Parent Functions and Transformations

Math Background

Previously, you

- Studied linear, absolute value, exponential and quadratic equations
- Graphed linear, absolute value, exponential and quadratic equations
- Recognized what patterns governed transformations of functions

In this unit you will

- Apply transformations to linear functions
- Apply transformations to quadratic functions
- Apply transformations to absolute value functions
- Apply transformations to exponential functions

You can use the skills in this unit to

- Identify graphs of parent functions
- Identify translations of functions from a graph and/or table
- Use technology to determine the transformation to the graph of a function

Vocabulary

- **Constant** – A quantity whose value stays the same and can be added or multiplied to a function.
- **Dilation** – A transformation in which a figure is enlarged (stretched) or reduced (shrunk).
- **Parent function** – The simplest function with the defining characteristics of the family. Functions in the same family are transformations of their parent function.
- **Reflection** – A transformation in which every point of a figure is mapped to a corresponding image across a line of symmetry.
- **Transformation** – A change in the position, size, or shape of a figure or graph.
- **Translation** – A figure is moved from one location to another on the coordinate plane without changing its size, shape or orientation.

Essential Questions

- How does multiplying by a constant to a function change the graph?
- How does adding a constant to a function change the graph?
- How does the knowledge of transformations help me to visualize and graph the function?

Overall Big Ideas

Graphs can be grouped into families that can be defined by a single parent function. The parent function is the most basic graph and can be transformed to create the other graphs in the family. Translation, reflection, and dilation are three types of transformations applied to the parent functions. Translations move the graph up, down, left, and right. Reflection flips the graph over the line of reflection, while dilation shrinks or enlarges the graph proportionally.

Skill

To apply transformations to parent functions (Absolute Value, Linear, Quadratic, and Exponential).

Related Standards

F.IF.B.5-1

Relate the domain of a linear, exponential, or quadratic function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. *(Modeling Standard)

F-BF.B.3-1

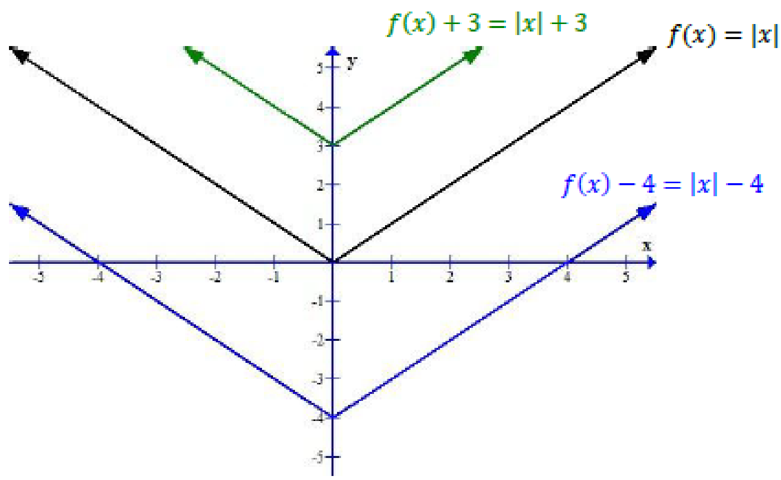
Identify the effect on the graph of replacing $f(x)$ by $f(x) + \text{k}$, $k\,f(x)$, $f(\text{k}x)$, and $f(x + \text{k})$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

Notes, Examples, and Exam Questions

In this unit, we will discuss how the graph of a function may be transformed either by shifting, stretching or compression, or reflection. The variables *k*, *h*, and *a* will be positive real numbers. A transformation changes a graph's size, shape, position, or orientation from the parent function. A translation is a transformation that shifts a graph horizontally and/or vertically; but does not change its size, shape or orientation.

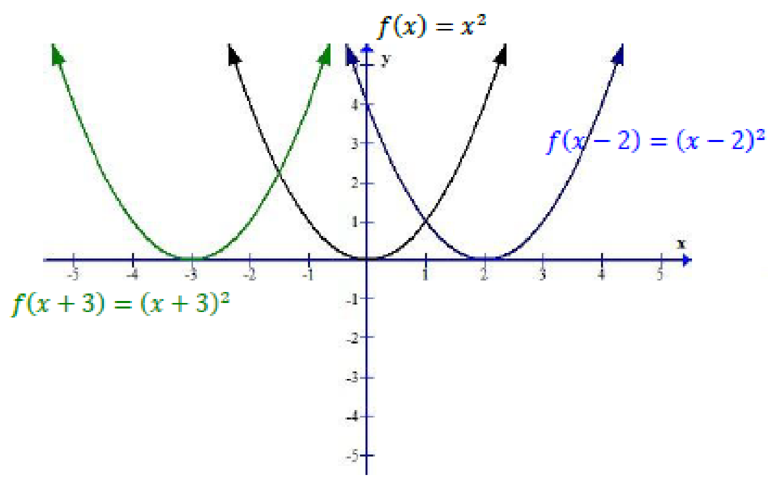
Vertical Translations

A shift may be referred to as a translation. If *k* is added to the function, where the function becomes $y = f(x) + k$, then the graph of $f(x)$ will vertically shift upward by *k* units. If *k* is subtracted from the function, where the function becomes $y = f(x) - k$, then the graph of $f(x)$ will vertically shift downward by *k* units.



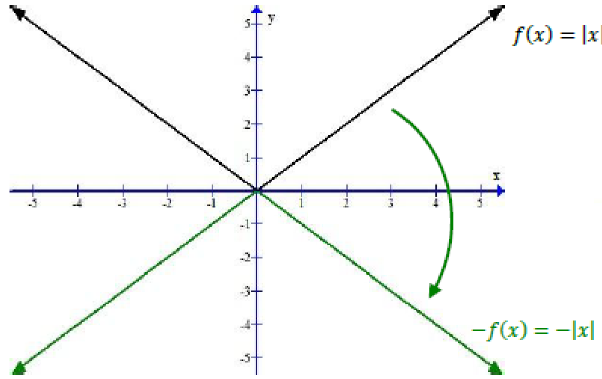
Horizontal Translations

If *h* is added to the variable of the function, where the function becomes $y = f(x+h)$, then the graph of $f(x)$ will horizontally shift to the left *h* units. If *h* is subtracted from the variable of the function, where the function becomes $y = f(x-h)$, then the graph of $f(x)$ will horizontally shift to the right *h* units.



Reflection

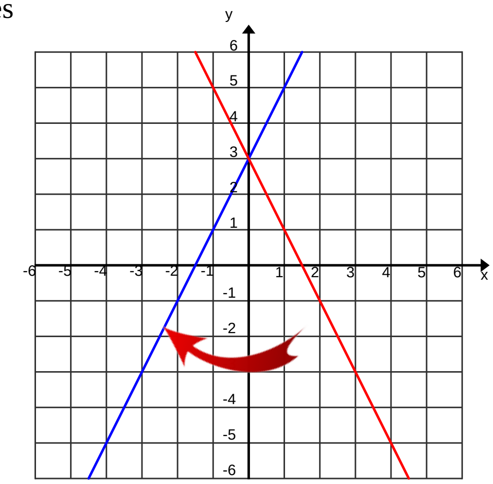
If the function or the variable of the function is multiplied by -1, the graph of the function will undergo a reflection. When the function is multiplied by -1 where $y = f(x)$ becomes $y = -f(x)$, the graph of $y = f(x)$ is reflected across the x-axis. This is a transformation as the graph's position is changed.



On the other hand, if the variable is multiplied by -1, where $y = f(x)$ becomes $y = f(-x)$, the graph of $y = f(x)$ is reflected across the y-axis.

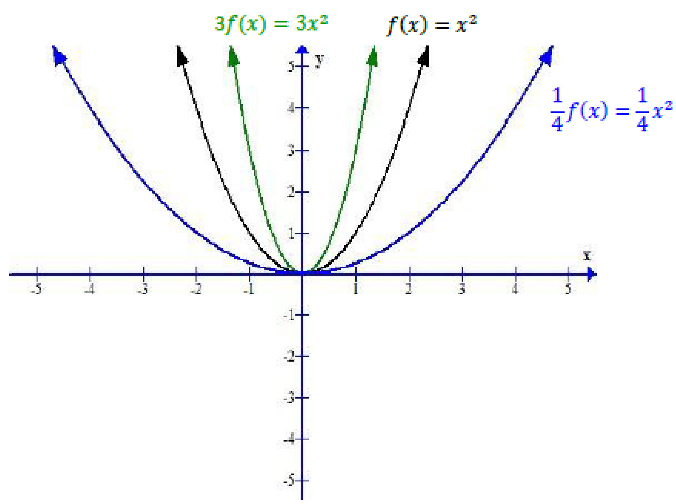
Red: $f(x) = 2x + 3$

Blue: $f(x) = 2(-x) + 3 = -2x + 3$

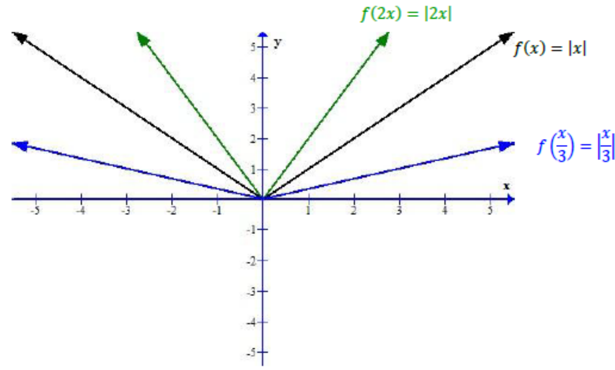


Vertical Stretching and Shrinking

If a is multiplied to the function then the graph of the function will undergo a vertical stretching or compression. So, when the function becomes $y = af(x)$ and $0 < a < 1$, a vertical shrinking of the graph of $y = f(x)$ will occur. Graphically, a vertical shrinking pulls the graph of $y = f(x)$ toward the x-axis. When $a > 1$ in the function $y = af(x)$, a vertical stretching of the graph of $y = f(x)$ will occur. A vertical stretching pushes the graph of $y = f(x)$ away from the x-axis. This is a transformation, not a translation as the graph's size is changed.



Horizontal Stretching and Shrinking



If a is multiplied to the variable of the function then the graph of the function will undergo a horizontal stretching or compression. So, when the function becomes $y = f(ax)$ and $0 < a < 1$, a horizontal stretching of the graph of $y = f(x)$ will occur. Graphically, a vertical stretching pulls the graph of $y = f(x)$ away from the y-axis. When $a > 1$ in the function $y = f(ax)$, a horizontal shrinking of the graph of $y = f(x)$ will occur. A horizontal shrinking pushes the graph of $y = f(x)$ toward the y-axis. This is a transformation, not a translation.

General Transformation Equations:

Linear: $y = a(x-h)+k$ **Quadratic:** $y = a(x-h)^2+k$ **Absolute Value:** $y = a|x-h|+k$

Exponential: $y = ab^{x-h}+k$

Multiple Transformations

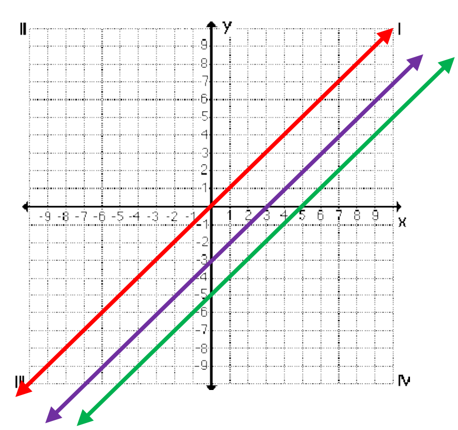
Transformations can be combined within the same function so that one graph can be shifted, stretched and reflected. If a function contains more than one transformation, it may be graphed using the following procedure:

Steps for Multiple Transformations

Use the following order to graph a function involving more than one transformation:

1. Horizontal Translation
2. Stretching or shrinking
3. Reflecting
4. Vertical Translation

Ex 1



Let $f(x) = x$. Graph $f(x)-3$ and $f(x-5)$.

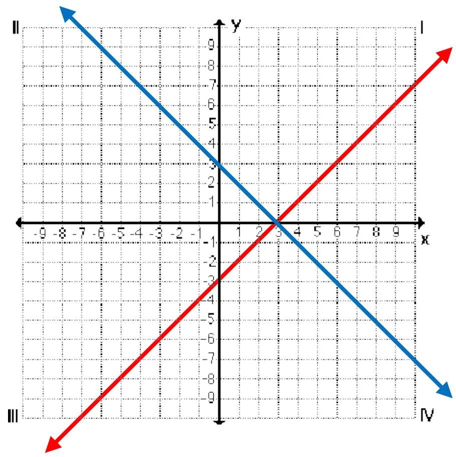
$f(x)-3 = x-3$, would translate the graph and all its points down three units from the parents function original location.

$f(x-5) = (x-5) = x-5$, would translate the graph and all its points 2 units to the right.

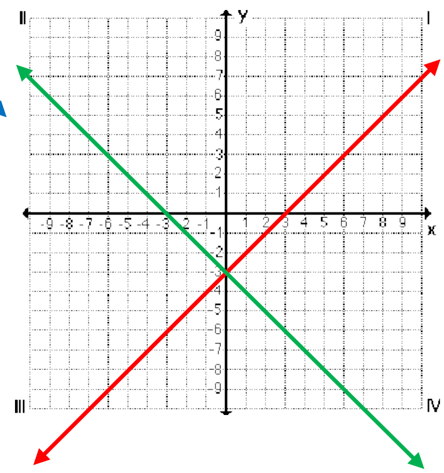
Ex 2

Let $f(x) = x - 3$. Graph $-f(x)$ which reflects the graph in the x-axis, and $f(-x)$ which reflects the graph in the y-axis.

Example: $-f(x) = -(x-3) = -x+3$



Example: $f(-x) = (-x) - 3 = -x - 3$

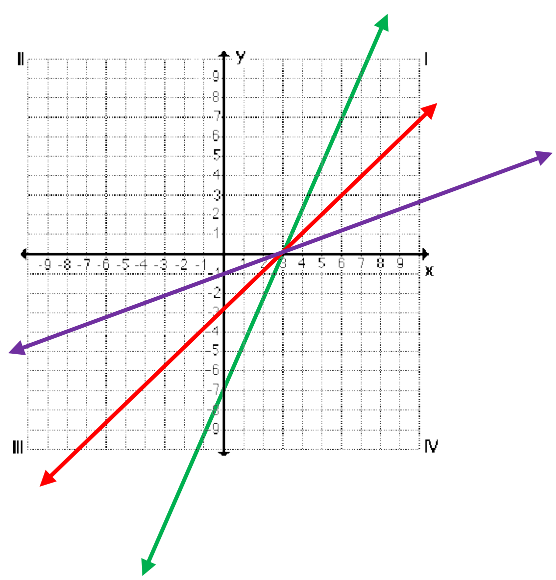


Ex 3

Let $f(x) = x - 3$. Graph $2f(x)$ which stretches the graph vertically, and graph $\frac{1}{3}f(x)$ which compresses the graph vertically.

Example: $2f(x) = 2(x-3) = 2x - 6$

Example: $\frac{1}{3}f(x) = \frac{1}{3}(x-3) = \frac{1}{3}x - 1$

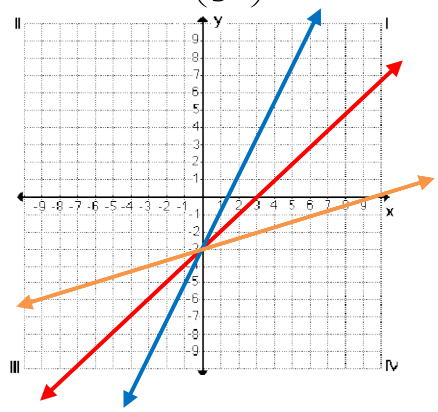


Ex 4

Additionally, the graph $f(2x)$ will compress the graph horizontally, and $f\left(\frac{1}{3}x\right)$ will stretch the graph horizontally.

Example: $f(2x) = (2x) - 3 = 2x - 3$

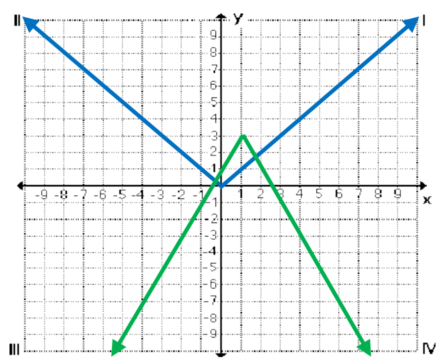
Example: $f\left(\frac{1}{3}x\right) = \left(\frac{1}{3}x\right) - 3 = \frac{1}{3}x - 3$



Ex 5

Graph $f(x) = |x|$ and $f(x) = -2|x-1| + 3$

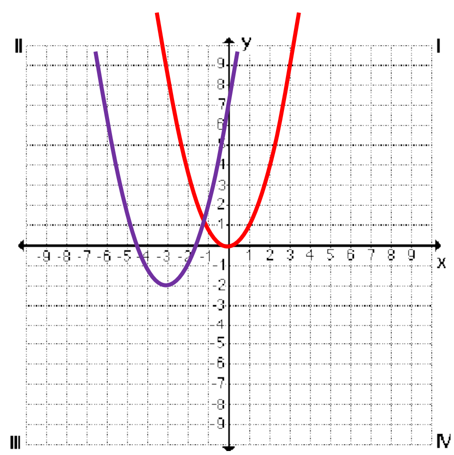
The graph is shifted one to the right, stretched by a factor of 2, flipped over the x-axis and shifted up 3.



Ex 6

Example: Graph $f(x) = x^2$ and $f(x) = (x+3)^2 - 2$

The graph is shifted three to the left and down 2.

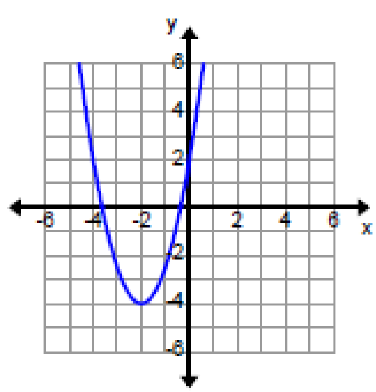


Ex 7

Describe the transformations performed on $f(x) = x^2$ to make it $f(x) = \frac{3}{2}(x+2)^2 - 4$. Then, graph the function.

Ans:

- Shifted two units to the left
- Vertical stretch of 3/2. The y-coordinates are multiplied by 3/2.
- Shifted down four units

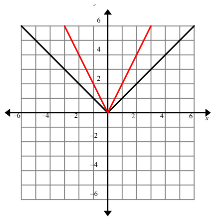


Ex 8

Graph the following on the same coordinate plane and describe the transformation of y_1.

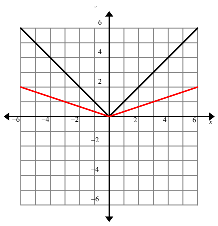
$$y_1 = |x|,\ y_2 = 2|x|,\ y_3 = \frac{1}{3}|x|$$

$y_1 = |x|,\ y_2 = 2|x|$



y_2 is a vertical stretch of y_1 by a factor of 2.

$y_1 = |x|,\ y_3 = \frac{1}{3}|x|$



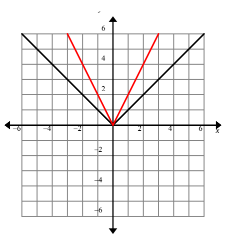
y_3 is a vertical shrink of y_1 by a factor of $\frac{1}{3}$.

Ex 9

Graph the following on the same coordinate plane and describe the transformation of y_1.

$$y_1 = |x|,\ y_2 = |2x|,\ y_3 = \left|\frac{x}{3}\right|$$

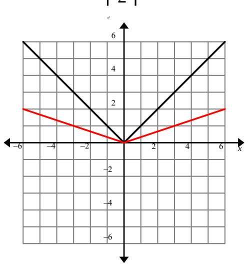
$y_1 = |x|,\ y_2 = |2x|$



y_2 is a horizontal shrink of y_1 by a factor of $\frac{1}{2}$.

Note: $y_2 = |2x| = \left|\frac{x}{\frac{1}{2}}\right|$

$y_1 = |x|,\ y_3 = \left|\frac{x}{3}\right|$



y_3 is a horizontal stretch of y_1 by a factor of 3.

**Note: A horizontal stretch causes a vertical shrink and a horizontal shrink causes a vertical stretch.

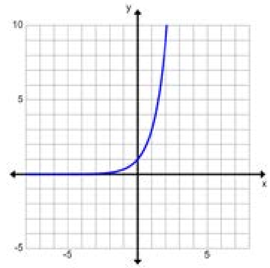
For exponential transformations, use the general exponential equation: $f(x) = ab^{x-h} + k$, where *a* indicates the vertical dilation, *h* is the horizontal translation and *k* is the vertical translation.

Ex 10

Let $f(x) = 3^x$.

The graph is shown at the right.

Graph the following functions:



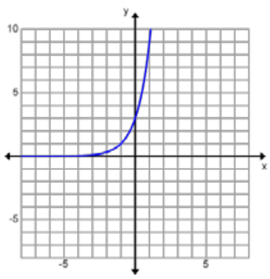
a) Shift the graph one unit to the left.

b) Shift the graph down one unit.

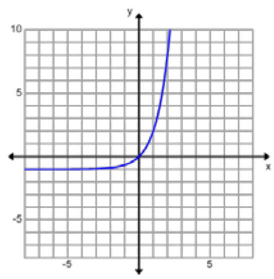
c) Reflect the graph over the x-axis.

d) Reflect the graph over the y-axis and shift up one unit.

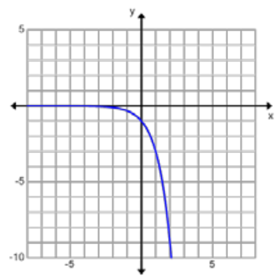
a) $f(x) = 3^{x+1}$



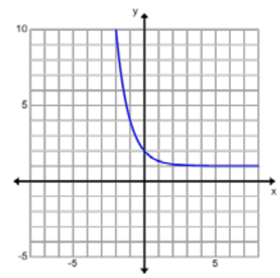
b) $f(x) = 3^x - 1$



c) $f(x) = -3^x$

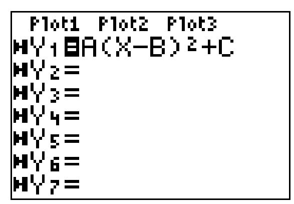


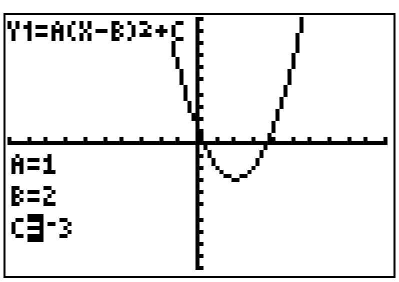
d) $f(x) = 3^{-x} + 1$

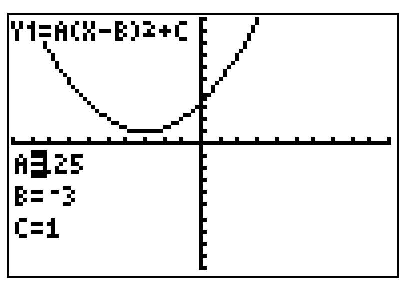


<u>Technology:</u>

You can use the Transfrm Application on the TI-84 Plus to investigate families of functions.
Step 1: Go to the APPS key, find Transfrm and hit enter.
Step 2: Press the Y= key and input the following into Y_1: $A(X - B)^2 + C$
Step 3: Set your window (ZOOM 6) and use the cursors to change the values of parameters A, B and C in the quadratic equation.







Students are able to see the effects of the parameters on the graph. This transform application can also be used with other functions. Try inputting the following to look at linear, absolute value and exponential.

$A(X - B) + C$

$A2\wedge(X - B) + C$

$Aabs(X - B) + C$

Note: You must use the parameters A, B and C on the TI-84 app. It will not recognize a, h and k.

SAMPLE EXAM QUESTIONS

1. How would the graph of the function $y = x^2 - 8$ be affected if the function were changed to $y = x^2 - 3$?

   a. The graph would shift 5 units to the left.
   b. The graph would shift 5 units down.
   c. The graph would shift 5 units up.
   d. The graph would shift 3 units down.

   Ans: C

2. How would you translate the graph of $y = -|2x|$ to produce the graph of $y = -|2x| - 4$

   a. translate the graph of $y = -|2x|$ down 4 units
   b. translate the graph of $y = -|2x|$ up 4 units
   c. translate the graph of $y = -|2x|$ left 4 units
   d. translate the graph or $y = -|2x|$ right 4 units

   Ans: A

3. Compare the graph of $g(x) = x^2 + 6$ with the graph of $f(x) = x^2$.

   a. The graph of g(x) is wider.
   b. The graph of g(x) is narrower.
   c. The graph of g(x) is translated 6 units down from the graph of f(x).
   d. The graph of g(x) is translated 6 units up from the graph of f(x).

   Ans: D

4. Compared to the graph of $f(x) = 2^x$, the graph of $g(x) = 2(2^x) - 5$ is _______.

   a. stretched and translated down
   b. stretched and translated up
   c. shrunk and translated down
   d. shrunk and translated up

   Ans: A

5. Four bowls with the same height are constructed using quadratic equations as their shapes. Which bowl has the narrowest opening?

   a. Bowl 1: $\frac{1}{8}x^2$
   b. Bowl 2: $\frac{1}{4}x^2$
   c. Bowl 3: $5x^2$
   d. Bowl 4: $7x^2$

   Ans: D

6. The points $\{(-3, 2), (0, 1), (4, 5)\}$ are on the graph of function *f*. What are the coordinates of these three points after a horizontal stretch by a factor of 3, followed by a reflection across the *x*-axis?

a. $\{(-9, -2), (0,-1), (12, -5)\}$
b. $\left\{(-1, -2), (0, -1), \left(\frac{4}{3}, -5\right)\right\}$
c. $\{(-9, 2), (0, 1), (12, 5)\}$
d. $\{(-3, -6), (0, -3), (4, -15)\}$

Ans: A

Use this description to write the quadratic function in vertex form:

7. The parent function $f(x) = |x|$ is vertically stretched by a factor of 2 and translated 14 units right and 6 units up.

a. $g(x) = \frac{1}{2}|x-14|+6$
b. $g(x) = 2|x-14|+6$
c. $g(x) = 2|x-14|-6$
d. $g(x) = 2|x+14|+6$

Ans: B

8. Which function is NOT a translation of $f(x) = x^2 + 17$?

a. $f(x) = (x-4)^2 + 17$
b. $f(x) = x^2 - 4$
c. $f(x) = -x^2 - 17$
d. $f(x) = \left(x + \frac{1}{2}\right)^2$

Ans: C

9. Which function's graph is the widest parabola?

a. $y = \frac{1}{8}x^2$
b. $y = \frac{1}{3}x^2$
c. $y = 3x^2$
d. $y = 8x^2$

Ans: A

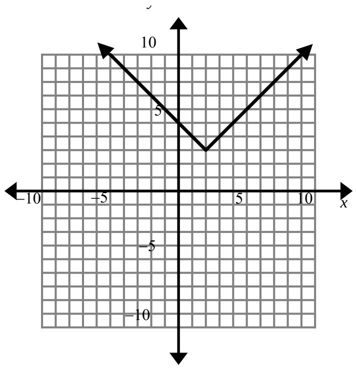
10. Which transformation from the graph of a function *f*(*x*) describes the graph of $10f(x)$?

a. horizontal shift left 10 units
b. vertical shift up 10 units
c. vertical stretch by a factor of 10
d. vertical shift down 10 units

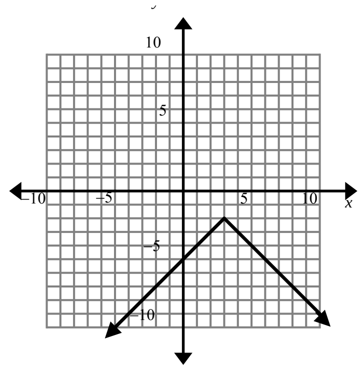
Ans: C

11. Which graph represents the equation $y = -|x-2| + 3$?

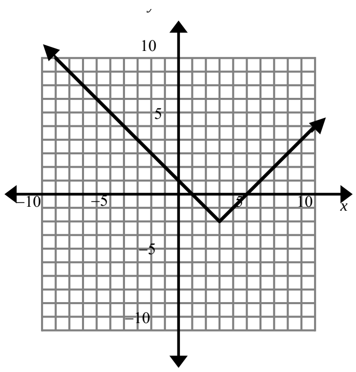
A.



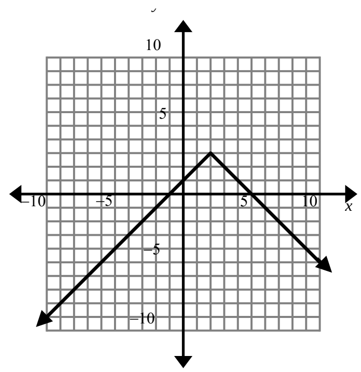
C.



B.

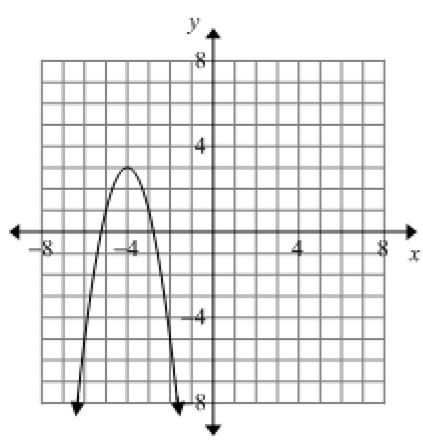


D.



Ans: D

12. Which equation represents the graph?



A. $y = 2(x+4)^2 + 3$

B. $y = 2(x-4)^2 - 3$

C. $y = -2(x+4)^2 + 3$

D. $y = -2(x-4)^2 - 3$

Ans: C