Piecewise linear functions

Math Background

Previously, you
- Related a table of values to its graph.
- Graphed linear functions given a table or an equation.

In this unit you will
- Determine when a situation requiring a step function will be necessary
- Must identify where discontinuities will occur in piecewise functions
- Must identify all key features for the types of functions that make up a piecewise function
- Interpret key features of functions in context of the real-world situation they model

You can use the skills in this unit to

- Graph piecewise functions with and without technology
- Write the equations for piecewise functions whose graph is shown

Vocabulary

- **Discontinuity** – A point at which the graph of a relation or function is not connected.
- **Piecewise Function** – A function defined piecewise, that is $f(x)$ is given by different expressions on various intervals.

Essential Questions

- How are appropriate inputs determined when modeling a real-world situation with a function?
- How do we model real-world situations when they cannot be described with a single function?

Overall Big Ideas

Functions are used to model real-world situations and their domains may be limited by the relationship between the quantities in the function. Piecewise functions have two or more parts, which may be any type of linear or non-linear functions and are used when a single function does not define a real-world situation well.

Skill

To write and graph linear piecewise functions.

Related Standards

F.IF.B.5-1

Relate the domain of a linear, exponential, or quadratic function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factor, then the positive integers would be an appropriate domain for the function. *(Modeling Standard)

F-IF.C.7b-1

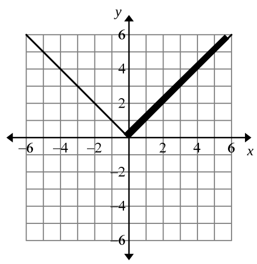
Graph piecewise-defined functions, including step functions and absolute value functions. *(Modeling Standard)

Notes, Examples, and Exam Questions

Piecewise Function: A function whose rule includes more than one formula. The formula for each piece of the function is applied to certain values of the domain, as specified in the definition of the function.

Ex 1 Graph the piecewise function: $f(x)=\begin{cases} x, & x\ge 0 \\ -x, & x<0 \end{cases}$

Graph each piece. The bolded portion is the first "piece".



Note: This is the Absolute Value Function!

Evaluating a Piecewise Function:

Ex 2 Find $f(-3)$ and $f(2)$ for the function $f(x)=\begin{cases} x^2-1, & x<0 \\ 4x+1 & x\ge 0 \end{cases}$.

Step One: Determine which equation to use based upon the value of *x*.

$f(-3)$: $x=-3<0$, so we will use the first equation.

$f(2)$: $x=2\ge 0$, so we will use the second equation.

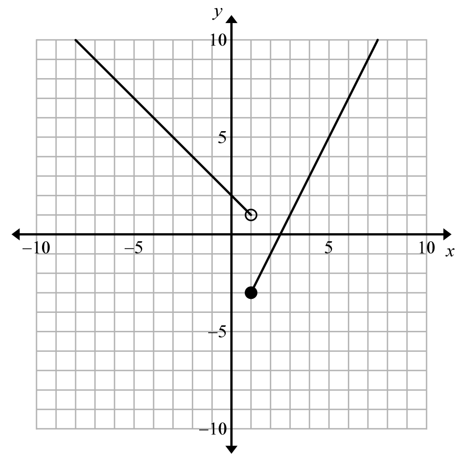
Step Two: Substitute the value of *x* into the appropriate equation.

$f(-3)=(-3)^2-1=9-1=8$

$f(2)=4(2)+1=9$

Graphing a Piecewise Function:

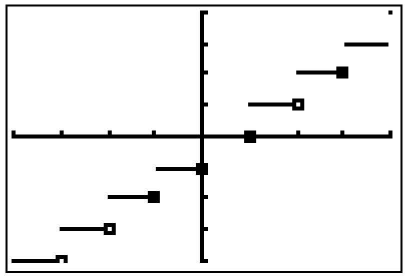
Ex 3 Graph the function $f(x)=\begin{cases}-x+2, & x<1\\ 2x-5 & x\geq 1\end{cases}$



Step One: Graph the function $y=-x+2$ for all values of *x* less than 1. There should be an open circle on the point $(1,1)$.

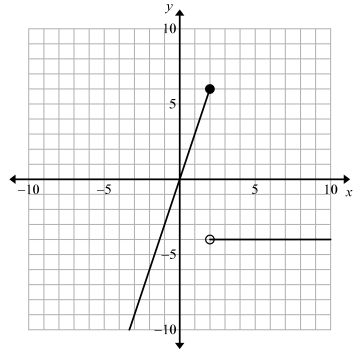
Step Two: Graph the function $y=2x-5$ for all values of *x* greater than 1 in the same coordinate plane. There should be a closed circle on the point $(1,-3)$.

The Greatest Integer Function: $g(x)=[\![x]\!]$ For every real number *x*, $g(x)$ is the greatest integer less than or equal to *x*. The greatest integer function is an example of a **step function**. Note: All points seen on the graph to the right are solid points. The left endpoints of each segment are open circles.



Ex 4

Write the equation of the piecewise function shown in the graph.

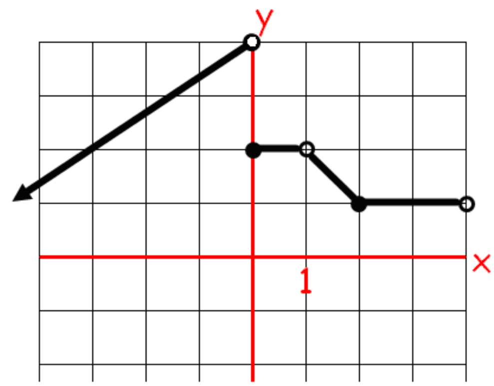


Our function is represented by two linear equations. The leftmost part of the function is a line with a slope of 3 and a y-intercept of 0. It can be represented as: $f(x)=3x$. The rightmost part of the function is a horizontal line and is represented by: $f(x)=-4$. Looking at the domains of the two pieces, we see that the first line goes to $x=2$ and includes that value (closed circle). The horizontal line has an open circle on the 2 so it is only for values greater than 2.

Ans: $f(x)=\begin{cases}3x, & x\leq 2\\ -4, & x>2\end{cases}$

Ex 5

Please write functions for the piecewise graph at the right.



Ans: $f(x)=\begin{cases} \frac{2}{3}x+4 & x<0 \\ 2 & 0\le x<1 \\ -x+3 & 1<x\le 2 \\ 1 & 2\le x<4 \end{cases}$

Modeling: Piecewise functions can be used in real-world applications and model real-life situations.

Ex 6

In 2005, the cost C (in dollars) to send U.S. Postal Service Express Mail up to 5 pounds depended on the weight *w* (in ounces) according to the following function:

$$C(w)=\begin{cases} 13.65, & \text{if } 0<w\le 8 \\ 17.85, & \text{if } 8<w\le 32 \\ 21.05, & \text{if } 32<w\le 48 \\ 24.20, & \text{if } 48<w\le 64 \\ 27.30, & \text{if } 64<w\le 80 \end{cases}$$

Ex 7

A parent drives from home to the grocery store at 0.9 mile per minute for 4 minutes, stops to buy snacks for the team for 2 minutes, and then drives to the soccer field at a speed of 0.7 mile per minute for 3 minutes. Write a piecewise function for the parent's distance from home to the soccer field during this time.

Let *d(t)* represent the distance traveled in time, *t*. Use the formula distance equals rate times time.

Ans: $d(t)=\begin{cases} 0.9t & 0\le t\le 4 \\ 3.6 & 4<t\le 6 \\ 3.6+0.7(t-6) & 6<t\le 9 \end{cases}$

← Parent travels at 0.9 mi/min for 4 minutes.
← Distance traveled is constant for 2 minutes.
← Add the distance traveled at 0.7 mi/min to the distance already traveled.

QOD: In a piecewise function, why must one part of the graph have an open circle as an endpoint, and the other have a closed circle as an endpoint?

SAMPLE EXAM QUESTIONS

1. Match the piecewise function with its graph.

13. $f(x) = \begin{cases} x - 4, & \text{if } x \le 1 \\ 3x, & \text{if } x > 1 \end{cases}$

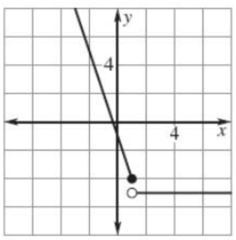
14. $f(x) = \begin{cases} x + 4, & \text{if } x \le 0 \\ 2x + 4, & \text{if } x > 0 \end{cases}$

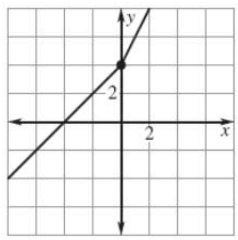
15. $f(x) = \begin{cases} 3x - 2, & \text{if } x \le 1 \\ x + 2, & \text{if } x > 1 \end{cases}$

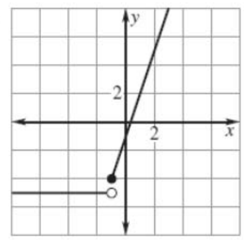
16. $f(x) = \begin{cases} 2x + 3, & \text{if } x \ge 0 \\ x + 4, & \text{if } x < 0 \end{cases}$

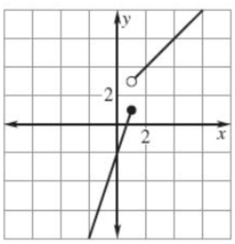
17. $f(x) = \begin{cases} 3x - 1, & \text{if } x \ge -1 \\ -5, & \text{if } x < -1 \end{cases}$

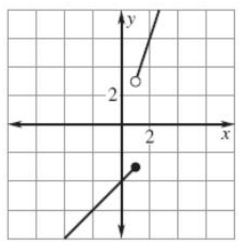
18. $f(x) = \begin{cases} -3x - 1, & \text{if } x \le 1 \\ -5, & \text{if } x > 1 \end{cases}$

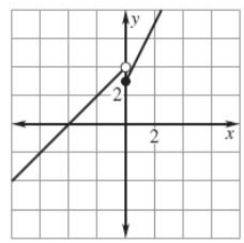
A.


B.


C.


D.


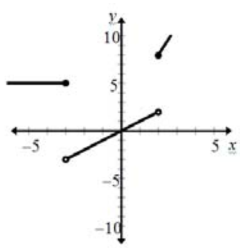
E.


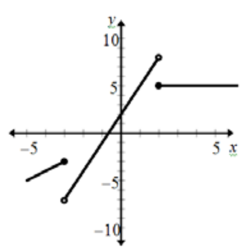
F.


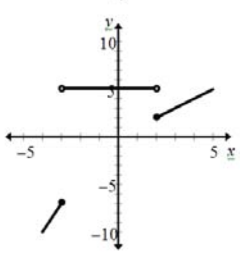
Ans: 13 - E, 14 – B, 15 – D, 16 – F, 17 – C, 18 - A

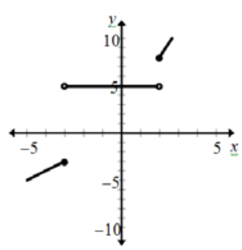
2. Which graph represents the piecewise function below?

$$f(x) = \begin{cases} x, & \text{if } x \ge 2 \\ 5, & \text{if } -3 < x < 2 \\ 3x + 2, & \text{if } x \le -3 \end{cases}$$

A.


C.


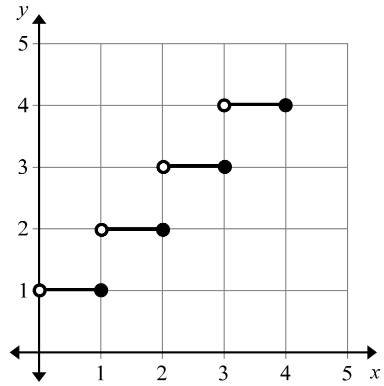
B.




D.

Ans: B

3. Which function represents the graph below?



A. $f(x)=\begin{cases} x+1, & \text{if } 0<x<4 \\ x, & \text{if } 0\le x\le 4 \end{cases}$

C. $f(x)=\begin{cases} 1, & \text{if } 0\le x<1 \\ 2, & \text{if } 1\le x<2 \\ 3, & \text{if } 2\le x<3 \\ 4, & \text{if } 3\le x<4 \end{cases}$

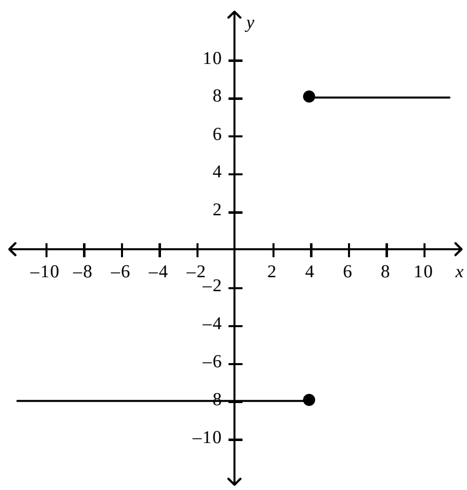
B. $f(x)=\begin{cases} x, & \text{if } x\in\{1,2,3,4\} \\ x+1, & \text{if } x\in\{0,1,2,3\} \end{cases}$

D. $f(x)=\begin{cases} 1, & \text{if } 0<x\le 1 \\ 2, & \text{if } 1<x\le 2 \\ 3, & \text{if } 2<x\le 3 \\ 4, & \text{if } 3<x\le 4 \end{cases}$

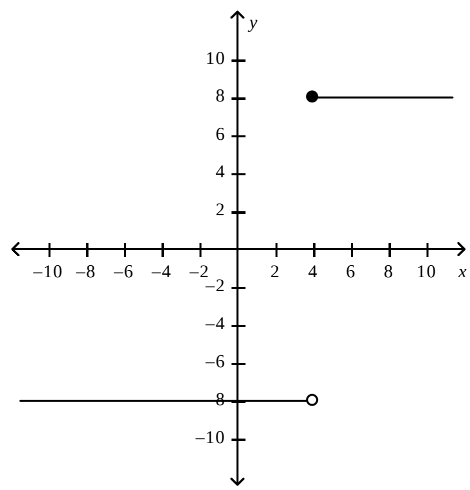
Ans: D

4. Graph the function: $y=\begin{cases} -8, & \text{if } x<4 \\ 8, & \text{if } x\ge 4 \end{cases}$

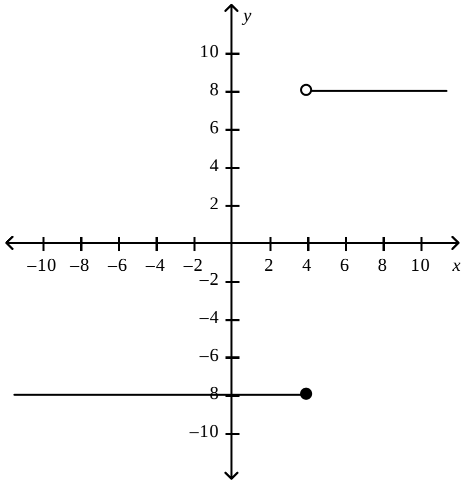
a.



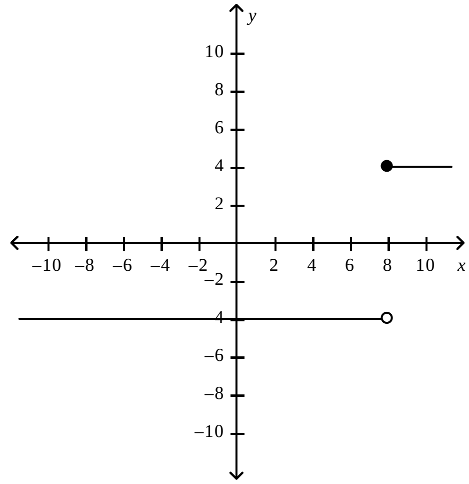
c.



b.



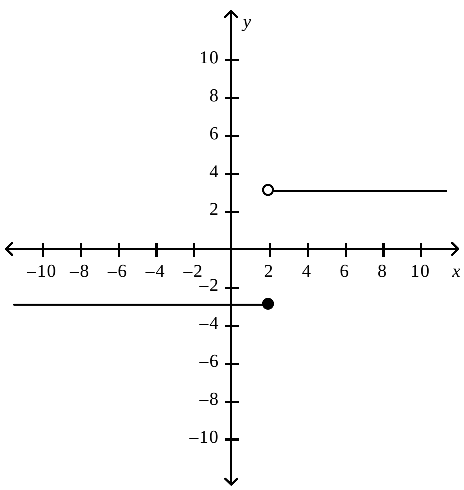
d.



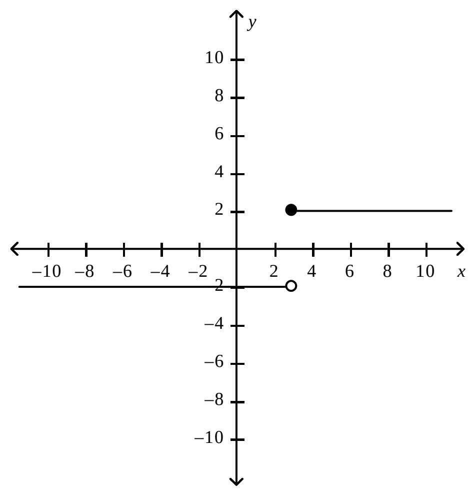
Ans: B

5. **Graph** $g(x) = \begin{cases} -3 \text{ if } x < 2 \\ 3 \text{ if } x \geq 2 \end{cases}$.

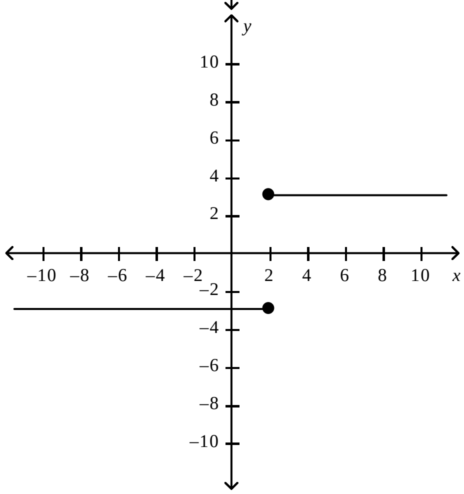
a.



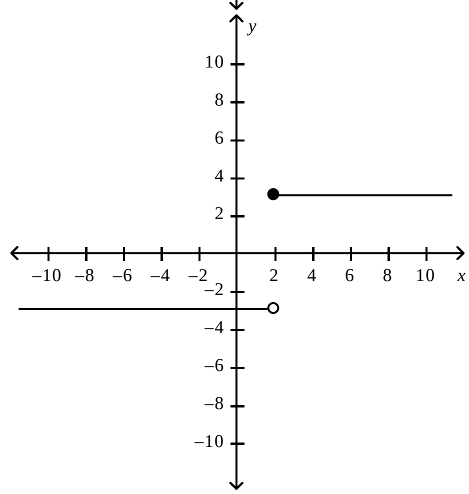
c.



b.



d.

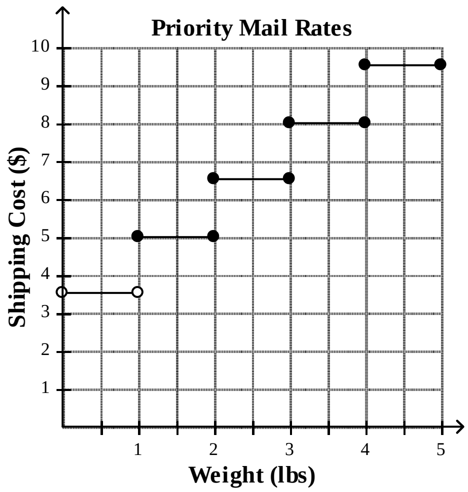


Ans: D

6. A shipping company charges \$3.50 to ship a package weighing one pound or less. Then they charge \$1.50 for each additional pound, or fraction of a pound, up to five pounds. Write a piecewise function that gives the price *P* for shipping a package weighing *w* pounds. Graph the function.

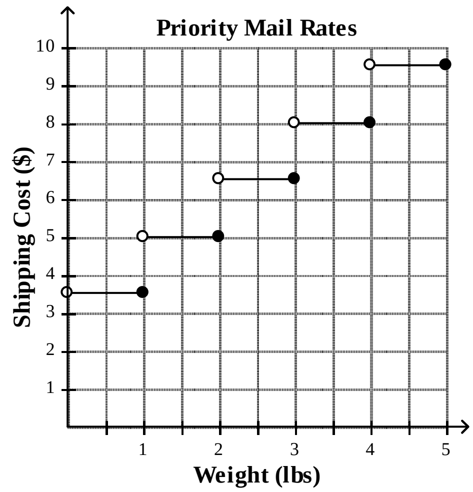
a.

$$P = \begin{cases} 3.5, & \text{if } 0 < x < 1 \\ 5, & \text{if } 1 \le x \le 2 \\ 6.5, & \text{if } 2 \le x \le 3 \\ 8, & \text{if } 3 \le x \le 4 \\ 9.5, & \text{if } 4 \le x \le 5 \end{cases}$$



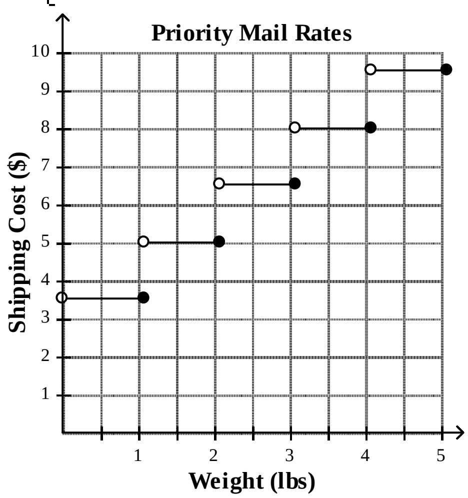
c.

$$P = \begin{cases} 3.5, & \text{if } 0 < x \le 1 \\ 5, & \text{if } 1 < x \le 2 \\ 6.5, & \text{if } 2 < x \le 3 \\ 8, & \text{if } 3 < x \le 4 \\ 9.5, & \text{if } 4 < x \le 5 \end{cases}$$



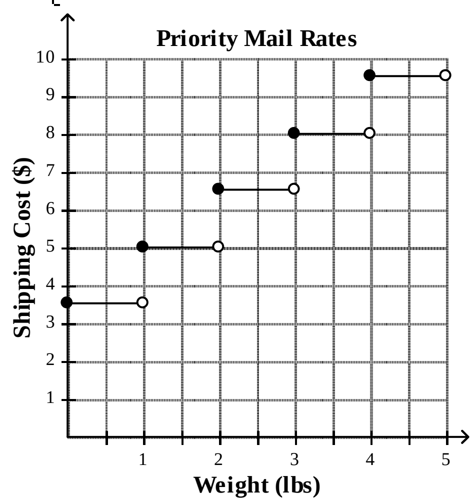
b.

$$P = \begin{cases} 3.5, & \text{if } 0 < x \le 1.1 \\ 5, & \text{if } 1.1 < x \le 2.1 \\ 6.5, & \text{if } 2.1 < x \le 3.1 \\ 8, & \text{if } 3.1 < x \le 4.1 \\ 9.5, & \text{if } 4.1 < x \le 5.1 \end{cases}$$



d.

$$P = \begin{cases} 3.5, & \text{if } 0 \le x < 1 \\ 5, & \text{if } 1 \le x < 2 \\ 6.5, & \text{if } 2 \le x < 3 \\ 8, & \text{if } 3 \le x < 4 \\ 9.5, & \text{if } 4 \le x < 5 \end{cases}$$

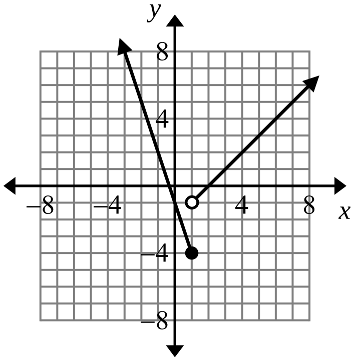


Ans: C

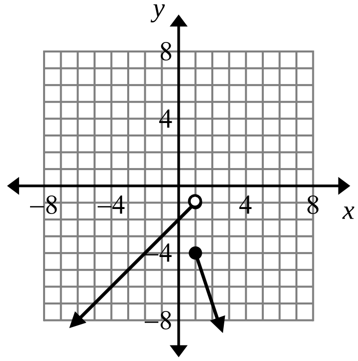
7. Which graph represents the piecewise function?

$$f(x)=\begin{cases}3x-1, & x\ge 1\\ -x-2, & x<1\end{cases}$$

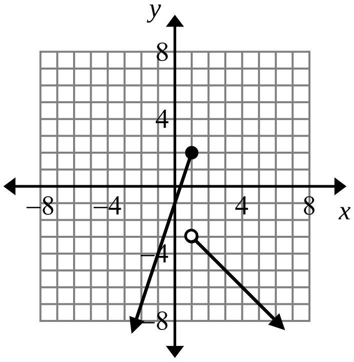
A.



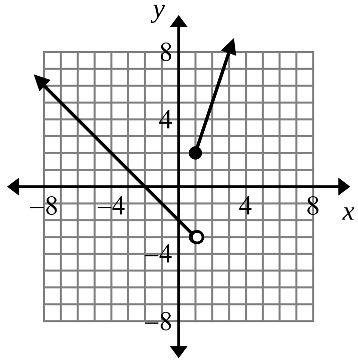
C.



B.



D.



Ans: D

8. Evaluate $f(-3)$ for the piecewise function:

$$f(x)=\begin{cases}x, & x\le 0\\ x^2-3x, & x>0\end{cases}$$

A. $f(-3)=-18$

B. $f(-3)=-3$

C. $f(-3)=0$

D. $f(-3)=18$

Ans: B