Math 7 GEOMETRY
Three Dimensional Figures will include:

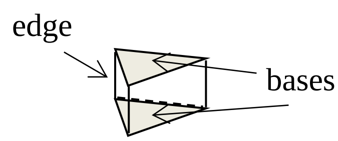
NACS Standard #	**Standard**
7.G.A.3	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
7.G.B.6	Solve real-world and mathematical problems involving area, volume and surface area for two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Math 7, Geometry
Three-Dimensional Figures *Notes*

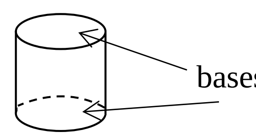
Prep for 7.G.A.3 and 7.G.B.6
A ***solid*** is a three-dimensional figure that occupies a part of space. The polygons that form the sides of a solid are called ***faces***. Where the faces meet in segments are called ***edges***. Edges meet at ***vertices***.

There are types of solids that students should be able to identify at the 7th grade level: prisms, cylinders, pyramids, spheres and cones.

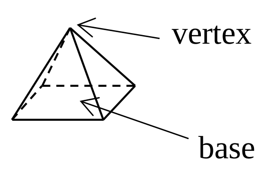
A ***prism*** is a solid formed by polygons. The faces are rectangles. The bases are congruent polygons that lie in parallel planes.



A ***cylinder*** is a solid with two congruent circular bases that lie in parallel planes.

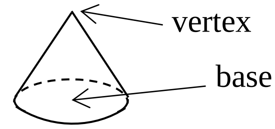


A ***pyramid*** is a solid whose base may be any polygon, with the other faces triangles.



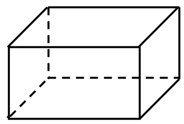
A ***sphere*** is a solid with all points in space equidistant from a given point, called its center.

A ***cone*** is a solid with one circular base and a surface that comes to a point called the vertex.



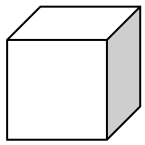
The most common **prisms** students should know are the:

Rectangular Prism



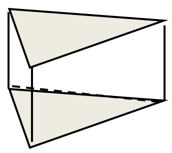
Notice there are two rectangular bases and 4 more rectangular faces.

Cube

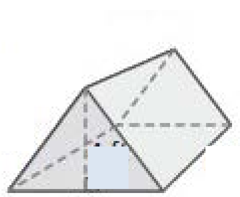


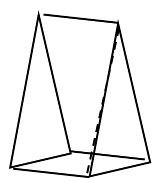
Notice there are 2 square bases and 4 more square faces.

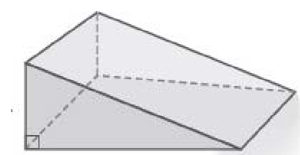
Triangular Prism Notice there are 2 triangular bases and 3 rectangular faces.



Note: Sometimes students have difficulty recognizing the triangular prism when the figure is orientated where the triangular bases are not at the 'bottom', like the following.



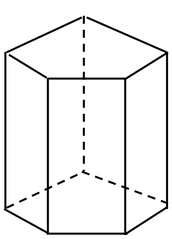




Be sure to give them exposure to these different orientations.

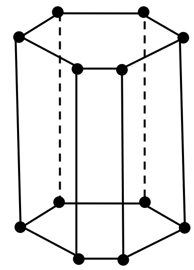
Naming Prisms

Although students are expected to know the common prisms above, prisms are not limited to those above. Note the **prism name is determined by the name of the base.**



Since the bases are pentagons, this is known as a pentagonal prism.

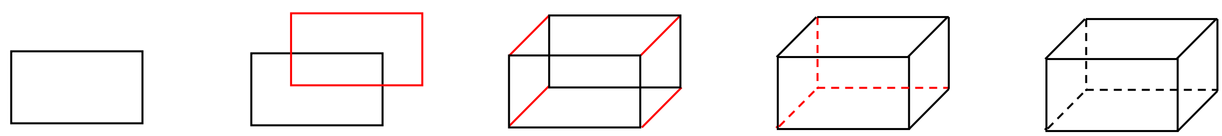
Here the bases are hexagons, this is known as a hexagonal prism.



Reflection?
What would a prism whose bases are octagons and faces are rectangles, be called?
What would a prism whose bases are decagons and faces are rectangles, be called?

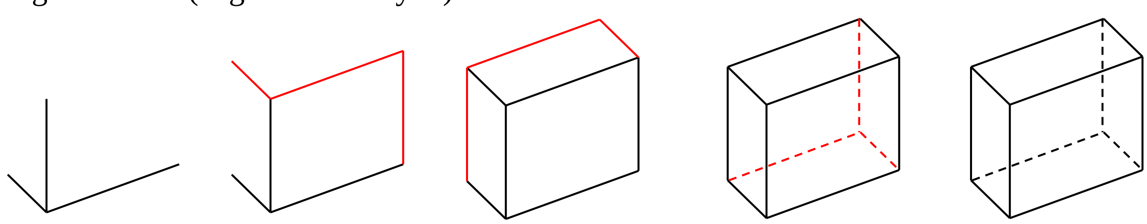
We further discuss ***polyhedrons:*** solids with all faces as polygons. Prisms and pyramids would meet this criterion, while cylinders and cones would *not*.

The ability to draw three-dimensional figures is an important visual thinking tool. "A picture is worth a thousand words." Here are some drawing tips:
Rectangular Prism (face closest to you):

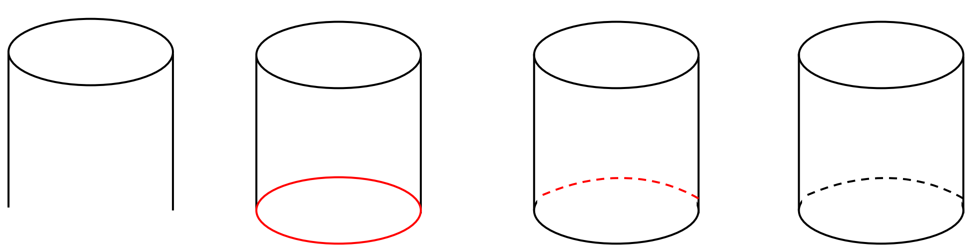


Draw the front rectangle.	Draw a congruent rectangle in another position.	Connect the corners of the rectangles.	Use dashed lines to show the edges you would not see.	Your rectangular prism!

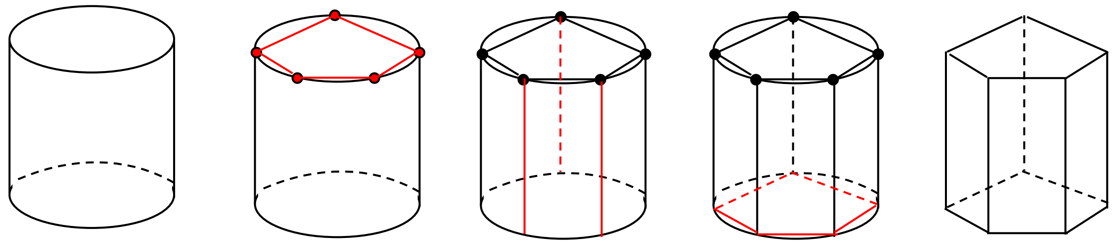
Rectangular Prism (edge closest to you):



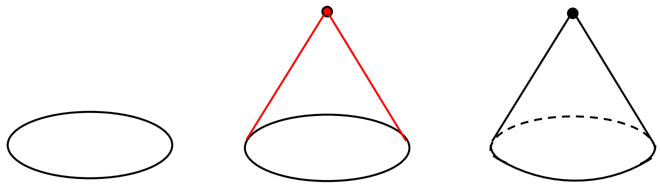
Cylinder:



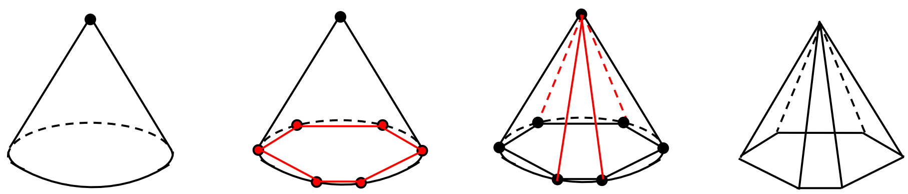
Pentagonal prism:



Cone:



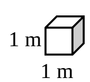
Hexagonal Pyramid:



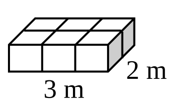
7.G.B.6 Solve real-world and mathematical problems involving area, volume and surface area for two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Volume

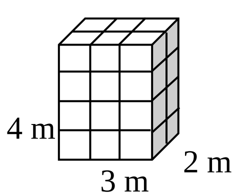
If you were to buy dirt for your yard, it's typically sold in cubic yards—that's describing volume. If you were laying a foundation for a house or putting in a driveway, you'd want to buy cement, and cement is often sold by the cubic yard. Carpenters, painters and plumbers all use volume relationships.



The volume of a three dimensional figure measures how many cubes will fit inside it. It's easy to find the volume of a solid if it is a rectangular prism with whole number dimensions. Let's consider a figure 3 m x 2 m x 4 m.



We can count the cubes measuring 1 meter on an edge. The bottom layer is 3 x 2—there are 6 square meter cubes on the bottom layer.



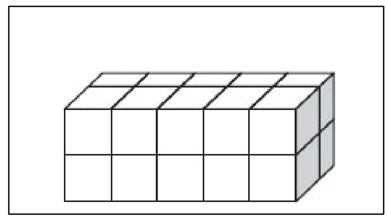
We have three more layers stacked above it (for a total of 4 layers), or
6 + 6 + 6 + 6 = 24.

Now we can reason that if I know how many cubes are in the first layer (6), then to find the total number of cubes in the stack, you simply multiply the number on the first layer by the height of the stack $(6 \cdot 4 = 24)$.

This is a way of finding volume. We find the area of the base (B) and multiply it times the height (h) of the object.

For **prisms and cylinders**, $V = Bh, \text{where } B \text{ is the area of the base and } h \text{ is the height}.$
We can also state that for **pyramids and cones**,
$V = \frac{1}{3}Bh, \text{where } B \text{ is the area of the base and } h \text{ is the height}.$ There are many activities that can be incorporated to drive home this formula. Using water, salt, sugar, or sand students can fill cones (or pyramids) and pour the contents into cylinders (or prisms) to see that it takes three full pours into the cylinder (or prism). (Of course the heights and base areas must be equal.) This activity provides great concept development as opposed to just telling students the formula.

Example: Find the volume of the rectangular prism shown.



Continuing from what students learned in 6th grade, they may begin by counting each square. Others will group, for example, along the front of the base we count 5 cubes in length, 2 cubes wide so that makes 10 cubes for the base. Since the stack is two cubes high , we double that for a total of 20 cubes.

For some students they may want to count the 10 cubes on the top (since they are visible) and then doubling that for the bottom layer they will still get 20 cubic units.

In each case, the volume is 20 cubic units.

After a number of examples we want to show a faster method. Using the formula, **Volume = length *x* width *x* height or *V=lwh*.** Using the figure above we would see:

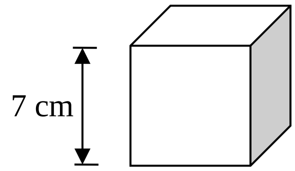
$$V = lwh$$
$$V = 5 \bullet 2 \bullet 2$$
$$V = 20$$

The volume is 20 cubic units.

Quickly we want to move students to use the formula **Volume = (Area of the base) height,** which makes sense since *Area of the base* is, in this case, the *length times width*.

Students may be given figures without dimensions that they must measure to find the length, width and height to the nearest millimeter, centimeter, inch, etc.

Example: Find the volume of the cube shown.

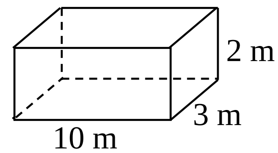


The bases of the cube are squares, so to find the area of the base we will use the formula $A = Bh$. *The height will be the distance between the two bases (7). We have:*

$$V = Bh$$
$$V = (bh)h$$
$$V = (7)(7)(7)$$
$$V = 343$$

The volume of the cube is 343 cubic cm.

Example: Find the volume of the rectangular prism shown.

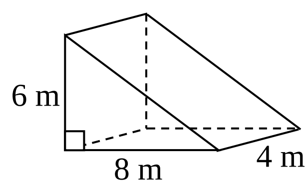


The bases of the prism are rectangles, so to find the area of the base we will use the formula $A = bh$. *The height will be the distance between the two bases (2). We have:*

$$V = Bh$$
$$V = (bh)h$$
$$V = (10)(3)(2)$$
$$V = 60$$

The volume of the rectangular prism is 60 cubic meters.

Example: Find the volume of the prism shown.



The bases of the prism are the triangles, so to find the area of the base we will use the formula $A = \frac{1}{2}bh$. *The height will be the distance between the two bases (4). We have:*

$$V = Bh$$
$$V = (\frac{1}{2}bh)h$$
$$V = \frac{1}{2}(8)(6)(4)$$
$$V = 96$$

The volume of the triangular prism is 96 cubic meters.

Reflection: What is the name of this prism? Why?

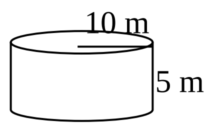
Example: Find the volume of a chocolate cake that has a diameter of 24 cm and a height of 14 cm. Use $\frac{22}{7}$ as an approximation for π.

$V = Bh$. *Our base is a circle, so we will need the radius. The radius is one-half the diameter so* $r = \frac{1}{2}(24)$ or 12. *We would now have*

$$V = Bh$$
$$V = \pi r^2 h$$
$$V = \left(\frac{22}{7}\right)(12)^2(14)$$
$$V = 6336$$

The volume of the chocolate cake is 6336 cubic centimeters.

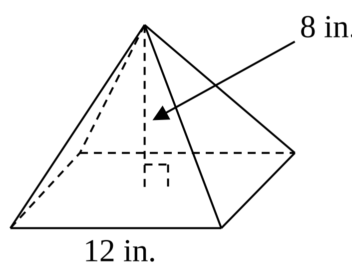
Example: Find the volume of the cylinder.
Use $\pi = 3.14$.



$$V = Bh$$
$$V = (\pi r^{2)}h$$
$$V = (3.14 \bullet 10^2)5$$
$$V = 3.14 \bullet 10^2 \bullet 5$$
$$V = 3.14 \bullet 100 \bullet 5$$
$$V = 314 \bullet 5$$
$$V = 1570$$

The volume of the cylinder is 1,570 cubic meters.

Example: Find the volume of the square pyramid. (Named for the base.)



$$V = \frac{1}{3}Bh$$
$$V = \frac{1}{3}(12)(12)(8)$$
$$V = 384$$

The volume of the square pyramid is 384 cubic inches.

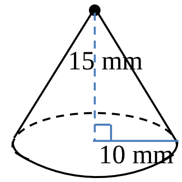
Reflection: Why isn't this figure a (rectangular) square prism?

Example: The volume of a rectangular prism is 24 cubic feet. If the base area is 6 square feet, find the height.

$$V = Bh$$
$$24 = 6h$$
$$4 = h$$

The height of the rectangular prism is 4 feet.

Example: Find the volume of the cone. (Use $\pi \approx 3.14$)



$$V = \frac{1}{3}Bh$$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}(3.14)(10^2)(15)$$

$$V = \frac{1}{3}(3.14)(100)(15)$$

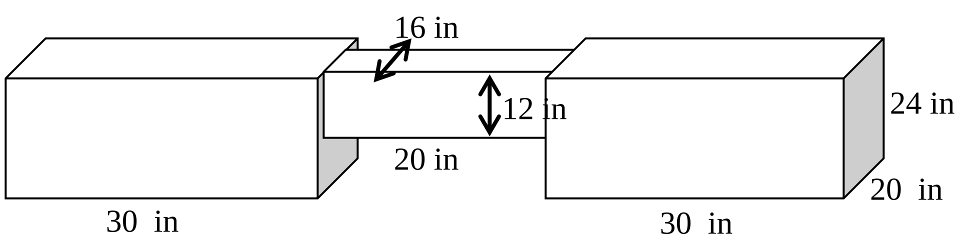
$$V = (3.14)(100)5$$

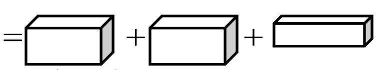
$$V = 1{,}570$$

$The\, volume\, is\, 1{,}570\ mm^3$

Composite Figures

Example: *John created and built a large aquarium to showcase his many tropical fish. He designed two identical large tanks connected by a smaller tank so the fish could swim from one large tank to the other. Using the diagram below, show how to find the volume of the entire aquarium.*



$V =$ 

$$V = 2(lwh) + lwh$$

$$V = 2(30 \bullet 20 \bullet 24) + (20 \bullet 12 \bullet 16)$$

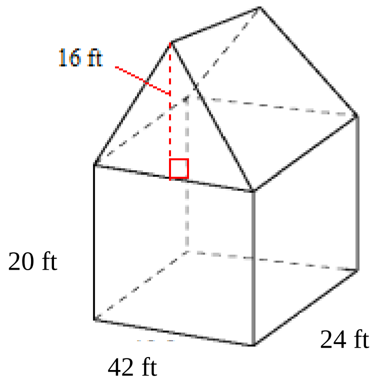
$$V = 2(14{,}400) + 3{,}840$$

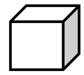
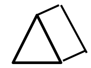
$$V = 28{,}800 + 3{,}840$$

$$V = 32{,}640$$

$The\, volume\, is\, 32{,}640\ in^3$

Example: *Mr. Kaiser is building a barn for his farm. The dimensions are shown at the right. Find the volume of the entire barn.*



$V =$  $+$ 

$$V = lwh + Bh$$

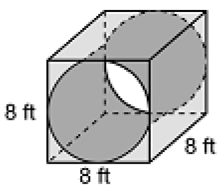
$$V = 42 \bullet 24 \bullet 20 + \left(\frac{1}{2} \bullet 42 \bullet 16\right)24$$

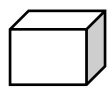
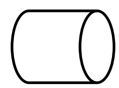
$$V = 20{,}160 + 8{,}064$$

$$V = 28{,}224$$

$$28{,}224 \ ft^3$$

Example: *A cement casing is poured around an 8 foot diameter pipeline. Find the volume of the composite figure shown below. Use* $\pi \approx 3.14$.



$V =$  $-$ 

$$V = lwh - \pi r^2 h$$

$$V = 8 \bullet 8 \bullet 8 - 3.14(4)^2 \bullet 8$$

$$V = 512 - 3.14(16)(8)$$

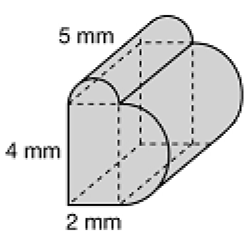
$$V = 512 - 3.14(128)$$

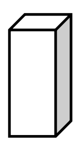
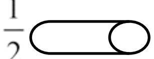
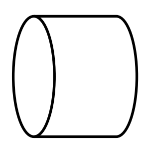
$$V = 512 - 401.92$$

$$V = 110.08$$

$$110.08 \ ft^3$$

Example: *Find the volume of the composite figure shown below. Use* $\pi \approx 3.14$.



$V =$  $+ \frac{1}{2}$  $+ \frac{1}{2}$ 

$$V = lwh + \frac{1}{2}\pi r^2 h + \frac{1}{2}\pi r^2 h$$

$$V = 4 \bullet 2 \bullet 5 + \frac{1}{2}(3.14)(1^2)(5) + \frac{1}{2}(3.14)(2^2)(5)$$

$$V = 40 + \frac{1}{2}(3.14)(1)(5) + \frac{1}{2}(3.14)(4)(5)$$

$$V = 40 + 7.85 + 31.4$$

$$V = 79.25$$

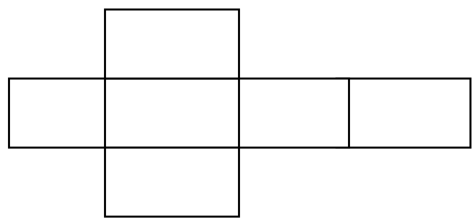
$$79.25 \ mm^3$$

Surface Area

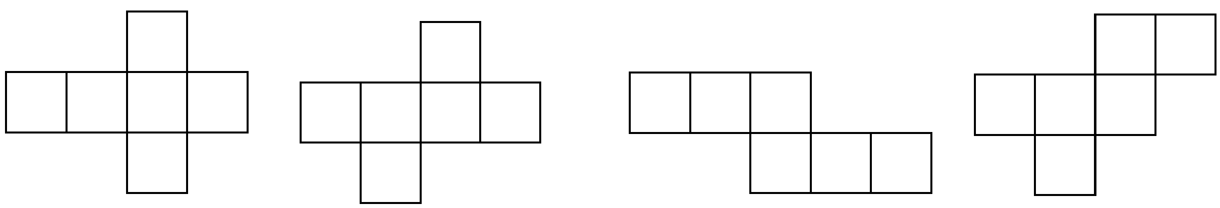
The ***surface area*** of a solid is the sum of the areas of all the surfaces that enclose that solid. To find the surface area, draw a diagram of each surface as if the solid was cut apart and laid flat. Label each part with the dimensions. Calculate the area for each surface. Find the total surface area by adding the areas of all of the surfaces. If some of the surfaces are the same, you can save time by calculating the area of one surface and multiplying by the number of identical surfaces.

Remind your students that "**nets**" are a way to break up these solids into figures for which we can easily find the area. Students may need to experiment with some nets. Here are a few nets students should know (remember there are other examples), not all are given for each solid:

<u>Rectangular Prism</u>

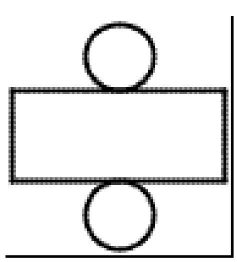


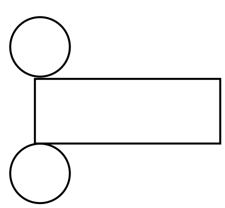
<u>Cube</u>



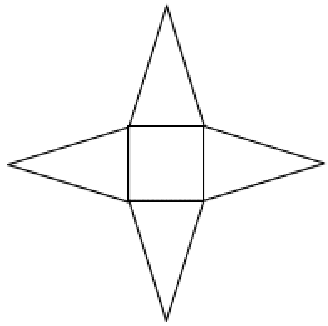
There are <u>numerous</u> ways the net of a cube can be drawn. This interactive site has a neat game for students to determine which nets create a cube.
http://gwydir.demon.co.uk/jo/solid/cube.htm#model

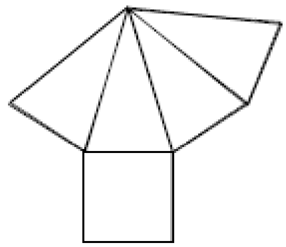
<u>Cylinder</u>

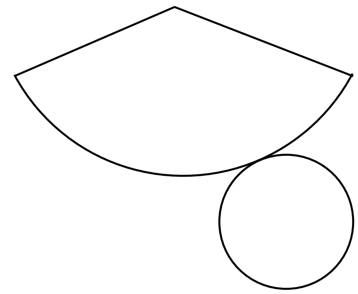




Triangular Prism **Cone**

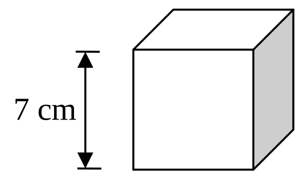






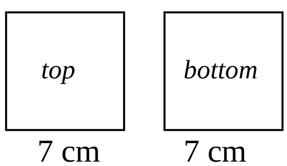
In addition to the link below there are numerous sites for teachers and students to explore the concept of nets.
http://www.learner.org/interactives/geometry/3d_prisms.html

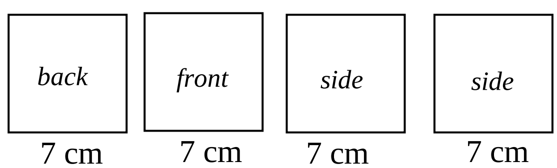
Example: Find the surface area of the prism shown.
All surfaces are squares.



Divide the prism into its parts. Label the dimensions.

Bases ***Lateral Faces***





Find the area of all the surfaces.

Bases		***Lateral Faces***			
$A = bh$	$A = bh$	$A = bh$	$A = bh$	$A = bh$	$A = bh$
$A = 7 \cdot 7$	$A = 7 \cdot 7$	$A = 7 \cdot 7$	$A = 7 \cdot 7$	$A = 7 \cdot 7$	$A = 7 \cdot 7$
$A = 49$	$A = 49$	$A = 49$	$A = 49$	$A = 49$	$A = 49$

Surface Area = Area of the top + bottom + front + back + side + side
Surface Area = 49 + 49 + 49 + 49 + 49 + 49
= 294

The surface area of the prism is 294 cm²^2***.***

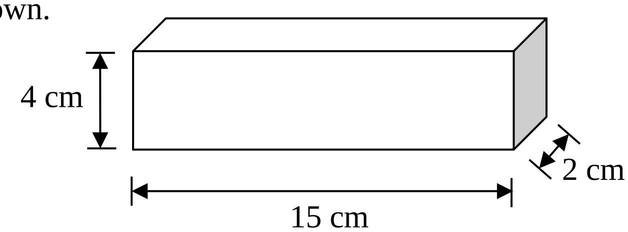
Since a cube has 6 congruent faces, a simpler method would look like

Surface Area = 6 · *the area of a face*
Surface Area = 6*B*
Surface Area = 6*bh*

$$= 6 \cdot 7 \cdot 7$$
$$= 42 \cdot 7$$
$$= 294\ cm^2$$

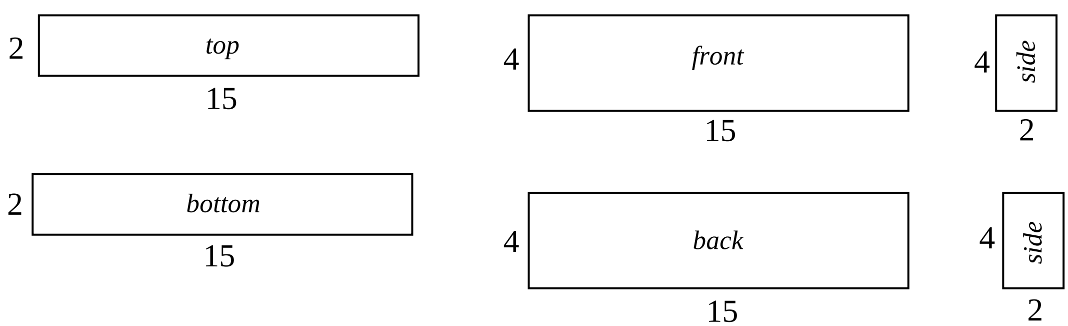
***The surface area of the prism is 294 cm²**.*

***Example**:* Find the surface area of the prism shown. All surfaces are rectangles.



Divide the prism into its parts. Label the dimensions.

Bases   ***Lateral Faces***



Add the area of all the surfaces.

Bases   ***Lateral Faces***

$$A = bh \qquad A = bh \qquad A = bh$$
$$A = 15 \cdot 2 \qquad A = 15 \cdot 4 \qquad A = 2 \cdot 4$$
$$A = 30 \qquad A = 60 \qquad A = 8$$

$$A = bh \qquad A = bh \qquad A = bh$$
$$A = 15 \cdot 2 \qquad A = 15 \cdot 4 \qquad A = 2 \cdot 4$$
$$A = 30 \qquad A = 60 \qquad A = 8$$

Surface Area = Area of the top + bottom + front + back + side + side
Surface Area = 30 + 30 + 60 + 60 + 8 + 8
= 196

***The surface area of the prism is 196 cm²**.*

Note: Since some of the faces were identical, we could multiply by 2 instead of adding the value twice. That work would look like

Surface Area = 2(top <u>or</u> bottom) +2(front <u>or</u> back) +2(side)
Surface Area = 2(30) + 2(60) + 2(8)
= 60 + 120 + 16
= 196
The surface area of the prism is 196 cm^2.

After some exposure to this method with rectangular prisms, you would introduce the formula, ***the Surface Area of rectangular prisms,*** $SA = 2lw + 2lw + 2wh$.

Remember students are expected to know their formulas so provide practice for that.

$$
\begin{aligned}
SA &= 2lw + 2lh + 2wh \\
&= 2 \bullet 15 \bullet 2 + 2 \bullet 15 \bullet 4 + 2 \bullet 2 \bullet 4 \\
&= \;\; 60 \;\; + \;\; 120 \;\; + \;\; 16 \\
&= \;\; 196\,cm^2
\end{aligned}
$$

Methods to Find Surface Area

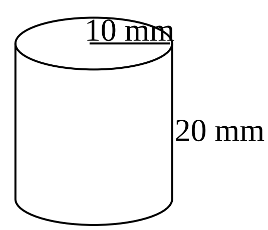
Method 1 – Count the faces.

Method 2 – Compute the area of each part and add parts.

Top and Bottom = $3 \bullet 2 = 6 \bullet 2 = 12$
Front and Back = $3 \bullet 2 = 6 \bullet 2 = 12$
Right and Left = $2 \bullet 2 = 4 \bullet 2 = 8$
$12 + 12 + 8 = 32\,cm^2$

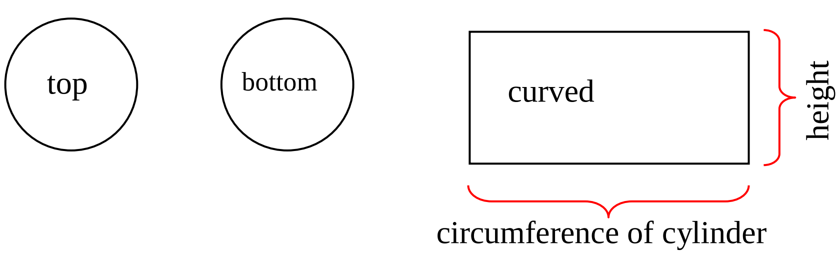
Method 3 – Use the formula.

Example: Find the surface area of the cylinder**.**



(Use $\pi = 3.14$)

Divide the prism into its parts. Label the dimensions.



Find the area of all the surfaces.

Top	**Bottom**	**Area of Curved Surface**
$A = \pi r^2$	$A = \pi r^2$	$A = 2\pi rh$
$A = 3.14 \bullet 10^2$	$A = 3.14 \bullet 10^2$	$A = 2 \bullet 3.14 \bullet 10 \bullet 20$
$A = 3.14 \bullet 100$	$A = 3.14 \bullet 100$	$A = 6.28 \bullet 10 \bullet 20$
$A = 314$	$A = 314$	$A = 62.8 \bullet 20$
		$A = 1,256$

Surface Area = Area of the top + bottom + curved surface
Surface Area = 314 + 314 + 1256
Surface Area = 1,884

$\boxed{\text{1,884 cubic mm}}$

Point out to students that the nets show there are $\underbrace{\text{2 congruent circles}}_{2 \bullet \pi r^2}$ *and the* $\underbrace{\text{curved area}}_{\text{Circumference of the cylinder} \bullet \text{height}}$*.*

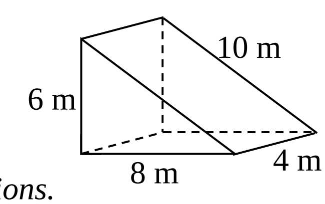
$$\underbrace{2 \bullet \pi r^2}_{2\pi r^2} \qquad \underbrace{\text{Circumference of the cylinder} \bullet \text{height}}_{2\pi r \bullet h}$$

So by formula we could show:

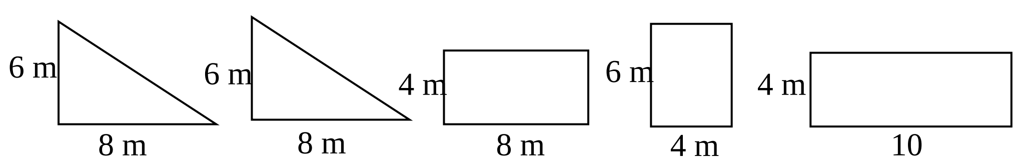
$$SA = 2\pi r^2 + 2\pi rh$$
$$SA = 2 \bullet 3.14 \bullet 10^2 + 2 \bullet 3.14 \bullet 10 \bullet 20$$
$$SA = 2 \bullet 3.14 \bullet 100 + 2 \bullet 31.4 \bullet 20$$
$$SA = 2 \bullet 314 + 2 \bullet 628$$
$$SA = 628 + 1256$$
$$SA = 1,884$$

The surface area of the cylinder is 1,884 cubic millimeters or 1,884 mm^3.

Example: *Find the surface area of the triangular prism.*



Divide the prism into its parts. Label the dimensions.



Find the area of all the surfaces.

$A=\frac{1}{2}bh$ $\quad A=\frac{1}{2}bh$ $\quad A=bh$ $\quad A=bh$ $\quad A=bh$

$A=\frac{1}{2}\bullet 8\bullet 6$ $\quad A=\frac{1}{2}\bullet 8\bullet 6$ $\quad A=8\bullet 4$ $\quad A=4\bullet 6$ $\quad A=10\bullet 4$

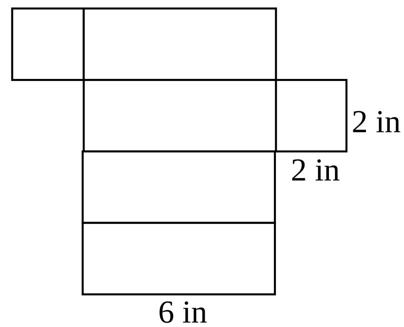
$A=4\bullet 6$ $\quad A=4\bullet 6$ $\quad A=32$ $\quad A=24$ $\quad A=40$

$A=24$ $\quad A=24$

$$SA=24+24+32+24+40$$
$$SA=144$$

The surface area of the triangular prism is 144 sq m or 144 m^2.

Example: Given the net for this rectangular prism, find the surface area and the volume.



Since this is the net for a rectangular prism, we know it has 4 congruent rectangles and, in this case, 2 congruent squares. So to find the surface area I could show:

$$SA=4bh+2bh$$
$$SA=4\bullet 6\bullet 2+2\bullet 2\bullet 2$$
$$SA=\ 48\ +\ 8$$
$$SA=\ 56$$

or

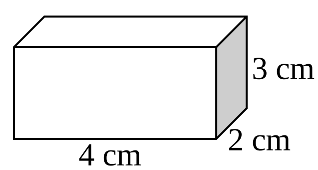
$$SA=2lw+2lh+2wh$$
$$SA=2\bullet 6\bullet 2+2\bullet 6\bullet 2+2\bullet 2\bullet 2$$
$$SA=\ 24\ +\ 24\ +\ 8$$
$$SA=\ 56$$

For the volume, we need to identify the base area and the height.

$$V=Bh$$
$$V=(bh)h$$
$$V=(6\bullet 2)\bullet 2$$
$$V=24$$

*The rectangular prism has a surface area of **56 square inches** and a volume of **24 cubic inches**.*

Example: Consider the rectangular prism shown to the right. Find its surface area.



SA= *2(top <u>or</u> bottom) +2(front <u>or</u> back) +2(side)*

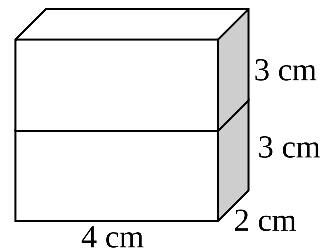
$$SA=\ 2(lw)\ +2(lh)\ +2(wh)$$
$$=2(4\bullet 2)+2(4\bullet 3)+2(3\bullet 2)$$
$$=2\bullet 8\ +\ 2\bullet 12\ +\ 2\bullet 6$$
$$=\ 16\ +\ 24\ +\ 12$$
$$=\ 52$$

The surface area is 52 cm.

Once your students have the understanding of finding volumes we need to explore different scenarios.

If we stacked an identical prism vertically (on top of it), what would be the new surface area?

Without showing a visual representation **for a moment**, many students will automatically jump to "double the original surface area". Consider showing a visual representation without dimensions and have students ponder the idea and generate some hypotheses. (Stack two identical boxes – even 2 tissue boxes or 2 storage boxes would work.)



Eventually show a visual like the one to the right. Students should notice only the height's dimension changed (in this case the height doubles). Now I could just 'plug and chug' the new numbers but let's look at this mentally.

We said only the height doubled so I would need to double the front/back dimension and the side dimension. So from above

$$
\begin{aligned}
& 16+2(24)+2(12) \ \ \textit{or} \\
SA \ = \ & 16+48+24 \\
& 88
\end{aligned}
$$

The surface area is 88 cm^2.

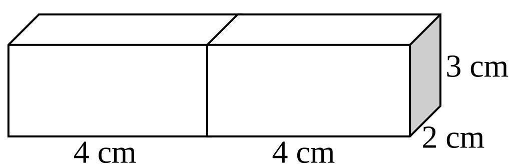
How are the two surface areas related?
Because we stacked two identical prisms vertically only the height dimension changed (double) so we doubled the part of our surface area that was affected – the front/back and the sides.

SO…What if we stacked 3 prisms vertically? Then we could triple the original front back and side dimensions….and get ….

SA= 16 +3(24)+3(12)
= 16 + 72+ 36
= 124

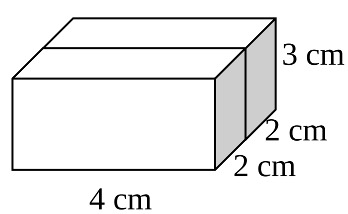
The surface area is 124 cm^2.

What if we were to stack two of them (two identical prisms) horizontally end-to-end instead of vertically?



Then the length would double so I would double the front and top dimension of my original surface area. So, SA= 2(16) + 2(24)+ 12= 32+48+12= 92 cm^2

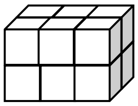
What if we were to stack two of them (two identical prisms) horizontally front-to-back?



Then the width would double, so I would double the top and side dimension of my original surface area. So, the SA= 2(16) + 24 + 2(12)= 32+24+24= 80 cm^2

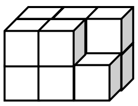
Example: Using centimeter cubes:

a. Find the surface area of the 3-by-2 cube.



SA = $32\ cm^2$

b. Find the surface area of the cube with one cube missing.



SA= $32\ cm^2$

c. Which figure has the greater surface area? ______________________________

____

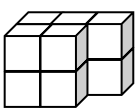
____

d. Does this make sense? Explain why or why not? __________________________

____

____

e. Now find the surface area of the cube with one corner stack missing.



SA= $30\ cm^2$

e. When comparing this figure and surface area to the figures and surface areas in a and b above, what do you notice? Does this make sense? Explain why or why not? ____

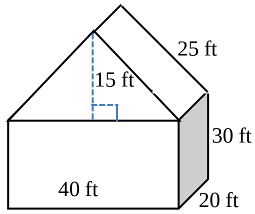
____

____

With regards to the example above, students must begin to see that the figures in parts a and b have the same surface area. Removing the unit cube from the vertex creates an inverted corner on the cube. When compared with the figure in part e, the surface area differs because a whole front stack or front corner stack is missing so there is no inverted corner on the cube. It would be interesting to **ask students to explain why** the surface area of the figure in part e differs by 2 cm^2 (from those in part a and b).

Composite Figures

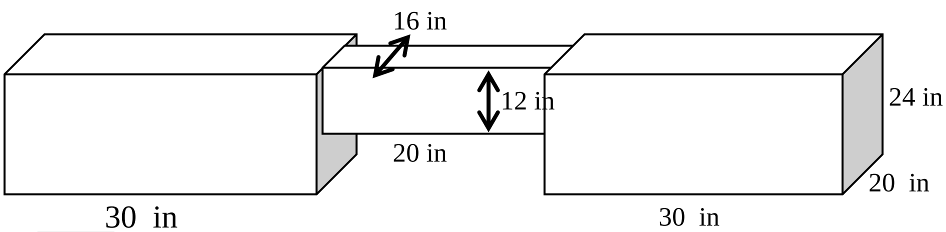
Example: *Find the surface area of the figure below.*

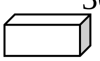
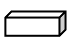
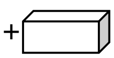


Front rectangle = $40 \bullet 20 = 800$
Back rectangle = $40 \bullet 20 = 800$
Front triangle = $\frac{1}{2} \bullet 40 \bullet 15 = 300$
Back triangle = $\frac{1}{2} \bullet 40 \bullet 15 = 300$
Right side = $20 \bullet 30 = 600$
Left side = $20 \bullet 30 = 600$
Right roof = $25 \bullet 20 = 500$
Left roof = $25 \bullet 20 = 500$
\+ Bottom = $40 \bullet 20 = 800$

Surface Area = 5,200 ft^2

Example: *Find the surface area of the figure below.*



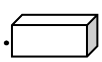
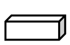
SA= ( **+**  **+**  **) – □ –□ =**

$$\left(\left(2 \bullet 30 \bullet 20 + 2 \bullet 20 \bullet 24 + 2 \bullet 30 \bullet 24\right) + \left(2 \bullet 20 \bullet 12 + 2 \bullet 12 \bullet 16 + 2 \bullet 20 \bullet 16\right) + \left(2 \bullet 30 \bullet 20 + 2 \bullet 20 \bullet 24 + 2 \bullet 30 \bullet 24\right)\right) - \left(12 \bullet 16\right) - \left(12 \bullet 16\right)$$

$$\left(\left(3{,}600\right) + \left(1{,}504\right) + \left(3{,}600\right)\right) - 192 - 192$$

$$8{,}320 \ in^2$$

OR

SA= (2 $\cdot$  **) +**  **– (2** $\cdot$ **□)**

$$2\left(2 \bullet 30 \bullet 20 + 2 \bullet 20 \bullet 24 + 2 \bullet 30 \bullet 24\right) + \left(2 \bullet 20 \bullet 12 + 2 \bullet 12 \bullet 16 + 2 \bullet 20 \bullet 16\right) - 2\left(12 \bullet 16\right)$$

$$2\left(3{,}600\right) + \left(1{,}504\right) - 2(192)$$

$$8{,}320 \ in^2$$

7.G.A.3 Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

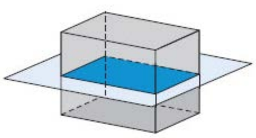
Cross Sections

When a plane intersects a three-dimensional figure (solid), the intersection is called a **cross section**. It can be thought of as a “slice” of a three-dimensional object. For example, when slicing an orange (sphere) in half, the cross section is a circle. Canned cranberry sauce, jelly style, (cylinder) when sliced parallel to its base yields a cross section that is a circle. Slicing a stick of butter (rectangular prism) perpendicular to its base yields a cross section that is a rectangle. Slicing a round cake (cylinder) in half perpendicular to its base, has a cross section that is a rectangle. These are a few examples that students have had some experience with.

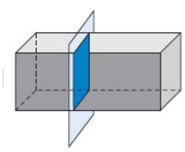
A three-dimensional figure can have many different cross sections depending on the angle of the intersection with the plane. Students will need to visualize, identify and draw horizontal cross sections, vertical cross sections, and oblique cross sections (when the plane intersects at an angle that is not parallel or perpendicular). This standard requires a great deal of hands-on experimentation. Students need to develop their spatial reasoning and visualization skills through this hands-on experimentation. It is only through these interactions and experiences with physical models that students will begin to make conjectures and generalizations about the solids and their cross sections.

The link below will take you to a site where a virtual rectangular prism is “sliced” and the cross sections are displayed. The viewer can control the rotation of the figure and the angle of the cuts made. This would be a great visual introduction to cross sections.

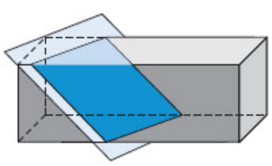
http://www.learner.org/courses/learningmath/geometry/session9/part_c/index.html



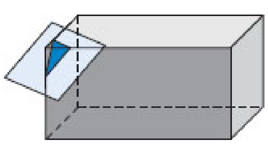
This figure shows the intersection of a rectangular prism and a plane. The **horizontal cross section** is a rectangle.



This **vertical cross section**, showing the intersection of a rectangular prism and a plane, is a rectangle.

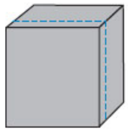


This **oblique cross section**, showing the intersection of a rectangular prism and a plane, is a parallelogram. Notice this “cut” intersects opposite sides of the rectangular prism.

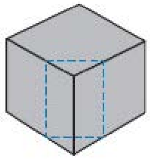


This **oblique cross section**, showing the intersection of a rectangular prism and a plane, is a triangle. Notice this “cut” intersects adjacent faces not opposite sides, or “cuts off the corner” of the prism.

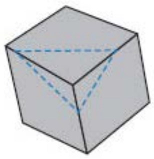
Other oblique cross sections on rectangular prisms can be explored using sticks of butter, styrofoam, modeling clay, cardboard boxes, etc.

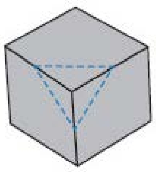


This figure shows the outline of the intersection of a cube and a plane. The **vertical cross section** is a square.

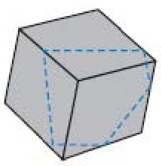


This **oblique cross section** of a cube is a rectangle. Notice the slice cuts opposite sides of the cube.

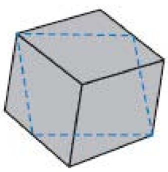




Both of these **oblique cross sections** of a cube form a triangle. Notice the slight differences in the cuts and the differences in the triangles.



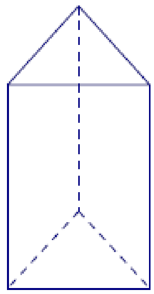
This **oblique cross section** of a cube is a pentagon.

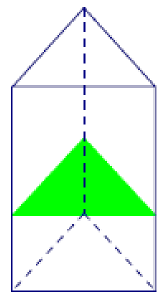


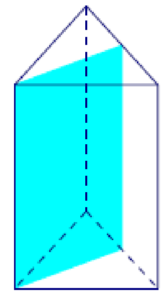
This **oblique cross section** of a cube is a parallelogram.

Other oblique cross sections on cubes can be explored using chunks of butter, styrofoam, modeling clay, cardboard boxes, etc.

Let us consider the following right prism and then draw some general conclusions.


Initial Diagram

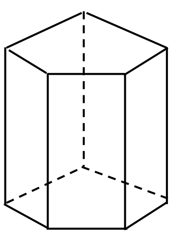

Cross Section Parallel
to the Bases


Cross Section Perpendicular
to the Bases

Using the triangular prisms above,answer the following:
the horizontal cross section is a ____triangle____.

the vertical cross section is a ____rectangle____.

using the intitial diagram above, sketch one oblique cross section and identify the figure.
____answers will vary____.



What would the vertical cross section of a pentagonal prism be?
____rectangle____

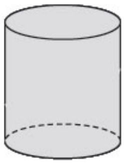
What would any horizontal cross section of a pentagonal prism be?
____pentagon____

Reflection: Looking at or visualizing the vertical cross sections of a rectangular prism, a cube, a triangular prism, a pentagonal prism, etc., can you make any conjectures about what figure the cross section formed? How could you test your conjectures?

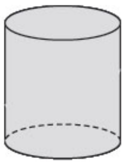
Reflection: Looking at or visualizing the horizontal cross sections of a rectangular prism, a cube a triangular prism, a pentagonal prism, etc., can you make any conjectures about what figure the cross section formed? How could you test your conjectures?

Reflection: Is it possible to cut any right rectangular prism and have a cross section that is a circle?

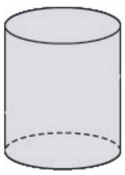
Cross sections on cylinders can be explored using canned jellied style cranberry sauce, styrofoam, modeling clay, oatmeal boxes, etc.



Sketch a **horizontal cross section** of a cylinder.
The cross section is a ____circle____.



Sketch a **vertical cross section** of a cylinder.
The cross section is a ____rectangle____.



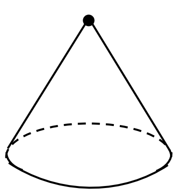
Challenge:
Sketch at least one **oblique cross section** of a cylinder.
Do you think you can name it? Answers may vary (but my include) – oval, half oval.

Reflection: Looking at or visualizing the horizontal cross sections of any prism, (a rectangular prism, a cube, a triangular prism, a pentagonal prism,a cylinder, etc.), can you make any conjectures about what figure the cross section formed? How could you test your conjectures?

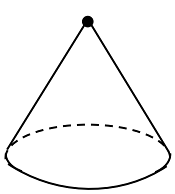
Reflection: Looking at or visualizing the vertical cross sections of any prism (a rectangular prism, a cube, a triangular prism, a pentagonal prism,a cytlinder, etc)., can you or make any conjectures about what figure the cross section formed? How could you test your conjectures?

Reflection: Is it possible to 'cut' any cylinder and have a cross section that is a rectangle? Why or why not?

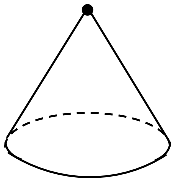
Cross sections on cones can be explored using cone shaped drink cups, styrofoam, modeling clay, etc.



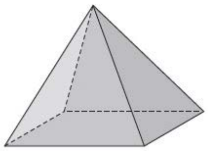
Sketch the **horizontal cross section** of a cone.
The cross section is a ____circle____.



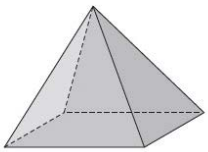
Sketch the **vertical cross section** of a cone through the vertex.
The cross section is a ____triangle____.



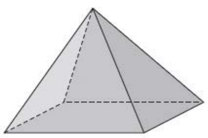
Challenge:
Sketch at least one **oblique cross section** of a cone.
Do you think you can name it? Answers may vary (but my include) – oval, half oval



Sketch the **horizontal cross section** of a right rectangular pyramid. The cross section is a rectangle.



Sketch the **vertical cross section** of a right rectangular pyramid. The vertical cross section through the vertex at the top of the pyramid is a triangle. The vertical cross section anywhere else is a trapezoid.



Sketch at least one **oblique cross section** of a right rectangular pyramid. The cross section is a answers will vary.

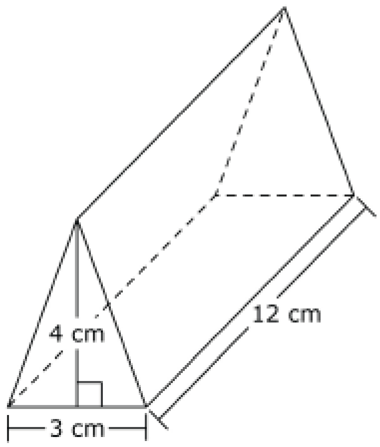
Reflection: Looking at or visualizing the horizontal cross sections of any right rectangular pyramid, (a cone, a right rectangular pyramid, etc.), can you make any conjectures about what figure the cross section formed? How could you test your conjectures?

Reflection: Looking at or visualizing the vertical cross sections of any pyramid, (a cone through the vertex, a right rectangular pyramid) can you make any conjectures about what figure the cross section formed? How could you test your conjectures?

Sample SBAC Questions

Standard: CCSS 7.G.6 ***DOK: 2*** ***Difficulty: M*** ***Question Type: TE (Technology Enhanced)***

Look at the triangular prism below. Each triangular face of the prism has a base of 3 centimeters (cm) and a height of 4 cm. The length of the prism is 12 cm.



What is the volume, in cm^3, of this triangular prism?

[   ] cm^3

Key:

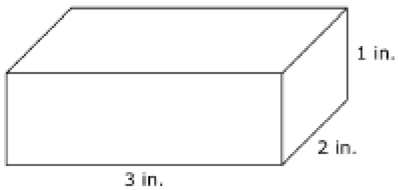
72 cm^3

$V = (\frac{1}{2} \times 3 \times 4) \times 12 = 6 \times 12 = 72$

Standard: CCSS 7.G.6 ***DOK: 2*** ***Difficulty: M*** ***Question Type: TE (Technology Enhanced)***

Using the rectangular prism shown below, create a new prism with a **surface area** of between 44 square inches and 54 square inches.

Click on the prism and drag it to the work area. Then stack additional prisms vertically to create the new prism. The prism may be used more than one time.



Work Area:

Key and Distractor Analysis:

4 prisms should be stacked vertically.

TE Information:

Item Code: MAT.07.TE.1.0000G.F.286

Template: Tiling

A. Interaction

i. Requires students to click on a prism and drag it to the work area

ii. The prism may be selected up to 10 times.

iii. The prism should not be able to rotate.

B. Interaction Space

i. Prisms may be stacked vertically.

ii. Dragged prisms should snap to figure in the work area so that it appears to be one figure.

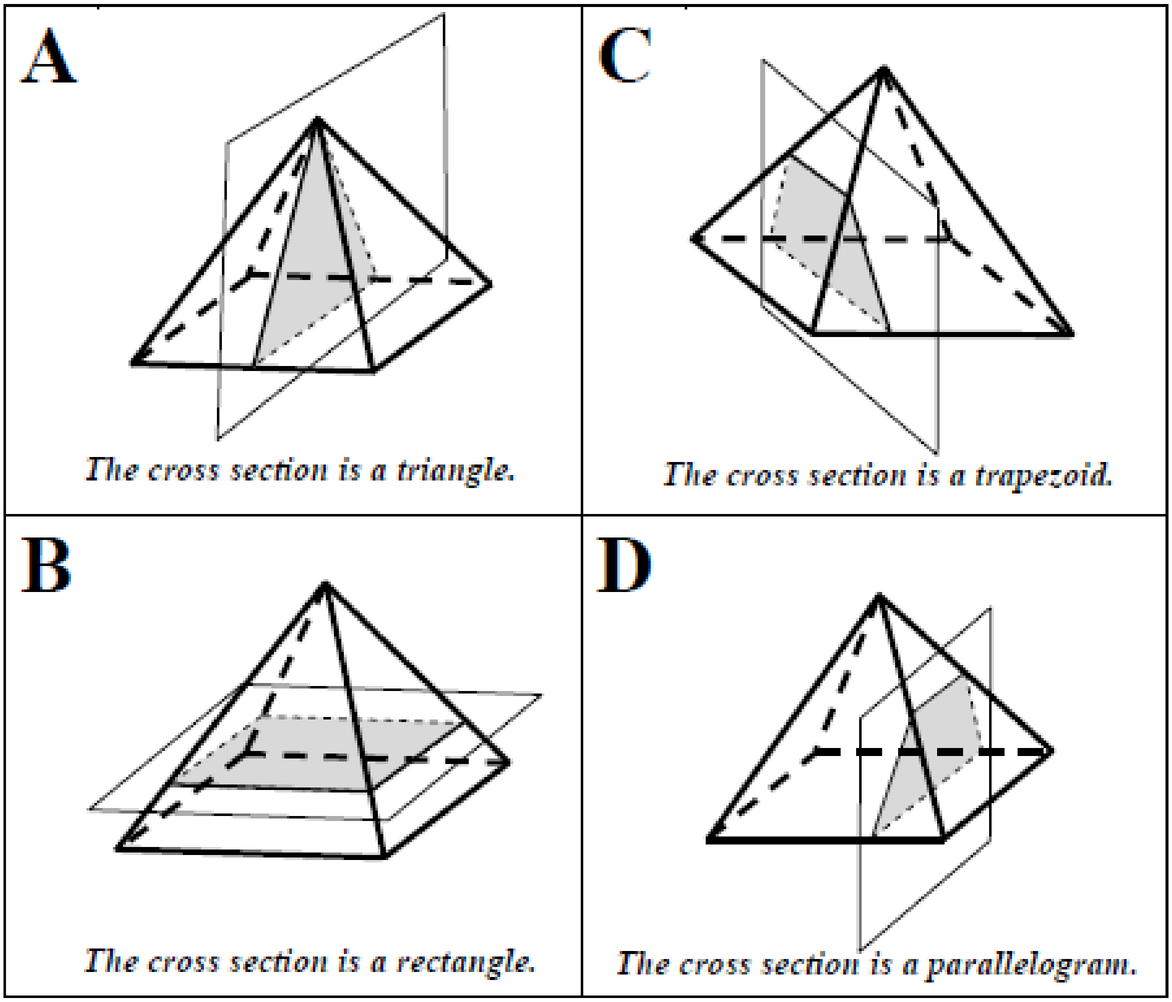
Scoring Data:

{4 prisms stacked}

{0 errors = 1 point}

Sample/possible CRT question from RPDP posters.

Which pyramid has a shaded area that represents a cross section cut parallel to the base? Name the figure of the cross section.

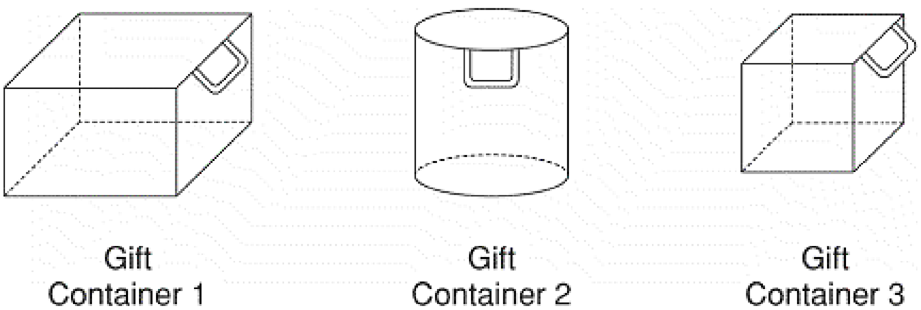


The correct solution is B.

Sample Explorations in CORE Math Questions

Find the surface area of the prism shown without a top.

Nifty Gifts wraps gifts for special occasions. There are different and unusual gift container shapes and sizes you can choose.



Small containers cost \$1.50 each; large ones cost \$3.50 each.

Gift Container 1	Gift Container 2	Gift Container 3
Small: 4 in. × 5 in. × 6 in.	Small: Diameter: 7 in. Height: 11 in.	Small: Side: 5 in.
Large: 12 in. × 8 in. × 10 in.	Large: Diameter: 12 in. Height: 24 in.	Large: Side: 12 in.

Choices for wrapping paper include:
• Foil paper at \$1.75 per square foot.
• Double-thick paper at \$1.50 per square foot.
• Economy paper at \$1.25 per square foot.
[Note: Each square foot = 144 square inches.]

Think of 3 gifts you could buy and have wrapped in one of each shape of container. Complete the chart for each.

Gift	Container Shape	Container Size	Wrapping Paper

Calculate the cost for wrapping each gift using the least amount of wrapping paper. Include the cost of the container for each. List the cost to wrap each gift.

. The coordinates of three vertices of a rectangle are $A(1, -5)$, $B(1, 3)$, and $C(10, 3)$. Find the coordinates of the fourth vertex. Then find the area of the rectangle.

a. $D(10, 1)$; area = 80 square units
b. $D(10, -5)$; area = 72 square units
c. $D(1, 10)$; area = 80 square units
d. $D(-5, 10)$; area = 72 square units

A square playground has an area of 175 m^2. What is the approximate length of each side of the playground? Round your answer to the nearest meter.

a. 13 m
b. 14 m
c. 87.5 m
d. 15 m

To determine the amount of wrapping paper needed for a rectangular box, Ryan finds the surface area of the box. How much wrapping paper is needed if the box measures 9 in. by 4 in. by 6 in?

a. 228 in^2 of wrapping paper
b. 114 in^2 of wrapping paper
c. 216 in^2 of wrapping paper
d. 180 in^2 of wrapping paper

If two pieces of ice have the same volume, the one with the greater surface area will melt faster because more of its surface area is exposed to the air, which is warmer than the ice. Four pieces of ice (P_1, P_2, P_3, and P_4) have the same volume. Each piece of ice is shaped like a rectangular prism. Find the piece of ice that melts faster than the others.

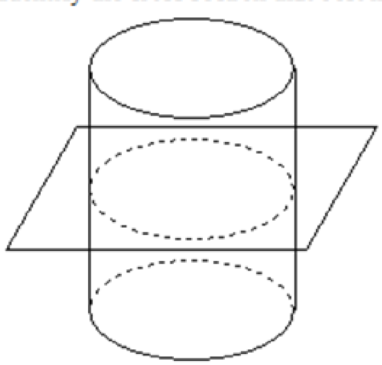
Piece	Length	Width	Height
P_1	32 ft	3 ft	3 ft
P_2	18 ft	4 ft	4 ft
P_3	16 ft	6 ft	3 ft
P_4	18 ft	8 ft	2 ft

a. P_1
b. P_2
c. P_3
d. P_4

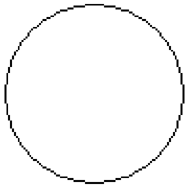
Multiple Choice Questions

Identify the choice that best completes the statement or answers the question.

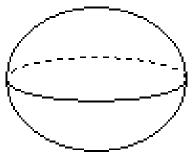
____ 1. Identify the cross section that best matches the given figure.



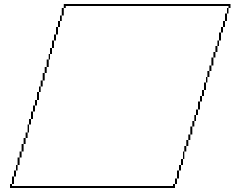
a.



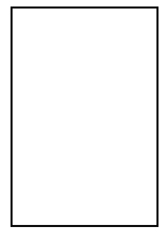
b.



c.



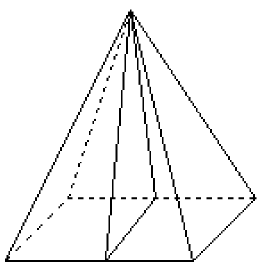
d.



Correct answer is A. DOK 1

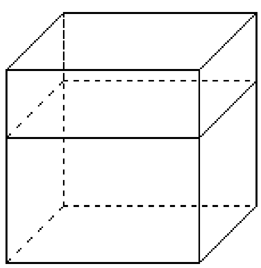
2. Which is a sketch of the cross section of a square pyramid that is cut parallel to its base? Describe the cross section.

a.



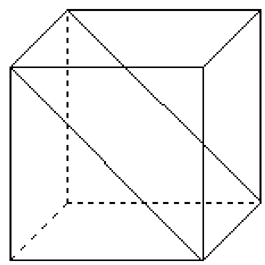
The cross section is a triangle.

b.



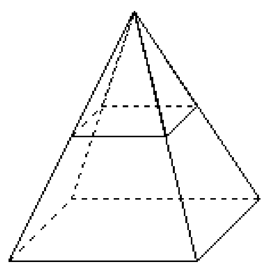
The cross section is a square.

c.



The cross section is a rectangle.

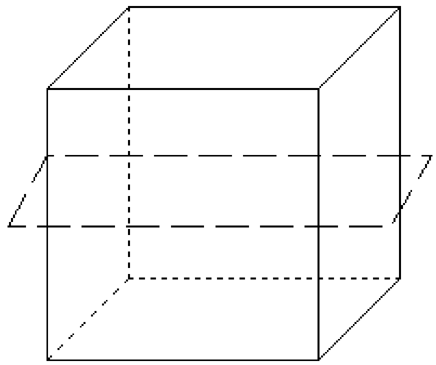
d.

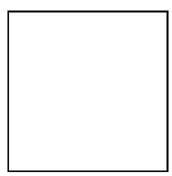


The cross section is a square.

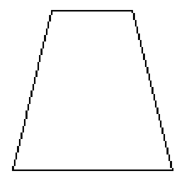
Correct answer is D. DOK 2

3. Draw the shape made by slicing the cube parallel to its bases.

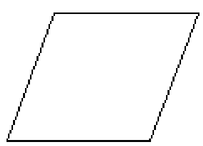




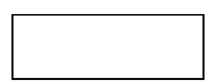
a.



c.



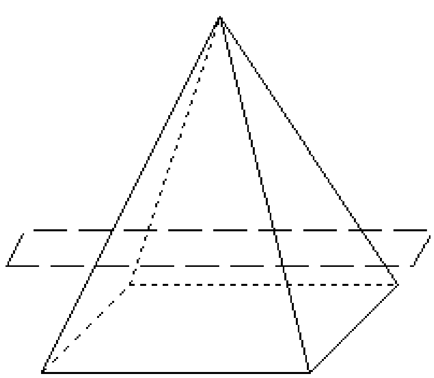
b.

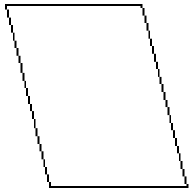


d.

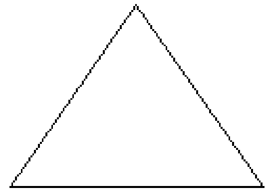
Correct answer is A. DOK 1

4. Draw the shape made by slicing the rectangular pyramid parallel to its bases.

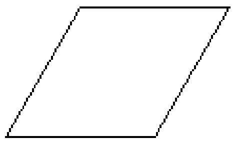




a.



c.

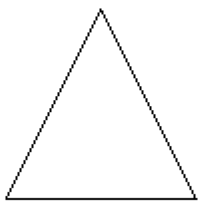


b.

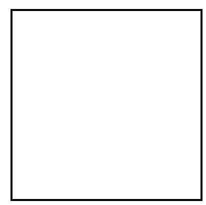
d.

Correct answer is D. DOK 1

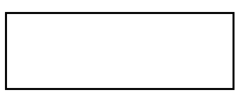
5. Draw the shape made by slicing a triangular prism parallel to its bases.



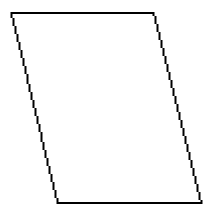
a.



c.



b.



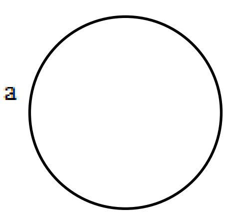
d.

Correct answer is A. DOK 1

Short Answer

1. Describe a two-dimensional cross section of a plane and a sphere. Sketch the figure.

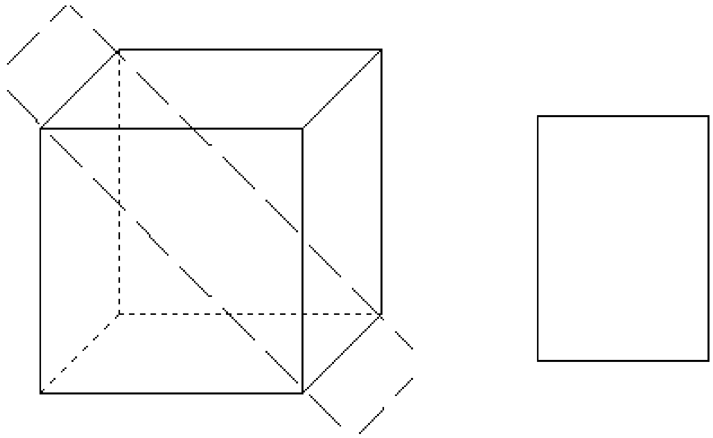
1. ANS:
Every two-dimensional cross section of a plane and a sphere is a circle. (A cross section could also be a point, which is not a two-dimensional figure.).



PTS: 1 NAT: NT.CCSS.MTH.10.7.7.G.3 KEY: cross section | sphere
DOK: DOK 2

2. Describe how you can slice a cube to produce a two-dimensional figure that is not a square. Make a sketch.

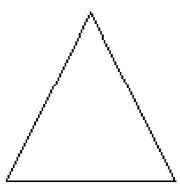
2. ANS:
Make a slice that is not parallel to a face of the cube.
In the sample below, the resulting figure is a rectangle, but not a square.



PTS: 1 NAT: NT.CCSS.MTH.10.7.7.G.3 KEY: cross section | cube
DOK: DOK 2

3. Describe the two-dimensional figure formed by slicing a triangular pyramid parallel to its base. Sketch the figure.

3. ANS:
A two-dimensional cross section formed by slicing a triangular pyramid parallel to its base is a triangle. (A cross section could also be a point, which is not a two-dimensional figure.).



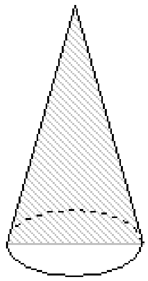
PTS: 1 NAT: NT.CCSS.MTH.10.7.7.G.3 KEY: cross section | triangular pyramid
DOK: DOK 2

4. A right cone with a height of 8 inches and a base radius of inches is sliced exactly in half through its vertex, perpendicular to the base.

Part A: What two-dimensional figure does the slice create? Draw a sketch to justify your answer.

Part B: Find the area of the two-dimensional figure. Explain how you found area.

4. ANS:
Part A:. The figure is a triangle.



Part B: The figure is a triangle. Its height is the same as the height of the cone, 8 inches. The length of its base is twice the radius of the cone, 4 inches.

$$A = \frac{1}{2}bh$$

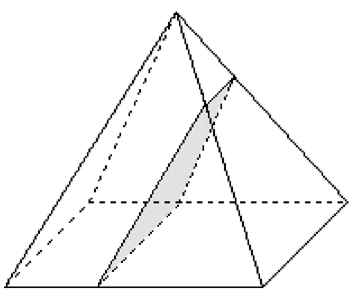
$$= \frac{1}{2}(4)(8)$$

$$= 16$$

The area of the triangle is 16 square inches.

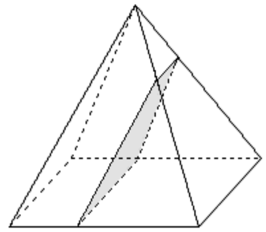
5. Explain how you can slice a square pyramid so the figure formed is a trapezoid. Draw a sketch to illustrate your answer.

5. ANS:
Sample Answer: Slice the pyramid so the cut is parallel to one side of the base so the cut passes through a triangular face.

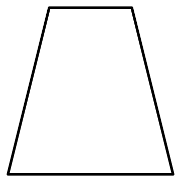


PTS: 1 NAT: NT.CCSS.MTH.10.7.7.G.3 DOK: DOK 3

6. Draw the shape made by slicing the square pyramid parallel to one side of the base so the cut passes through a triangular face.



6. ANS:
Sample answer:

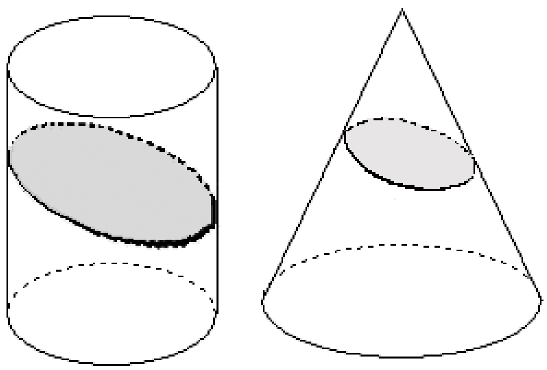


7. A rectangular pyramid has one base corner sliced off, perpendicular to the base. It also has the top sliced off, parallel to the base. Compare the two resulting 2-dimensional shapes shown by the slices.

7. ANS:
The top slice shows a rectangle. The corner slice shows a triangle.

8. A solid is sliced so that the slice is a 2-dimensional figure in the shape of an oval. Name two solids that could have been used. Draw sketches to illustrate your answers.

8. ANS:
Cone or cylinder

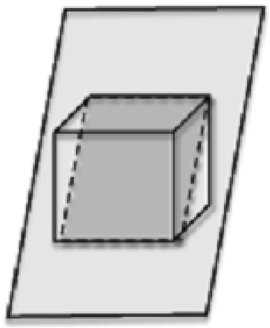
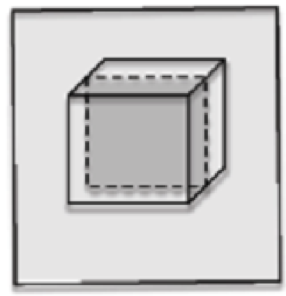


PTS: 1 NAT: NT.CCSS.MTH.10.7.7.G.3 DOK: DOK 3

Essay Questions

1. Draw and describe two different resulting cross-sections that are possible from slicing a cube with a plane.

1. ANS:
Answers may vary. Sample answer:

Rectangle	**Square**
	
The plane cuts through opposite faces of the cube but does not form right angles with the opposite faces.	The plane cuts through opposite faces of the cube at right angles with the opposite faces.

PTS: 1 NAT: NT.CCSS.MTH.10.7.7.G.3 DOK: DOK 2

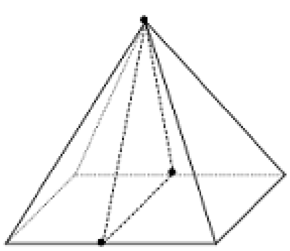
2. A cylinder is 8 inches tall and has a diameter of 3 inches. Is it possible for a cross section of this cylinder to be square? Explain. If not, under what circumstances is a square cross section of a cylinder possible?

ANS:
No, it is not possible. The cross section will be a rectangle with dimensions 8 inches by 3 inches. If the cylinder has a height and diameter of equal measure, a cross section perpendicular to and through the center of the bases makes a square.

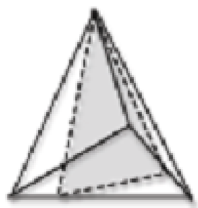
PTS: 1 NAT: NT.CCSS.MTH.10.7.7.G.3 DOK: DOK 3

3. Jes makes a cross section of a rectangular pyramid and a triangular pyramid. Both cross sections are perpendicular to the base and go through the vertex. Describe each cross section and include a drawing for each.

ANS:
A slice of a rectangular pyramid through the vertex and perpendicular to the base is a triangle.

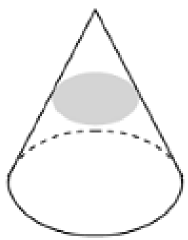


A slice of a triangular pyramid through the vertex and perpendicular to the base is a triangle.

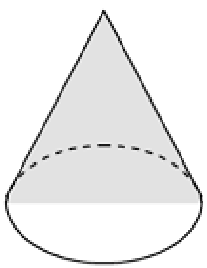


4. Compare a cross section of a cone that is parallel to its base with a cross section of a cone that is perpendicular to the base and goes through the vertex. Justify your answer and include a drawing for each shape.

ANS:
The slice through the cone parallel to the base would result in a round or circular cross-sectional shape.



The slice perpendicular to the base and through the point of the cone would result in a triangular cross-sectional shape.

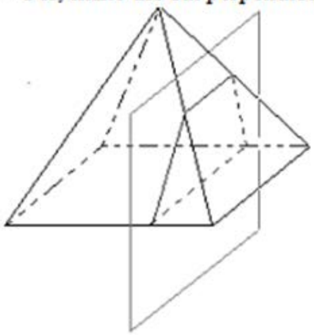


PTS: 1 NAT: NT.CCSS.MTH.10.7.7.G.3 DOK: DOK 3

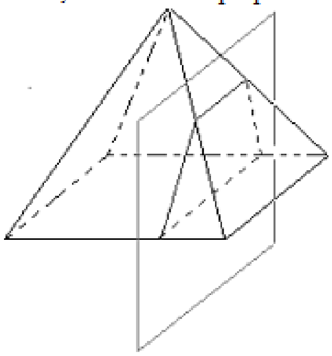
5. Is it possible to slice a rectangular pyramid so that the resulting cross-section is the given polygon? Answer for each shape. Explain your answer and if the answer is yes, justify your answer and include a drawing.

- trapezoid
- square
- parallelogram

ANS:
• Yes, make the cut perpendicular to the base, and parallel to the edge of the base, but not through the vertex.

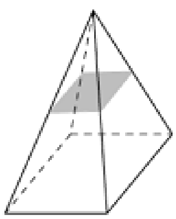


ANS:
• Yes, make the cut perpendicular to the base, and parallel to the edge of the base, but not through the vertex.

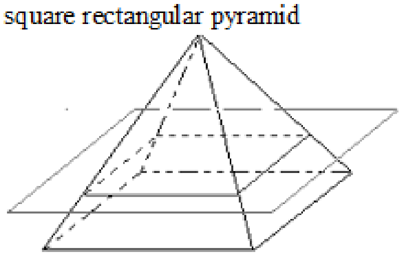


• Yes, but only if the rectangular pyramid is a square pyramid. Then a cut a slice parallel to the base.

non-square rectangular pyramid



square rectangular pyramid

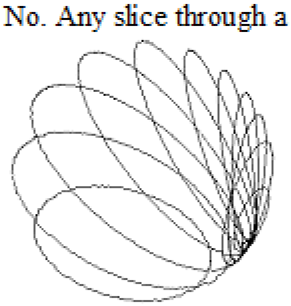


• Yes, a square and a rectangle are parallelograms, so both cross-sections in part b show that a cross-section can be a rectangular parallelogram.

PTS: 1 NAT: NT.CCSS.MTH.10.7.7.G.3 DOK: DOK 3

6. Can a sphere be sliced in any way so that the resulting cross-section would NOT be a circle? Explain.

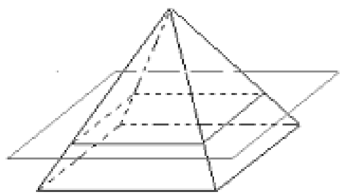
ANS:
No. Any slice through a sphere would result in a circle.



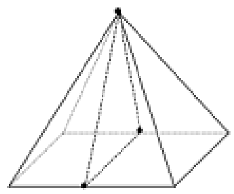
7. List all the two-dimensional cross-sections of a rectangular pyramid. Justify your answer and include a drawing for each shape.

ANS:
A cross-section of a rectangular pyramid may be a rectangle, a triangle, or a trapezoid.

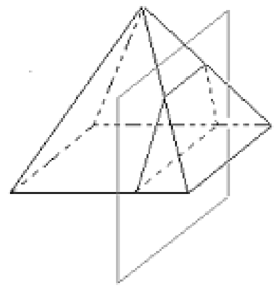
A slice parallel to the base will make a rectangle.



A slice perpendicular to the base and through the vertex will make a triangle.



A slice through the base and three of the four faces will make a trapezoid.

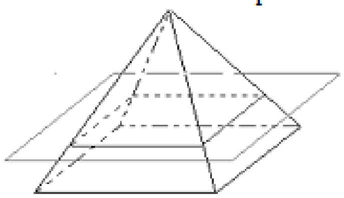


PTS: 1 NAT: NT.CCSS.MTH.10.7.7.G.3 DOK: DOK 4

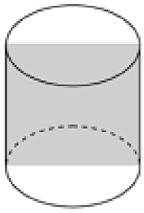
8. Kelly has four 3-dimensional shapes made of clay: a cone, a rectangular prism, a cylinder, and a triangular prism. With which of the four shapes can she find a cross section that will be square? Name all that apply, and justify your answer with a drawing for each shape.

ANS:

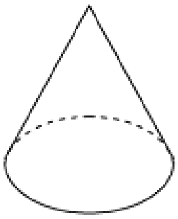
A rectangular prism can have a square cross section if the base is square.



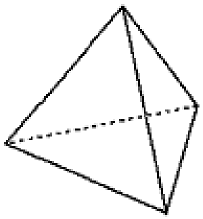
A cylinder can have a square cross section if the height is equal to the diameter.



A cone cannot have a square cross section.



A triangular pyramid cannot have a square cross section.

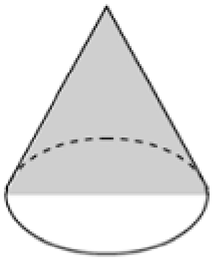


PTS: 1 NAT: NT.CCSS.MTH.10.7.7.G.3 DOK: DOK 4

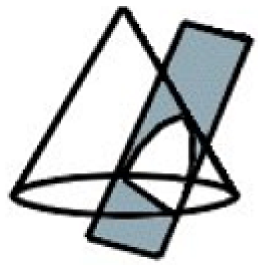
9. What 2-dimensional shapes can be created from a cross section of a cone? Justify your answer and include a drawing for each shape.

ANS:

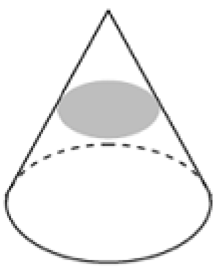
A slice through the base and through the vertex of the cone makes a triangle.



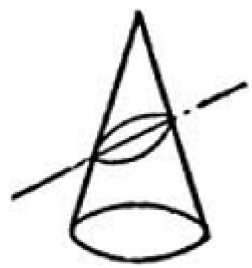
A slice through the base and the side of the cone makes a shape resembling a half-oval.



A slice parallel to the base of the cone makes a circle.



A slice that is not parallel to the base and does not intersect the base makes an oval.

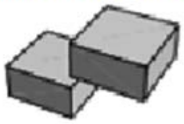


PTS: 1 NAT: NT.CCSS.MTH.10.7.7.G.3 DOK: DOK 4

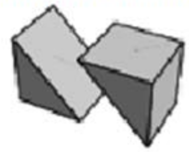
10. List at least two different two-dimensional shapes that can be created by slicing a cube. Justify your answer and include a drawing for each shape.

ANS:
Possible answers:

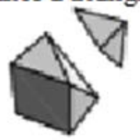
A cross section through four faces and parallel to the base makes a square.



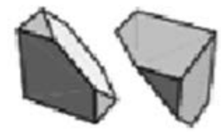
A cross section through four faces that is not parallel to the base makes a rectangle.



A cross section through three faces (cutting off a vertex) makes a triangle.



A cross section that intersects all six faces makes a hexagon.

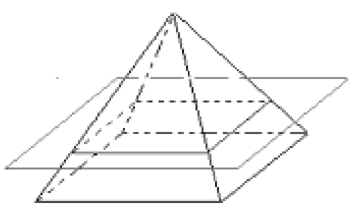


PTS: 1 NAT: NT.CCSS.MTH.10.7.7.G.3 DOK: DOK 4

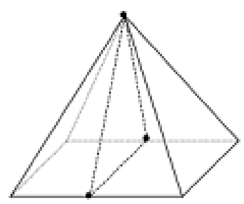
11. List all the two-dimensional cross-sections of a square pyramid. Justify your answer and include a drawing for each shape.

ANS:
A cross-section of a square pyramid may be a square, a triangle, or a trapezoid.

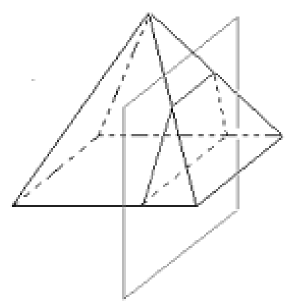
A slice parallel to the base will make a rectangle.



A slice perpendicular to the base and through the vertex will make a triangle.



A slice through the base and three of the four faces will make a trapezoid.



PTS: 1 NAT: NT.CCSS.MTH.10.7.7.G.3 DOK: DOK 4

12. Describe all the possible two-dimensional cross-sections of a cylinder.

ANS:
A cylinder could be sliced to make a cross-section of a rectangle, a circle, a shape resembling a half-oval, and possibly a square.
- A cross section perpendicular to the bases makes a rectangle.
- A cross section parallel to the bases makes a circle.
- A cross section through only one of the bases makes a shape resembling a half-oval.
- A cross section of a cylinder can only be a square if the height and diameter of the cylinder are equal. In this case, a cross section through the center of the bases and perpendicular to the bases makes a rectangle.

PTS: 1 NAT: NT.CCSS.MTH.10.7.7.G.3 DOK: DOK 4