figures have in common. **intersection**
- 32) A(n) _____ is a trapezoid that has two congruent legs. **isosceles**
- 33) A(n) _____ triangle is a triangle that has at least two congruent legs. **isosceles**
- 34) A _____ is a quadrilateral that has two pairs of consecutive congruent sides, but in which opposite sides are not congruent. **kite**
- 35) _____ is the distance between the endpoints of a segment. **Length**
- 36) A(n) _____ is an undefined term that extends in one dimension, which is usually represented by a straight line with two arrowheads. **line**
- 37) A(n) _____ is a portion of a line that consists of two endpoints and all points in between the two endpoints. **line segment**
- 38) The point that divides, or bisects, a segment into two congruent parts is called the _____.
midpoint

- 39) A(n) _____ angle is an angle whose measure is greater than 90 degrees but less than 180 degrees. **obtuse**
- 40) A(n) _____ triangle is a triangle with exactly one obtuse angle. **obtuse**
- 41) Two rays that share a common endpoint, but extend in opposite directions, thus forming a straight line are called _____. **opposite rays**
- 42) _____ lines are lines that are coplanar and do not intersect. **Parallel**
- 43) A quadrilateral with both pairs of opposite sides parallel is called a _____. **parallelogram**
- 44) If two lines/segments/rays intersect to form right angles, then they are _____. **perpendicular**
- 45) A(n) _____ is a line/segment/ray that is perpendicular to a given segment at its midpoint. **perpendicular bisector**
- 46) A(n) _____ is an undefined term that extends in two dimensions, and is usually represented by a parallelogram. **plane**
- 47) A(n) _____ is an undefined term that has no dimension, and is usually represented by a dot indicating a specific location. **point**
- 48) A(n) _____ is a closed figure created by line segments. **polygon**
- 49) The _____ is the distance from the center of a circle to a point on the circle. **radius**
- 50) A(n) _____ is a portion of a line that consists of a point called an initial point, and all points on the line that extend in one direction. **ray**
- 51) A(n) _____ is a parallelogram with four right angles. **rectangle**
- 52) A convex polygon with all angles congruent and all sides congruent is called _____. **regular**
- 53) A _____ is a parallelogram with four congruent sides. **rhombus**
- 54) A(n) _____ angle is an angle whose measure is exactly 90 degrees. **right**
- 55) A(n) _____ triangle is a triangle with exactly one right angle. **right**
- 56) A(n) _____ triangle is a triangle with no congruent sides. **scalene**
- 57) A _____ is a segment/line/ray/plane that intersects a segment at its midpoint. **segment bisector**
- 58) A(n) _____ is the segment formed by connecting two adjacent vertices of a polygon. **side**
- 59) The _____ of an angle are the two rays that share a common endpoint and form the angle. **sides**
- 60) A _____ is a parallelogram with four right angles and four congruent sides. **square**
- 61) A(n) _____ angle is an angle whose measure is exactly 180 degrees. **straight**
- 62) A(n) _____ is a quadrilateral with exactly one pair of parallel sides. **trapezoid**
- 63) The _____ of an angle is the initial point formed by the two rays (sides of the angle). **vertex**

True or False



- 64) X, Y and Z are collinear. **True**
- 65) Points V, Z and X lie on line m. **False**
- 66) Points X, Z and W are coplanar **True**
- 67) Point W lies on line l. **True**

Multi-Step Problems

- 1) Suppose J is between H and K . Solve for x . Then find the length of each segment.

$$HJ = 5x - 3$$

$$x = 11$$

$$JK = 8x - 9$$

$$HJ = 52$$

$$KH = 131$$

$$JK = 79$$

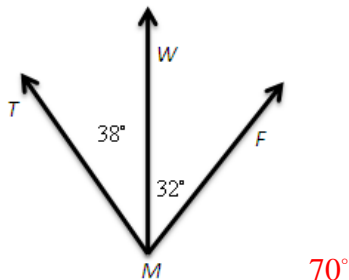
- 2) Find the distance between $(-1, 0)$ and $(2, -4)$.

$$5$$

- 3) Find the distance between $(2, -4)$ and $(1, 3)$.

$$\sqrt{50} = 5\sqrt{2}$$

- 4) Find the $m\angle TMF$.



- 5) Let Q be in the interior of $\angle POR$. Solve for x . Find the measure of each angle

$$m\angle POQ = (3x + 7)^\circ$$

$$x = 7$$

$$m\angle QOR = (5x - 2)^\circ$$

$$m\angle POQ = 28^\circ$$

$$m\angle POR = 61^\circ$$

$$m\angle ROQ = 33^\circ$$

- 6) Find the coordinates of the midpoint of a segment with the given endpoints.

$$A(-3, 5)$$

$$(1, 2)$$

$$B(5, -1)$$

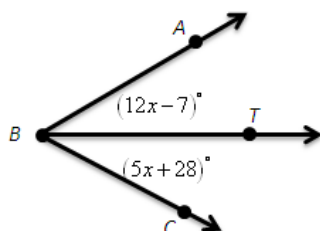
- 7) Find the coordinates of the other endpoint of a segment with the given endpoint and midpoint M .

$$A(-4, 3)$$

$$(2, -5)$$

$$M(-1, -1)$$

- 8) \overline{BT} bisects $\angle ABC$. Find the value of x . Then find the measures of the angles.



$$x = 5$$

$$m\angle ABT = 53^\circ$$

$$m\angle CBT = 53^\circ$$

$$m\angle ABC = 106^\circ$$