



## Graphing Quadratic Functions

### Math Background

#### Previously, you

- Identified and graphed linear functions
- Applied transformations to parent functions
- Graphed quadratic functions

#### In this unit you will

- Graph quadratic functions using transformations
- Graph quadratic functions using standard form

#### You can use the skills in this unit to

- Graph parabolas using translations and identify translations of quadratic functions from a graph and/or table.
- Show the maximum and minimum (vertex) on the graph of a quadratic function.
- Show intercepts and the axis of symmetry on the graphs of quadratic functions.
- Graph parabolas with and without technology.

#### Vocabulary

- **Axis of symmetry** – A line that passes through a figure in such a way that the part of the figure on one side of the line is a mirror reflection of the part on the other side of the line.
- **Dilation** – A transformation in which a figure is enlarged (stretched) or reduced (shrunk).
- **Domain** – The input values of the function. For a quadratic function, the domain is all real numbers.
- **Maxima** – The largest value (highest point) of the function.
- **Minima** – The smallest value (lowest point) of the function.
- **Parabola** – The U-shaped graph of a quadratic function.
- **Parent Quadratic function** – The simplest quadratic function,  $f(x) = x^2$ .
- **Quadratic function** – A function where the highest exponent of the variable is a square.
- **Range** – The output values of a function.
- **Reflection** – A transformation in which every point of a figure is mapped to a corresponding image across a line of symmetry.
- **Transformation** – A change in the position, size, or shape of a figure or graph.
- **Translation** – A figure is moved from one location to another on the coordinate plane without changing its size, shape or orientation.
- **Vertex** – The point of intersection of a parabola and its line of symmetry.
- **X-intercept** – Where the graph of a function intersects the x-axis.
- **Y-intercept** – Where the graph of a function intersects the y-axis.

**Essential Questions**

- How does multiplying by and/or adding a constant to a function change the graph?
- What are the key features of the graphs of quadratic functions?

**Overall Big Ideas**

The location and value of a constant within a function affects its graph. Quadratic equations can be written in vertex form  $y = a(x - h)^2 + k$ , where the vertex of the graph of the equation is at  $(h, k)$  and the axis of symmetry is the line  $x = h$ . The value of  $k$  determines the graph's vertical translation. The value of  $h$  determines the graph's horizontal translation. The value of  $a$  determines the direction of opening of the graph and the shape of the parabola.

A quadratic function is represented by a U-shaped curve, called a parabola, intercepts one or both axes and has one maximum or minimum value.

**Skill**

**To use transformations to graph quadratic functions.**

**To use standard form to graph quadratic functions.**

**Related Standards****F.IF.C.7a**

Graph linear and quadratic functions and show intercepts, maxima, and minima. \*(Modeling Standard)

**F.BF.B.3-1**

Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ ,  $f(x + k)$  for special values of  $k$  (both positive and negative) and find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include linear, exponential, quadratic, and absolute value functions.



### Notes, Examples, and Exam Questions

**Parent Function:** The most basic quadratic function is  $f(x) = x^2$ . This function is often called the parent quadratic function. Once you understand the parent function you can shift, reflect, and stretch the parent graph to get graphs of other quadratic functions.

The graph of the function  $y = f(x)$  is the set of all points  $(x, f(x))$ ,  $x$  is in the domain of  $f$ . We match domain values along the  $x$ -axis with their range values along the  $y$ -axis to get the ordered pairs that yield the graph of  $y = f(x)$ .

#### Graphing a Parabola Using a Table

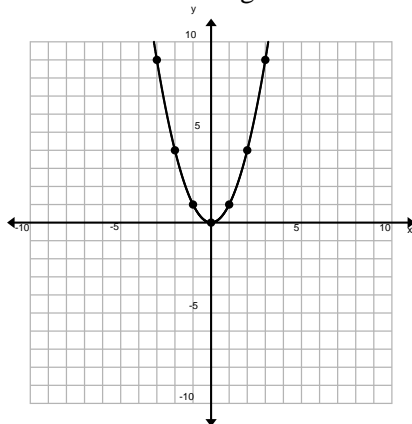
##### Ex 1

Graph the **parent quadratic function**  $y = x^2$ .

**Step One:** Make a table of values (t-chart).

<b>x</b>	-3	-2	-1	0	1	2	3
<b>y</b>	9	4	1	0	1	4	9

**Step Two:** Plot the points on a coordinate grid and connect to draw the parabola.



Note: The vertex is a minimum at  $(0, 0)$ , and the axis of symmetry (the vertical line that passes through the vertex) is  $x = 0$ .

**What is the domain (the input values) of  $f(x) = x^2$ ?** {all real numbers} **What is the range (the output values)?**  
 $\{y \mid y \geq 0\}$

**For what value of  $x$  is  $f(x) = x^2$  increasing? For what values of  $x$  is it decreasing?**

$f(x) = x^2$  is increasing for  $x > 0$  and decreasing for  $x < 0$ .

**Why is  $(0, 0)$  the vertex of the graph of  $f(x) = x^2$ ?** It is the lowest point on the parabola.



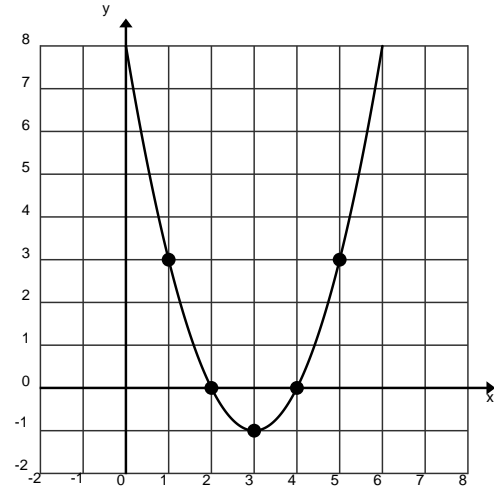
### Graphing Quadratic Functions Using a Table

#### Ex 2

Graph  $f(x) = x^2 - 6x + 8$  by using a table.

Make a table. Plot enough ordered pairs to see both sides of the curve.

$x$	$f(x) = x^2 - 6x + 8$	$(x, f(x))$
<b>1</b>	$f(1) = 1^2 - 6(1) + 8 = 3$	$(1, 3)$
<b>2</b>	$f(2) = 2^2 - 6(2) + 8 = 0$	$(2, 0)$
<b>3</b>	$f(3) = 3^2 - 6(3) + 8 = -1$	$(3, -1)$
<b>4</b>	$f(4) = 4^2 - 6(4) + 8 = 4$	$(4, 0)$
<b>5</b>	$f(5) = 5^2 - 6(5) + 8 = 3$	$(5, 3)$



**Vertex:**  $(3, -1)$     **Axis of Symmetry:**  $x = 3$

**Domain:** All real numbers    **Range:**  $y \geq -1$

#### Ex 3

Graph the parabola  $y = -x^2$

**Step One:** Make a table of values (t-chart).

<b>x</b>	-3	-2	-1	0	1	2	3
<b>y</b>	-9	-4	-1	0	-1	-4	-9

**Step Two:** Plot the points on a coordinate grid and connect with a smooth curve to draw the parabola.

Note: The vertex is a maximum at  $(0, 0)$ , and the axis of symmetry is  $x = 0$ .

**Describe the transformation of the parent graph  $f(x) = x^2$ .**

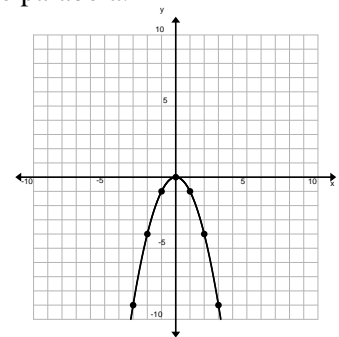
It is a reflection across the x axis.

**What is the domain of  $f(x) = x^2$ ?** {all real numbers}    **What is the range?**  $\{y \mid y \leq 0\}$

**For what value of  $x$  is  $f(x) = x^2$  increasing?** For what values of  $x$  is it decreasing?

$f(x) = x^2$  is increasing for  $x < 0$  and decreasing for  $x > 0$ .

**Why is  $(0, 0)$  the vertex of the graph of  $f(x) = -x^2$ ?** It is the highest point on the parabola.





## Reflection, Shrinking and Stretching, Vertical and Horizontal Translations

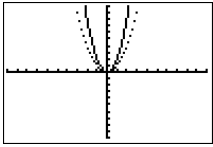
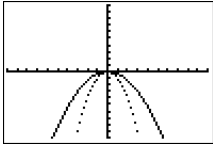
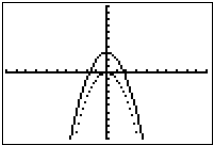
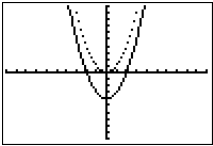
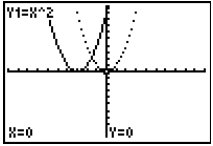
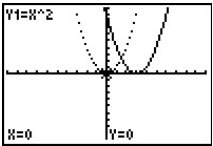
Comparing  $y = x^2$  and  $y = -x^2$ : The vertex is  $(0,0)$ , and the axis of symmetry is  $x = 0$  for both graphs. When  $a$  is **positive**, the parabola opens **up** and its vertex is a **minimum**; when  $a$  is **negative**, the parabola opens **down** and its vertex is a **maximum**.

### DISCOVERY:



**Calculator Exploration: Transformations with  $y = x^2$  and  $y = -x^2$ .**

Use the graphing calculator to investigate the graphs of quadratic functions. Describe the effect on the graphs of  $y = x^2$  and  $y = -x^2$ . (Note: In the calculator graphs shown,  $y = x^2$  or  $y = -x^2$  is graphed as a dotted line.)

- |                          |   |   |
|--------------------------|---|---|
| 1. $y = 2x^2$            |    | Compare to $y = x^2$<br>Vertex: Same $(0,0)$ Opens <i>narrower</i> than $y = x^2$         |
| 2. $y = -\frac{1}{3}x^2$ |   | Compare to $y = -x^2$<br>Vertex: Same $(0,0)$ Opens <i>wider</i> than $y = -x^2$          |
| 3. $y = -x^2 + 3$        |  | Compare to $y = -x^2$<br>Vertex: <i>Up 3</i> $(0,3)$ Opens <i>the same as</i> $y = -x^2$  |
| 4. $y = x^2 - 4$         |  | Compare to $y = x^2$<br>Vertex: <i>Down 4</i> $(0,-4)$ Opens <i>the same as</i> $y = x^2$ |
| 5. $y = (x+3)^2$         |  | Compare to $y = x^2$<br>Vertex: <i>left 3</i> $(-3,0)$ Opens <i>the same as</i> $y = x^2$ |
| 6. $y = (x-3)^2$         |  | Compare to $y = x^2$<br>Vertex: <i>right 3</i> $(3,0)$ Opens <i>the same as</i> $y = x^2$ |



### Transformations of Quadratic Functions

$$y = a(x - h)^2 + k$$

$a$  indicates a reflection across the x-axis and/or a vertical stretch or compression.

$h$  indicates a horizontal translation

$k$  indicates a vertical translation

Vertical Stretch:  $|a| > 1$

Vertical Shrink:  $0 < |a| < 1$

Reflection over x-axis:  $-a$

Reflection over y-axis:  $-x$

Horizontal Translation:  $h < 0$  moves left       $h > 0$  moves right

Vertical Translation:  $k < 0$  moves down       $k > 0$  moves up

**Other:** Axis of Symmetry  $x = h$       Vertex is  $(h, k)$

If  $a$  is positive the parabola opens up. If  $a$  is negative the parabola opens down.

**Ex 4** Describe the shift, reflect and stretch of the parent function.  $f(x) = x^2$

a.  $f(x) = \frac{1}{4}(x+5)^2$

- Vertical Shrink by a factor of  $\frac{1}{4}$  (wider)
- Horizontal Shift 5 units left.

b.  $f(x) = (x-2)^2 - 1$

- Horizontal Shift 2 units right
- Vertical Shift 1 units down

c.  $f(x) = -2(x-3)^2 + 4$

- Reflect across the x axis
- Vertical Stretch by a factor of 2
- Horizontal Shift 3 units right
- Vertical Shift 4 units up



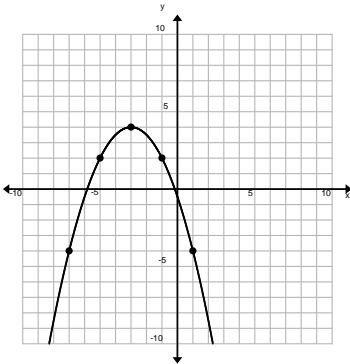
### Ex 5 Graphing a Quadratic Function in Vertex Form

Graph the quadratic function  $y = -\frac{1}{2}(x+3)^2 + 4$ . State the vertex and axis of symmetry.

**Step One:** Determine if the graph opens up or opens down. Because  $a$  is  $-\frac{1}{2}$  the graph opens down.

**Step Two:** Identify the vertex and axis of symmetry. Note: Another way of writing the function is  $y = -\frac{1}{2}(x - (-3))^2 + 4$ . So the vertex is  $(-3, 4)$  and the axis of symmetry is  $x = -3$ .

**Step Three:** Plot the vertex and sketch the graph. Notice the parent graph is reflected over the  $x$  axis, is wider (vertical shrink), shifted to the left 3 (opposite of what you think), and is translated up four units.



**Optional:** Make a table of values. When choosing  $x$ -values for the T-table, use the vertex, a few values to the left of the vertex, and a few values to the right of the vertex. (Note: Because of the fraction, you may want to choose values that will guarantee whole numbers for the  $y$ -coordinates.) Five to seven points will give a nice graph of the parabola.

$x$	-9	-7	-5	-3	-1	1	3
$y$	-14	-4	2	4	2	-4	-14

### Ex 6 Writing the Equation of a Quadratic Function in Vertex Form

Write an equation for the parabola in vertex form.

The vertex is at  $(1, -7)$ . So the vertex form of the equation is  $y = a(x-1)^2 - 7$ .

To solve for  $a$ , we will choose a point on the parabola and substitute it into the equation for  $(x, y)$ .

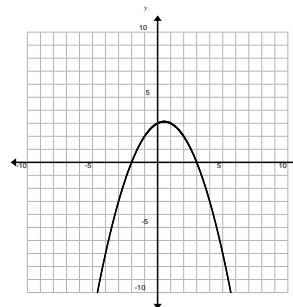
$$2 = a(4-1)^2 - 7$$

Choose  $(4, 2)$ .

$$2 = 9a - 7$$

$$1 = a$$

So the vertex form of the equation is  $y = (x-1)^2 - 7$ .



### Ex 7 Automotive Application

The minimum braking distance  $d$  in feet for a vehicle on dry concrete is approximated by the function  $d(v) = .035v^2$ , where  $v$  is the vehicle's speed in miles per hour. If the vehicle's tires are in poor condition, the braking-distance function is  $d_p(v) = .065v^2$ . What kind of transformation describes this change and what does the transformation mean?



Examine both functions in vertex form.

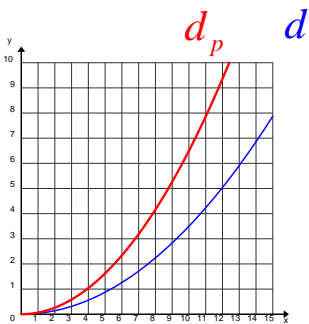
$$d(v) = .035(v-0)^2 + 0$$

$$d_p(v) = .065(v-0)^2 + 0$$

The value  $a$  has increased from .035 to .065. The increase indicates a vertical stretch.

Find the stretch factor by comparing the new  $a$ -value to the old  $a$  value:

$$\frac{d_p(v)}{d(v)} = \frac{0.065}{0.035} \approx 1.857$$



The function  $d_p$  represents a vertical stretch of  $d$  by a factor of approximately 1.9. Because the value of each function approximates braking distance, a vehicle with tires in poor condition takes about 1.9 times as many feet to stop as a vehicle with good tires does.

**Graph:** Graph both functions on a graphing calculator. The graph of  $d_p$  appears to be vertically stretched compared with the graph of  $d$ .

**Explore:** The height  $h$  in feet of a baseball on Earth after  $t$  seconds can be modeled by the function

$$h(t) = -16(t-1.5)^2 + 36 \text{ where } -16 \text{ is a constant in } ft/s^2 \text{ due to Earth's gravity.}$$

- What if...? The gravity on Mars is only 0.38 times that on Earth. If the same baseball were thrown on Mars, it would reach a maximum height 59 feet higher and 2.5 seconds later than on Earth. Describe the transformations that must be applied to make the function model the height of the baseball on Mars.
- Write a height function for the baseball thrown on Mars.

**QOD:** Describe the shift, reflect, and stretch of a quadratic equation in vertex form.

**Write About It:** Describe the graph of  $f(x) = 999,999(x+5)^2 + 5$  without graphing it.

The graph would be a very narrow parabola opening upward with its vertex at  $(-5, 5)$

**Closure:** What type of graph would a function of the form  $y = a(x-h)^2 + k$  have if  $a = 0$ ? What type of function would it be?





## Graphing a Quadratic Function in Standard Form

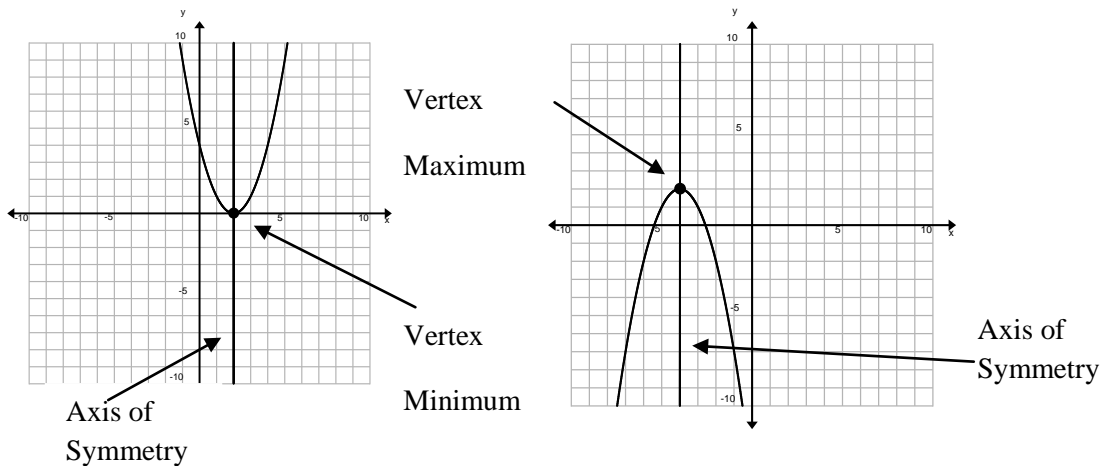
Standard Form of a Quadratic Function  $y = ax^2 + bx + c$ , when  $a \neq 0$ ;  $a$ ,  $b$ , and  $c$  are real numbers

Vertex: the vertex is the point  $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$       Axis of Symmetry:  $x = -\frac{b}{2a}$       y-intercept:  $c$

Minimum Value: When a parabola opens upward, the y-value of the vertex is the minimum value.

Maximum Value: When a parabola opens downward the y-value of the vertex is the maximum value.

Axis of Symmetry: the vertical line that passes through the vertex of a quadratic function.



$D: \{x \mid x \in \mathbb{R}\}$  The domain is all real numbers  
 $R: \{y \mid y \geq k\}$  The range is all values greater than or equal to the minimum.

$D: \{x \mid x \in \mathbb{R}\}$  The domain is all real numbers  
 $R: \{y \mid y \leq k\}$  The range is all values less than or equal to the maximum.

### Ex 8

Graph the quadratic function  $y = x^2 - 6x - 1$ . State the vertex and axis of symmetry.

**Step One:** Determine whether the graph opens upward or downward. If  $a$  is positive the graph opens upward. If  $a$  is negative the graph opens downward. Since  $a=1$  the parabola opens upward.

**Step Two:** Find the axis of symmetry. (the  $x$ -coordinate of the vertex)  $x = -\frac{b}{2a}$

$$a = 1, b = -6 \quad x = -\frac{-6}{2(1)} = 3 \quad \text{The axis of symmetry is the line } x = 3$$



**Step Three:** Find the vertex.

The vertex lies on the axis of symmetry, so the  $x$ -coordinate is 2. The  $y$ -coordinate is the value of the function at this  $x$ -value, or  $f(3)$ .

$$f(3) = (3)^2 - 6(3) - 1 = -10$$

The vertex is  $(3, -10)$

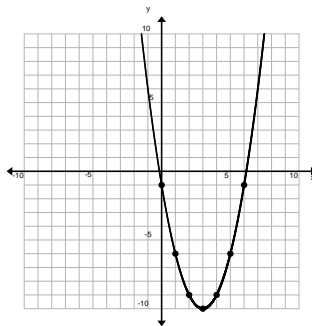
**Step Four:** Find the  $y$ -intercept.

Because  $c = -1$ , the  $y$ -intercept is  $-1$ .

**Step Five:** Graph by sketching the axis of symmetry and then plotting the vertex and the intercept point  $(0, -1)$ . Use the axis of symmetry to find another point on the parabola. Notice that  $(0, -1)$  is 3 units left of the axis of symmetry. The point on the parabola symmetrical to  $(0, -1)$  is 3 units to the right of the axis at  $(6, 0)$ . Connect points with a smooth curve to draw the parabola.

Vertex:  $(3, -10)$

Axis of Symmetry:  $x = 3$



**Optional:** To check, make a table of values. When choosing  $x$ -values for your T-table, use the vertex, a few values to the left of the vertex, and a few values to the right of the vertex.

$x$	0	1	2	3	4	5	6
$y$	-1	-6	-9	-10	-9	-6	-1

Note: When calculating the  $y$ -coordinate of points to the right and left of the vertex, notice the symmetry.

### Ex 9

Find the minimum or maximum value of  $f(x) = 2x^2 - 2x + 5$ . State the domain and range.

**Step One:** Determine whether the function has a minimum or maximum value.

Because  $a$  is positive, the graph opens upward and has a minimum value.



**Step Two:** Find the x-value of the vertex.

$$x = \frac{-b}{2a} = \frac{-(-2)}{2(2)} = \frac{2}{4} = \frac{1}{2}$$

**Step Three:** Then find the y-value of the vertex,  $f\left(\frac{-b}{2a}\right)$ .

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 5 = \frac{9}{2} = 4\frac{1}{2}$$

Minimum Value:  $4\frac{1}{2}$  or 4.5.

Domain: all real numbers,  $\mathbb{R}$ . Range: all real numbers greater than or equal to  $4\frac{1}{2}$   $\{y \mid y \geq 4.5\}$  or  $[4.5, \infty)$

Check with Graphing Calculator. Graph  $f(x) = 2x^2 - 2x + 5$ . The graph and table support the answer.



### Using a Quadratic Model

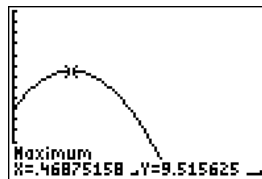
#### Ex 10

A basketball's path can be modeled by  $y = -16x^2 + 15x + 6$ , where  $x$  represents time (in seconds) and  $y$  represents the height of the basketball (in feet). What is the maximum height that the basketball reaches?

Press the “Y=” key and graph the function in  $Y_1$  and find the maximum (in the CALC menu). The maximum is the vertex. The maximum height of the basketball is the y-coordinate of the vertex, which is approximately 9.5 ft.

```

WINDOW
Xmin=0
Xmax=2
Xscl=1
Ymin=0
Ymax=15
Yscl=1
Xres=1
  
```



#### Ex 11

A baseball is thrown with a vertical velocity of 50 ft/sec from an initial height of 6 ft. The height  $h$  in feet of the baseball can be modeled by  $h(t) = -16t^2 + 50t + 6$ , where  $t$  is the time in seconds since the ball was thrown.

**Approximately how many seconds does it take the ball to reach its maximum height?**

About 1.6 seconds

**What is the maximum height that the ball reaches?**

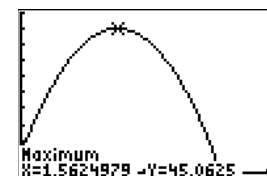
About 45 ft.

```

Plot1 Plot2 Plot3
Y1=-16X^2+50X+6
Y2=
Y3=
Y4=
Y5=
  
```

```

WINDOW
Xmin=0
Xmax=4
Xscl=1
Ymin=-1
Ymax=50
Yscl=5
Xres=
  
```





**Closure:** Show equivalent quadratic functions in both standard form and vertex form. Review how to use each form to determine the y-intercept, axis of symmetry, vertex, and maximum/minimum value. Use a graph to check.

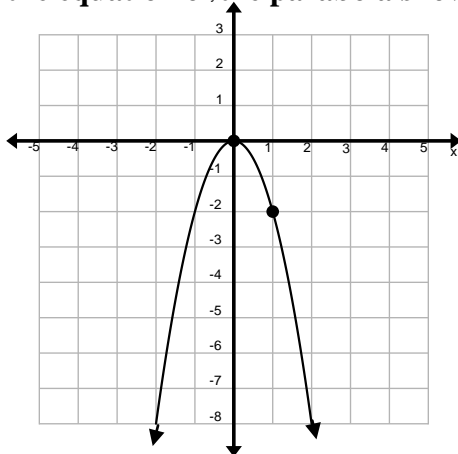
$$f(x) = 2x^2 - 4x + 6$$

$$f(x) = 2(x-1)^2 + 4$$

<b>y-intercept:</b> 6	<b>Axis of Symmetry:</b> $x = 1$
<b>Vertex:</b> (1, 4)	<b>Minimum Value:</b> 4

### SAMPLE EXAM QUESTIONS

1. What is the equation of the parabola shown?



- |                           |                           |
|---------------------------|---------------------------|
| (A) $y = -4x^2$           | (B) $y = -2x^2$           |
| (C) $y = -\frac{1}{2}x^2$ | (D) $y = -\frac{1}{4}x^2$ |

Ans: B

2. Find the vertex of  $y = x^2 + 2x - 3$  and state if it is a maximum or a minimum.

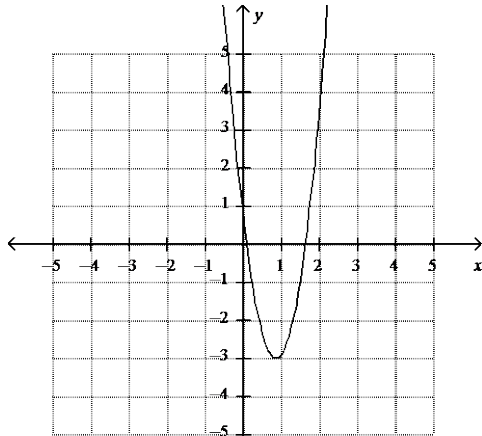
- (A) (-1, -4); maximum
- (B) (-1, -4); minimum
- (C) (-4, -1); maximum
- (D) (-4, -1); minimum

Ans: B

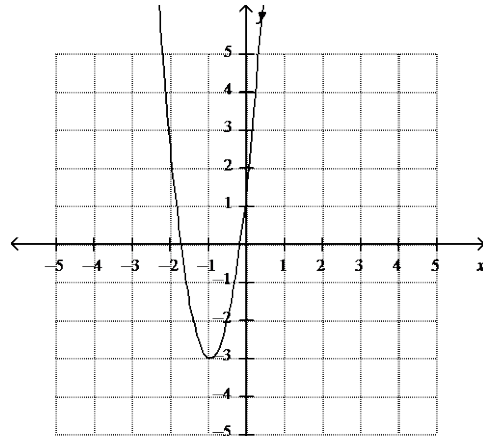


3. Create a table for the quadratic function  $f(x) = 5x^2 + 9x + 1$ , and use it to graph the function.

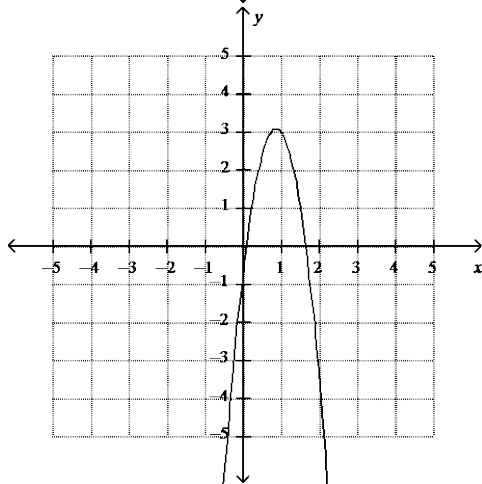
a.



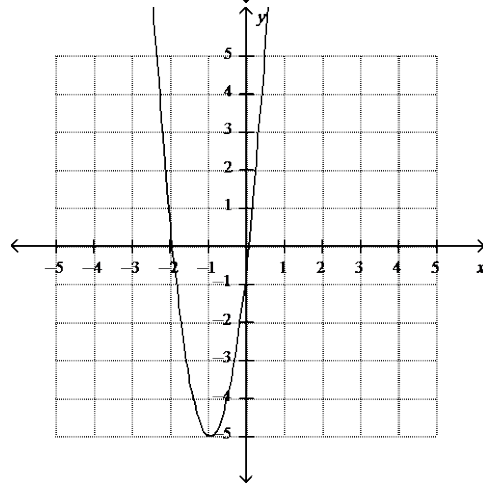
c.



b.

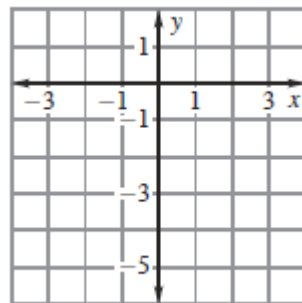


d.

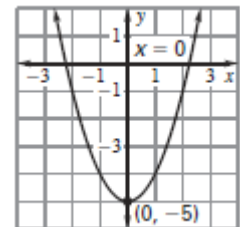


Ans: C

4. Graph the function  $y = x^2 - 5$ . Label the vertex and axis of symmetry.



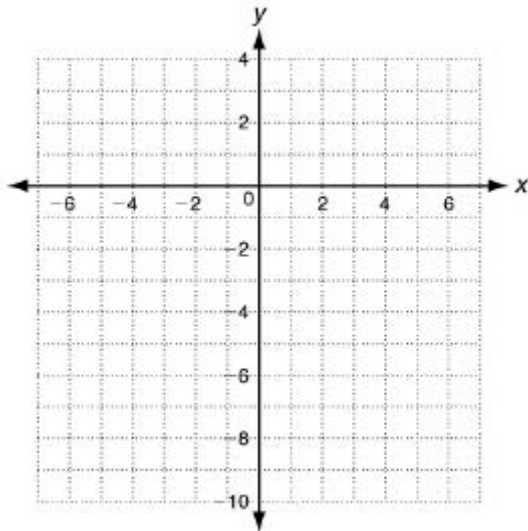
Ans:





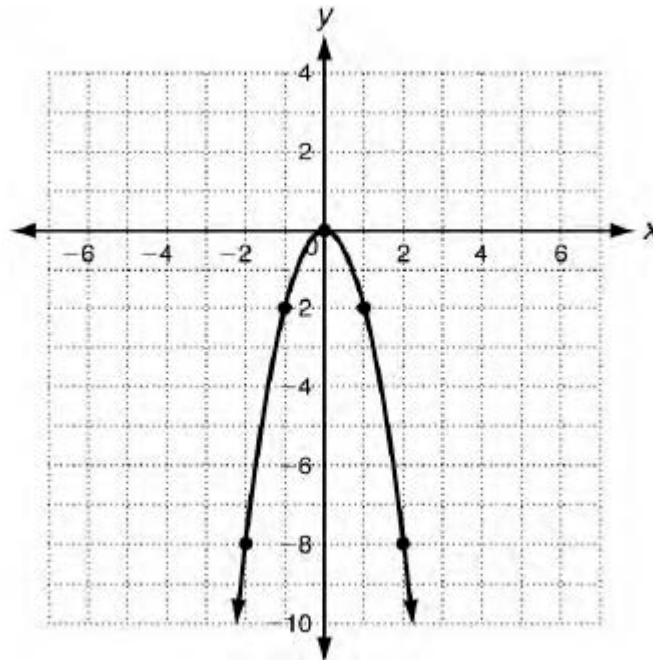
5. Use a table of values to graph  $y = -2x^2$

<b>x</b>	-2	-1	0	1	2
<b>y</b>					



Ans:

<b>x</b>	-2	-1	0	1	2
<b>y</b>	-8	-2	0	-2	-8





6. How would the graph of the function  $y = x^2 - 3$  be affected if the function were changed to  $y = x^2 - 2$ ?

- The graph would shift 2 units up.
- The graph would shift 5 units up.
- The graph would shift 2 units to the right.
- The graph would shift 5 unit down.

Ans: B

7. How would you translate the graph of  $y = -x^2$  to produce the graph of  $y = -x^2 - 4$ ?

- translate the graph of  $y = -x^2$  down 4 units
- translate the graph of  $y = -x^2$  up 4 units
- translate the graph of  $y = -x^2$  left 4 units
- translate the graph of  $y = -x^2$  right 4 units

Ans: A

8. Which transformation from the graph of a function  $f(x)$  describes the graph of  $\frac{1}{3}f(x)$ ?

- |   |  |
|---|--|
| a. horizontal shift left $\frac{1}{3}$ unit | c. vertical compression by a factor of $\frac{1}{3}$ |
| b. vertical shift up $\frac{1}{3}$ unit     | d. vertical shift down $\frac{1}{3}$ unit            |

Ans: C

9. Identify the vertex of  $g(x) = (x + 14)^2 - 8$ .

- |                |               |
|----------------|---------------|
| a. $(-14, -8)$ | c. $(14, -8)$ |
| b. $(-14, 8)$  | d. $(14, 8)$  |

Ans: A

10. Use this description to write the quadratic function in vertex form:

The parent function  $f(x) = x^2$  is vertically stretched by a factor of 3 and translated 8 units right and 1 unit down.

- |                                      |                            |
|--------------------------------------|----------------------------|
| a. $g(x) = \frac{1}{3}(x - 8)^2 - 1$ | c. $g(x) = 3(x - 8)^2 - 1$ |
| b. $g(x) = \frac{1}{3}(x + 8)^2 - 1$ | d. $g(x) = 3(x + 8)^2 - 1$ |

Ans: C

11. What is the maximum of the quadratic function  $f(x) = -2x^2 + 4x + 6$ ?

- |               |               |
|---------------|---------------|
| A. $f(x) = 1$ | C. $f(x) = 3$ |
| B. $f(x) = 6$ | D. $f(x) = 8$ |

Ans: D



**12. Use this description to write the quadratic function in vertex form:**

The parent function  $f(x) = x^2$  is vertically compressed by a factor of  $\frac{1}{2}$  and translated 11 units left and 5 units down.

a.  $g(x) = \frac{1}{2}(x - 11)^2 - 5$

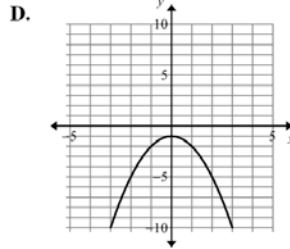
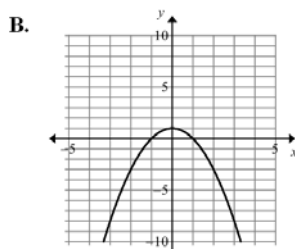
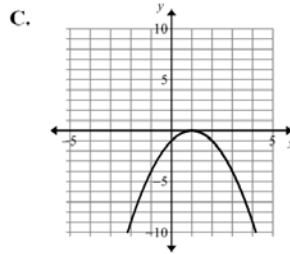
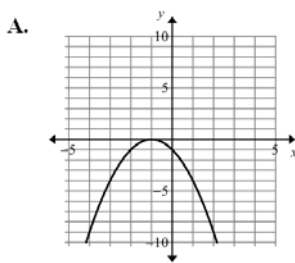
c.  $g(x) = 2(x - 11)^2 - 5$

b.  $g(x) = \frac{1}{2}(x + 11)^2 - 5$

d.  $g(x) = 2(x + 11)^2 - 5$

Ans: B

**13. Which graph represents  $f(x) = -x^2 + 2x - 1$ ?**



Ans: C

**14. Consider  $h(x) = 3x^2 + 6x + 2$ . What are its vertex and y-intercept?**

a. vertex: (-1, -1), y-intercept: (0, 2)

c. vertex: (1, 1), y-intercept: (0, 2)

b. vertex: (-2, 2), y-intercept: (0, -2)

d. vertex: (-2, 1), y-intercept: (0, -2)

Ans: A

**Sample Essay Question:**

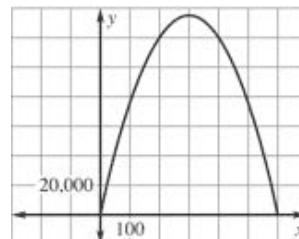
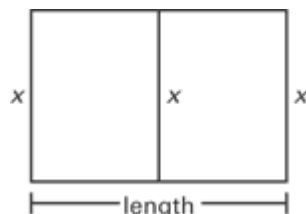
**15.** Rick uses 1800 feet of fencing to build a rectangular pen. He divides the pen into two sections that have the same area. Let  $x$  be the width (in feet) of the pen, as shown in the drawing.

**Part A:** Write an expression to represent the length of the pen in terms of  $x$ . Justify your work.

**Part B:** Write an equation for the area  $y$  of the pen in terms of  $x$ . Graph the equation using a graphing calculator.

**Part C:** Does the function in **Part B** have a maximum or a minimum value? Explain.

**Part D:** Rick wants the pen to have the largest possible area. What width  $x$  should he use? What is the area of the pen with the largest area?







Ans:

**Part A:**  $900 - 1.5x$ ; Rick has 1800 feet of fencing. He uses  $3x$  feet to make two sides and a divider for the pen. So, the amount of fencing he has to make the other two sides of the fence is  $1800 - 3x$ . Divide  $1800 - 3x$  by 2 to get the length of one of the two sides:  $900 - 1.5x$ .

**Part B:**  $y = x(900 - 1.5x)$ , or  $y = -1.5x^2 + 900x$ ; see graph below.  $y = -1.5x^2 + 900x$ ; use  $x$ -scale  $0 \leq x \leq 700$  with intervals of 50, and  $y$ -scale  $0 \leq y \leq 160,000$  with intervals of 20,000.]

**Part C:** The function has a maximum value because the value of the coefficient of  $x^2$  is negative.

**Part D:** 300 feet; 135,000 square feet