



OBJECTIVE #: G.CO.12

OBJECTIVE

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line

BIG IDEA (Why is this included in the curriculum?)

- Students explore geometric ideas using key tools of geometry. This objective focuses on classical Euclidean compass and straightedge constructions, both by hand and in dynamic geometry environments. The goal is to both create constructions and explore why they work.
- G.CO.12 is an objective that will be used throughout the course. Students will be using constructions to extend, discover, and prove geometric concepts through the entire course.

PREVIOUS KNOWLEDGE (What skills do they need to have to succeed?)

- Students will need a basic grasp of previous vocabulary terms: circle, arc, line, line segment, ray, angle, distance.

VOCABULARY USED IN THIS OBJECTIVE (What terms will be essential to understand?)

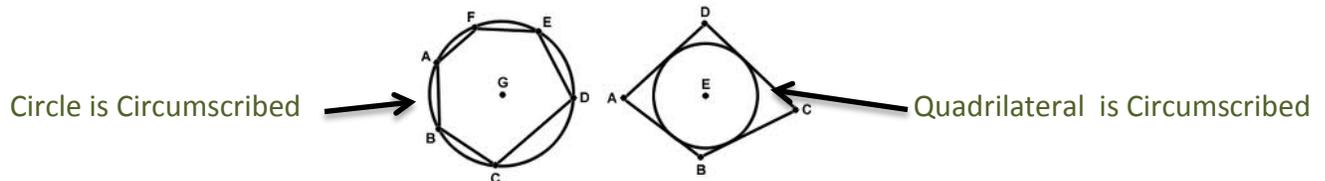
PREVIOUS VOCABULARY (Terms used but defined earlier)

- Angle: A geometric figure that consists of two different rays that have the same initial point.
 - Acute Angle: An angle whose measure is less than 90° .
 - Right Angle: An angle whose measure is exactly 90° .
 - Obtuse Angle: An angle whose measure is greater than 90° but less than 180° .
 - Straight Angle: An angle whose measure is exactly 180° .
- Angle Bisector: A line/segment/ray that divides an angle into two congruent parts.
- Arc: A portion of circle.
- Bisect: To divide into two congruent parts.
- Circle: The set of all points in a plane that are equidistant from a given point, called the center.
- Distance: The length of a line segment between two points.
- Endpoint: A point marking either end of a line segment.
- Equidistant: Two points that are the same distance from a given point.
- Line: An undefined term that extends in one dimension, which is usually represented by a straight line with two arrowheads.
- Line Segment: A portion of a line that consists of two endpoints and all points in between the two endpoints.
- Parallel Lines: Two lines that are coplanar and do not intersect.
- Perpendicular: Two lines/segments/rays that intersect to form right angles.
- Perpendicular Bisector: A perpendicular line/segment/ray that intersects a segment at its midpoint.
- Plane: An undefined term that extends in two dimensions, which is usually represented by a parallelogram.
- Point: An undefined term that has no dimension which is usually represented by a dot indicating a specific location.
- Polygon: A closed figure created by line segments.

- Radius: The distance from the center of a circle to a point on the circle.
- Ray: A portion of a line that consists of a point called an initial point, and all points on the line that extend in one direction.
- Regular Polygon: A convex polygon with all angles congruent and all sides congruent.
- Sides of an Angle: Two rays that share a common endpoint and form the angle.
- Triangle: A figure formed by three segments joining three non-collinear points, called vertices.
 - Equiangular Triangle: A triangle with three congruent angles.
 - Equilateral Triangle: A triangle with three congruent sides.
- Vertex of an Angle: The initial point formed by the two rays (sides of an angle).

NEW VOCABULARY (New Terms and definitions introduced in this objective)

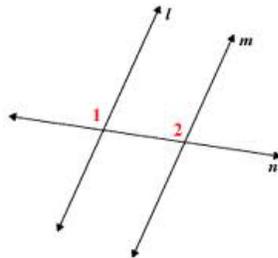
- Circumscribed Figure: A geometric figure that is drawn around and enclosing another figure.



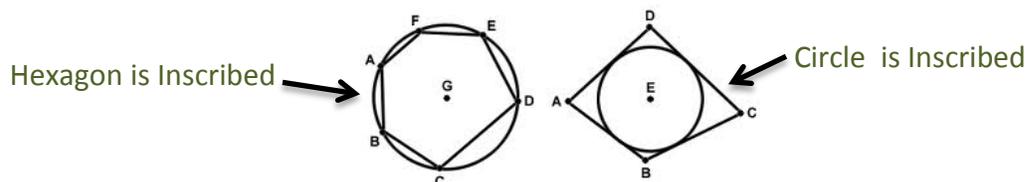
- Compass: A drawing tool used to make circles of different radii.



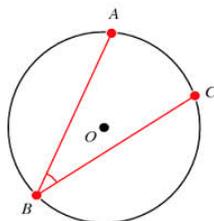
- Construction: A precise way of drawing using a variety of tools, which usually includes the straightedge and the compass.
- Corresponding Angles: Two angles that are formed by two lines and a transversal which lie in the same position with respects to the lines and the transversal. [G.CO.9p]



- Inscribed Figure: A geometric figure that is drawn inside another geometric figure.



- Intercepted Arc: The part of the circle that lies between the sides of a given angle.



Notation: \widehat{AC}

- Straightedge: An unmarked tool used to draw straight lines.

SKILLS (What will they be able to do after this objective?)

The student will be able to use a compass and straightedge to create classical Euclidean constructions.

These constructions will include:

- copy a given line segment.
- copy a given angle.
- bisect a line segment.
- bisect an angle.
- construct a line perpendicular to a given line through a given point .
- construct the perpendicular bisector of a line segment.
- construct a line parallel to a given line through a point not on the line.
- construct geometric figures using a variety of tools and methods.

SHORT NOTES (A short summary of notes so that a teacher can get the basics of what is expected.)

No real changes to how the constructions are taught. Constructions can be linked to the properties of rhombi and reviewed when those quadrilateral properties are taught for reinforcement/linkage.

<http://www.mathopenref.com/constructions.html> is a great resource; videos of constructions can be run on auto-repeat while students practice and teacher circulates. Site includes worksheets that can be copied for practice.

<http://www.mathsisfun.com/geometry/constructions.html> is another resource for construction videos.

Essential Questions:

- What is the difference between drawing and construction?
- How is “Construction” connected to Euclidean Geometry?
- What are the different mathematical methods to approach the same problem?
{*Numerical (arithmetic), Analytical (algebra) , and Geometrical*}

MISCONCEPTIONS (What are the typical errors or difficult areas? Also suggest ways to teach them.)

If a ruler is used as a straightedge, students should not use it to measure. In constructions a ruler is only used to draw straight lines. The only tool that is used to measure in classic compass/straightedge constructions is the compass. Constructions should be taught by the formal compass/straightedge method but also introduced and/or reinforced by an alternative method. (i.e. string, reflective devices, paper folding, dynamic geometric software, etc.)

FUTURE CONNECTIONS (What will they use these skills for later?)

- Learning the basic constructions will help students understand the geometric relationships of figures and allow them to “prove” theorems without formal two-column proofs.

ADDITIONAL EXTENTIONS OR EXPLANATIONS (What needs greater explanation?)

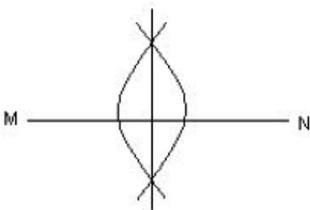
- Students should be given time to practice with the tools of construction in order to find a method of handling them that works best.
- Each of these constructions is utilizing one or more characteristics of a rhombus.
- This unit also considers other tools, such as string, paper folding, etc. As with the compass and straightedge constructions, it is important to both mechanically use these tools and to work toward mathematical explanations and justifications.
- In dynamic geometry environments, a key distinction is between drawing and constructing figures with particular characteristics. For example, if a rectangle is merely drawn to look like a rectangle, then it is

easy to mess up the figure by dragging parts of the diagram. But a rectangle that is “constructed” as a rectangle will remain rectangular even when parts of the figure are dragged about.

- By using the “compass and straightedge” Euclid and the Greeks solved problems graphically, by drawing shapes, as a substitute for using arithmetic.
- Because the steps for making a construction, along with their justifications, function like the statements and reasons in a proof, the result of the construction is true in the same way that the conclusion of a deductive proof must be true.

ASSESSMENTS (Questions that get to the heart of the objective – multiple choice, short answer, multi-step)

Multiple Choice



What geometric construction is shown in the diagram above?

- A) An angle bisector
- B) A line parallel to the line MN
- C) An angle congruent to a given angle
- D) A perpendicular bisector of segment MN

Short Answer

- 1) What constructions would be necessary to copy a given line segment and construct a 45° angle at its midpoint?
 - copy a segment
 - perpendicular bisector
 - bisect an angle.

Multi-Step Problems

- 1) Copy a given line segment and construct a 45° angle at its midpoint. Show all construction marks and label all congruent parts.
See students' work.
- 2) Explain how you would construct a line segment congruent to a given segment.
Use a compass to transfer the length of the given segment.
- 3) Explain how you would find the midpoint of a given segment.
Using a compass, make arcs above and below the approximate location of the midpoint from both ends of the segment. Connect the arc intersections both above and below the segment with a straightedge. The point at which these two segments intersect is the midpoint of the given segment.
- 4) Explain how you would construct the perpendicular bisector of a given segment.
Using a compass, make arcs above and below the approximate location of the midpoint from both ends of the segment. Connect the arc intersections both above and below the segment with a straightedge. The newly drawn segment is the perpendicular bisector of the given segment.
- 5) Explain how you would construct a line segment perpendicular to a given line segment that goes through a given point.
Using a compass, choose an arbitrary length from the point to the line segment. Create an arc that is this length from the line segment on the side opposite the point. Do this once again from the other side of the segment. With a straightedge, connect the given point to the intersection of the arcs.

- 6) Explain how you would construct a segment perpendicular to a given segment that goes through a point on the given segment.
Select a point clearly not on the given segment and not perpendicular. Using this point as the center, use a compass to draw a circle with the radius as the distance from the chosen point to the given point. It should intersect with the given segment at another location. Draw the diameter of the circle with a straightedge from the circle and segment intersection to the other side of the circle. Connect this final point (on the circle but not on the given segment) with the original point to construct a segment perpendicular to the given segment at that point.
- 7) Explain how you would construct a line segment parallel to a given segment that goes through a given point.
Construct a segment from the given point to the given segment at an arbitrary angle. Using a compass, mark the angle made at the intersection of the segments with an arc and do the same at the given point, where the desired segment will be. Adjusting the compass, mark the distance from the intersections of the arc with the segments. On the intersection between the drawn segment and the arc at the given point, draw an arc that intersects with the arc made above the given segment. These two arcs intersect at a point that, when connected to the given point with a straightedge, is on a line segment parallel to the original.
- 8) Explain how you would construct an angle congruent to a given angle.
Draw a ray. Using a compass, draw an arc that intersects the sides of the given angle. Keeping the compass at the same measure, draw an arc at the endpoint of the drawn ray. Adjusting the compass, measure the distance between the intersections of the arc with the sides of the given angle. Using the intersection of the arc and the drawn ray as the center, draw an arc that intersects with the previous arc. With a straightedge, draw a segment from the endpoint of the ray to the intersection of the two arcs to form a congruent angle.
- 9) Explain how you would construct the angle bisector of a given angle.
Using a compass, draw an arc that intersects the sides of the given angle. Keeping the same measurement, draw arcs on the interior of the angle using the intersections of the arcs with the angle's sides as vertices. The two arcs intersect a point. The segment that connects the vertex of the angle and this point is the angle bisector.
- 10) Explain how you would construct the angle bisectors of each of the three angles in the triangle.
Choose an angle. Using a compass, draw an arc that intersects the sides of the given angle. Keeping the same measurement, draw arcs on the interior of the angle using the intersections of the arcs with the angle's sides as vertices. The two arcs intersect a point. The segment that connects the vertex of the angle and this point is the angle bisector. Repeat this for the two remaining angles.
- 11) Explain how you would construct the orthocenter of a triangle.
This is identical to constructing three perpendicular segments to each of the three the sides of the triangle using the opposite vertex as the given point. The three perpendicular segments should intersect at a single point called the orthocenter.

Why we learn about constructions (Below text taken from Math Open Reference Website - by John Page)



The ancient Greek mathematician **Euclid** is the acknowledged inventor of geometry. He did this over 2000 years ago, and his book "Elements" is still regarded as the ultimate geometry reference. In that work, he uses these construction techniques extensively, and so they have become a part of the geometry field of study. They also provide a greater insight into geometric concepts and give us tools to draw things when direct measurement is not appropriate.

Why did Euclid do it this way?

Why didn't Euclid just measure things with a ruler and calculate lengths? For example, one of the basic constructions is bisecting a line (dividing it into two equal parts). Why not just measure it with a ruler and divide by two?

The answer is surprising. The Greeks could not do arithmetic. They had only whole numbers, no zero, and no negative numbers. This meant they could not for example divide 5 by 2 and get 2.5, because 2.5 is not a whole number - the only kind they had. Also, their numbers did not use a positional system like ours, with units, tens, hundreds, etc., but more like the Roman numerals. In short, they could perform very little useful arithmetic.

So, faced with the problem of finding the midpoint of a line, they could not do the obvious - measure it and divide by two. They had to have other ways, and this led to the constructions using compass and straightedge or ruler. It is also why the straightedge has no markings. It is definitely not a graduated ruler, but simply a pencil guide for making straight lines. Euclid and the Greeks solved problems graphically, by drawing shapes, as a *substitute* for using arithmetic.