Inverse of a Linear Function

Math Background

Previously, you
- Performed operations with linear functions
- Identified the domain and range of linear functions

In this unit you will
- Solve a linear equation for a given y value
- Solve a literal equation for a given variable
- Find the inverse of a linear function
- Describe how the domain of a function must be restricted so the function has an inverse

You can use the skills in this unit to

- Find the inverse of a linear function and recognize restrictions on its domain
- Solve for the independent variable in terms of the dependent variable in real-world situations
- Read values of an inverse function from a graph or a table, given that the function has an inverse
- Graph the inverse of a function by reflecting the graph over the line y = x

Vocabulary

- **Domain** – The input values of a relation.
- **Horizontal line test** – The inverse of a function *f* is also a function if and only if no horizontal line intersects the graph of *f* more than once.
- **Inverse functions** – Two functions are inverse functions if the domain of the original function matches the range of the second function.
- **Inverse relation** – Interchanges the input and output values of the original relation**.**
- **Literal equation** – An equation that involves two or more variables.
- **One-to-one function** – A function whose inverse is a function. Both must pass the vertical and horizontal line tests
- **Range** – The output values of a relation.
- **Vertical line test** – A relation is a function if and only if no vertical line intersects the graph of the function more than once.

Essential Questions

- How is the inverse of a relation related to the relation?
- How do you find the inverse relation of a given function?

Overall Big Ideas

The inverse of a relation is “created” by reflecting the ordered pairs about the line y = x. Relations can be restricted to force the inverse to be a function. Using Algebra we can find the inverse function for any line.

Skill

To find and graph the inverse of a linear function.

Related Standards

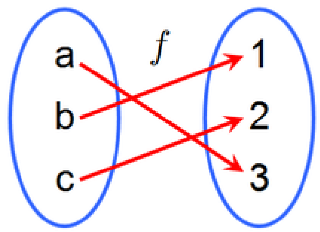
F.BF.B.4a-1

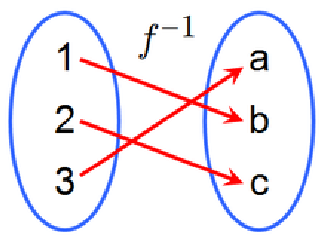
Solve an equation of the form $f(x)=c$ for simple linear and quadratic functions *f* that have an inverse and write an expression for the inverse. For example, $f(x)=2x^3$ or $f(x)=\frac{(x+1)}{(x-1)}$ for $x \neq 1$

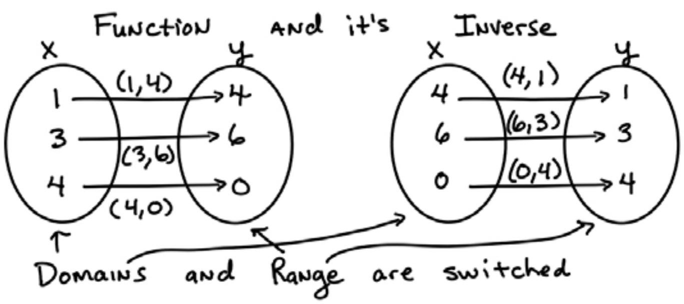
Notes, Examples, and Exam Questions

In this unit, we will find the inverse of a linear function and see how the original function is related to its inverse. Let's take a look at a mapping diagram. The one on the right top shows our original function, *f*. The mapping on the bottom shows the inverse of f, f^{-1}, the original outputs become the inputs, and the original inputs become the outputs.

The notation $f^{-1}(x)$ indicates the inverse of a function $f(x)$. The domain of $f^{-1}(x)$ is the range of $f(x)$, and the range of $f^{-1}(x)$ is the domain of $f(x)$.







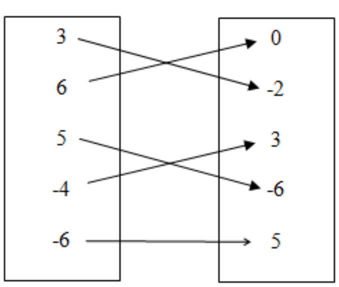
**Note: The symbol -1 in f^{-1} is not to be interpreted as an exponent. In other words, $f^{-1}(x) \neq \dfrac{1}{f(x)}$.

The procedure for finding the inverse of a linear function is fairly basic.

1) Substitute y for f(x)
2) Switch the "x" and the "y" in the equation
3) Solve for y. You have now solved for the inverse of the function.

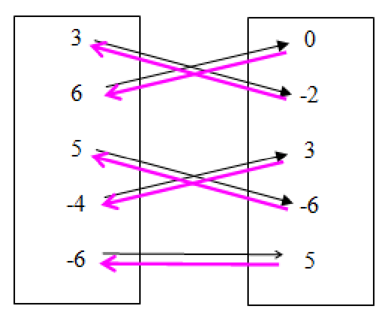
Ex 1

Find the ordered pairs for the function and then find the ordered pairs for the inverse function. Show the inverse mapping on the figure below as well.



Ans: Function:
{(3, -2), (6, 0), (5,-6), (-4, 3), (-6, 5)}

Inverse:
{(-2, 3), (0, 6), -6, 5), (3, -4), (5, -6)}



Inverse mapping is in pink.

Ex 2

Find the inverse of $f(x)=3x-2$.

Here's the original function:	$y=3x-2$
Interchange the x and the y:	$x=3y-2$
Solve for y:	$x+2=3y$ $\frac{x+2}{3}=y$
Rewrite in function notation:	$f^{-1}(x)=\frac{x+2}{3}$

If you need to find the domain and range, look at the original function and its graph and since it is a linear function, the domain was "all real numbers" and the range is "all real numbers". To find the domain and range of the inverse, just swap the domain and range from the original function. For linear functions, domain and range will always be "all real numbers" for the original function and the inverse function.

Ex 3

Find the inverse of the given function. $f(x)=-\frac{2}{3}x+4$

$y=-\frac{2}{3}x+4$ switch the variables and solve

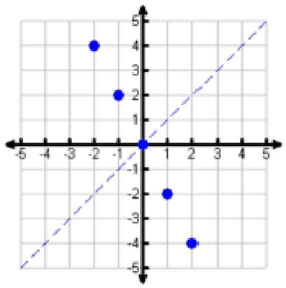
$x=-\frac{2}{3}y+4 \Rightarrow x-4=-\frac{2}{3}y \Rightarrow -\frac{3}{2}(x-4)=y$

$\frac{-3x+4}{2}=y$

Ans: $f^{-1}(x)=\frac{4-3x}{2}$

The graph of an inverse relation is a *reflection* of the graph of the original relation. The line of reflection is $y=x$. Graph the original function, graph the line $y=x$, and reflect the figure over that line.

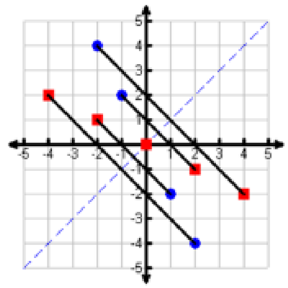
The original relation is the set of ordered pairs: {(-2, 1), (-1, 2), (0, 0), (1, -2), (2, -4)}. The inverse relation is the set of ordered pairs: {(1, -2), (2, -1), (0, 0), (-2, 1), (-4, 2)}. Notice that for the inverse relation the domain (x) and the range (y) reverse positions.



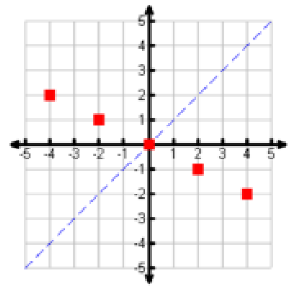
Original Relation

Domain: {-2, -1, 0, 1, 2}

Range: {-4, -2, 0, 1, 2}



The points are reflected over the line $y=x$. Notice that each point is the same distance away from the line, but on the opposite side of the line.



Inverse Relation

Domain: {-4, -2, 0, 1, 2}

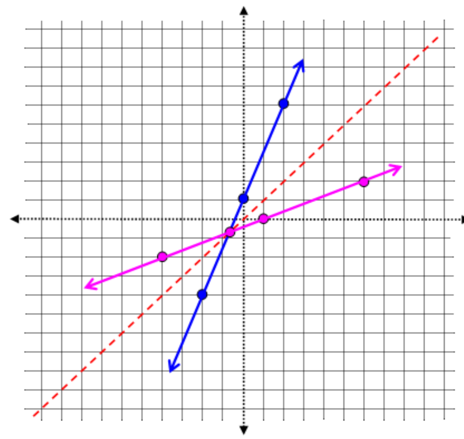
Range: {-2, -1, 0, 1, 2}

If no vertical line intersects the graph of a function f more than once, then f is a function. This is called the **vertical line test.** If no horizontal line intersects the graph of a function f more than once, then the inverse of f is itself a function. This is called the **horizontal line test.** A function is a **one-to-one function** if and only if each second element corresponds to one and only one first element. In order for the inverse of a function to be a function, the original function must be a one-to-one function and meet the criteria for the vertical and horizontal line tests.

Ex 4

Graph the inverse of $f(x)=\frac{5}{2}x+1$.

To graph the function, three ordered pairs were found: (0, 1), (2, 6), and (-2, -4). The original function is in blue. The red dashed line is the reflection line y = x and the ordered pairs were then reflected over this line: (1, 0), (6, 2), (-4, -2). The pink line represents the inverse graph of f(x).



Ex 5

The formula $F = f(C) = 1.8C + 32$ converts temperatures in degrees Celsius, *C*, to degrees Fahrenheit, *F*.

a) What is the *input* to the function f? What is the *output* ?

Ans: input: temperature in degrees Celsius, output: temperature in degrees Fahrenheit

b) Find a formula for the inverse function giving Celsius as a function of Fahrenheit.

$$F = 1.8C + 32 \Rightarrow F - 32 = 1.8C \Rightarrow \frac{F-32}{1.8} = C$$

***Note that you do not switch the variables when you are finding inverses of models. This would be confusing because the letters are chosen to remind you of the real-life quantities they represent.

c) Use inverse notation to write your formula. $f^{-1}(F)=\frac{F-32}{1.8}$

d) What is the *input* to the function f^{-1}? the *output* ?

Ans: input: temperature in degrees Fahrenheit, output: temperature in degrees Celsius

e) Interpret the meaning of the notation: $f(50)=122$ Ans: 50° Celsius is 122° Fahrenheit

f) Interpret the meaning of the notation: $f^{-1}(200)=93.3$ Ans: 200° Fahrenheit is 93.3° Celsius

Sample Exam questions

1. **What is the inverse of** $f(x) = 2x + 9$**?**

   A. $f^{-1}(x) = \frac{x}{2} + 9$ **C.** $f^{-1}(x) = \frac{x-9}{2}$

   B. $f^{-1}(x) = \frac{1}{2x+9}$ **D.** $f^{-1}(x) = \frac{2}{x-9}$ Ans: C

2. **If** $f^{-1}(x) = \frac{4}{3}x + 8$**, what is** $f(x)$**?**

   A. $f(x) = \frac{3}{4}(x-8)$ **C.** $f(x) = \frac{4}{3}x - 6$

   B. $f(x) = \frac{3}{4}x - 8$ **D.** $f(x) = \frac{4}{3}(x-8)$ Ans: A

3. **If the point (a, b) lines on the graph** ***y= f(x)*, the graph of** ***y =*** $f^{-1}(x)$ **must contain point**

   A. (0, b) **B.** (a, 0) **C.** (b, a) **D.** (-a, -b) Ans: B

4. **The inverse function of** {(2, 6), (-3, 4), (7, -5)} **is**

   A. {(-2,6),(3,4),(-7,-5)} **C.** {(6,2),(4,-3),(-5,7)}

   B. {(2,-6),(-3,-4),(7,5)} **D.** {(-6,-2),(-4,3),(5,-7)} Ans: C

5. **Given the relation** ***A*:** {(3, 2), (5, 3), (6, 2), (7,4)}

   A. Both *A* and A^{-1} are functions. **C.** Only A^{-1} is a function.

   B. Neither *A* nor A^{-1} are functions. **D.** Only *A* is a function. Ans: D

6. **The inverse of the function** $2x + 3y = 6$ **is**

   A. $y = -\frac{2}{3}x + 2$ **C.** $y = \frac{3}{2}x + 2$

   B. $y = -\frac{3}{2}x + 3$ **D.** $y = \frac{2}{3}x + 3$ Ans: B