Precision and Accuracy

Skill: To read the appropriate measurement tool to the required degree of accuracy.

N.Q.A.3 - Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

We often use numbers that are not exact. Measurements are approximate—there is no such thing as a perfect measurement. The precision of a number refers to its exactness—to the level of detail to which the tool can measure. Measurements cannot be more precise than the measuring tool. This is very important in science!

Example: To what degree of precision can you measure a length using this ruler?

The smaller the unit of measurement, the more precise the measurement. Consider some measures of time, such as 15 seconds and 15 hours. A measure of 15 seconds implies it is precise to the nearest second, or a time interval between 14.5 and 15.5 seconds. The time of 15 hours is far less precise: it suggests a time between 14.5 and 15.5 hours. The potential error in the first interval is 0.5 seconds; the potential error in the 15 hour scenario is 0.5 hour or 1800 seconds. Because the potential for error is greater, the 15-hour-measure is less precise.

Ex 1: Choose the more precise measurement in the given pair.

(a) 3 m, 3.5 km  
(b) 12 inches, 1 foot  
(c) 1 pound, 1 ounce

3 m is more precise (meters are smaller than km)  
12 inches (inches are smaller than a foot)  
1 ounce (an ounce is smaller than a pound)

The number of decimal places in a measurement can also affect precision. Using time again, a measure of 5.1 seconds is more precise than 5 seconds. The 5.1 measurement implies a measure precise to the nearest tenth of a second. The potential error in 5.1 seconds is 0.05 seconds, compared to the potential error of 0.5 seconds with the measure of 5 seconds.

Ex 2: Choose the more precise measurement in the given pair.

(a) 5.4 m, 5.67 m  
(b) 3 yards, 3.6 yards  

5.67 is more precise (hundredths of a meter is smaller than tenths)  
3.6 is more precise (tenths of a yard is more precise than yards)
Skill: To will round to the appropriate significant digit.

This leads us to a discussion about significant digits. All the digits that are known with certainty are called significant digits. Below are the rules for determining significant digits. The only tricky digits are the zeros:

- All non-zero digits are significant digits.
  - 3 has one significant digit
  - 2.5 has two significant digits
  - 356.491 has six significant digits

- Zeros that occur between significant digits are significant digits.
  - 207 has 3 significant digits
  - 6.005 has 4 significant digits
  - 20.006 has 5 significant digits

- Zeros to the right of the decimal point AND to the right of a non-zero digit are significant digits.
  - 0.10 has 2 significant digits (the 0 before the decimal is not significant while the 0 to the right of the decimal point and the digit 1 are significant)
  - 0.0040 has 2 significant digits (just the last two)
  - 4.60 has 3 significant digits
  - 460 has 2 significant digits (zero is to the left of the decimal point)
  - 46.00 has 4 significant digits
  - 460.00 has 5 significant digits (the two zeros to the right of the decimal point are significant—this makes the zero to the left of the decimal point significant because it lies between significant digits)

Ex 3: Determine the number of significant digits in each measurement.

(a) 32.75  4 significant digits (all nonzero digits)
(b) 43.023  5 significant digits (zero is between significant digits)
(c) 0.0240  3 significant digits (zero after the last nonzero digit and to the right of the decimal point is significant)
(d) 0.007  1 significant digit

Differing levels of precision can cause us a problem when dealing with arithmetic operations. Suppose I wish to add 11.1 seconds to 13.47 seconds. The answer, 24.57 seconds, is misleading. That is:

11.1 seconds implies the time is between 11.05 and 11.15 seconds
13.47 seconds implies the time is between 13.465 and 13.475 seconds
The sum should imply the time is between 24.515 and 24.625 seconds

But the sum 24.57 seconds implies the time is between 24.565 and 24.575, which is more precise than the actual result.

So it is generally accepted that when you add or subtract, you report your answer to the same precision as the least precise measure. In other words, the answer should have the same number of digits to the right of the decimal point as the measurement with the least number of digits to the right of the decimal point. In our example, we would report 24.57 as 24.6 seconds.
**Ex 4:** Calculate. Use the correct number of significant digits.

(a) \( 4.5 + 2.17 = 6.67, \)
\[ \approx 6.7 \quad \text{Answer rounded to 1 digit to the right of the decimal} \]
\[ \text{(tenths)} \]

(b) \( 15 - 5.6 = 9.4 \quad \text{Answer rounded to no digits to the right of the decimal} \)
\[ \approx 9 \]

Multiplying or dividing measures creates a different type of problem. For instance, I want to find the area of a rectangle that measures 2.7 cm by 4.6 cm. When I multiply, I obtain the answer 12.42 cm\(^2\). However,

2.7 implies 2.65 cm to 2.75 cm
4.6 implies 4.55 cm to 4.65 cm

The product should imply 12.0575 cm\(^2\) to 12.7875 cm\(^2\)
But the product 12.42 cm\(^2\) implies 12.415 cm\(^2\) to 12.425 cm\(^2\), which is more precise than the actual result.

The accepted practice when multiplying or dividing is to report the result using the fewest number of significant digits in the original measures given. Or, when you multiply or divide measurements, the answer should have the same number of significant digits as the measurement with the least number of significant digits. In our example, there are two significant digits in 2.7 cm and 4.6 cm, so the result is rounded to two significant digits, 12 cm\(^2\).

**Ex 5:** Calculate. Use the correct number of significant digits.

(a) Find the area of a parallelogram with height of 12.1 inches and base of 6 inches.
\[ \text{Area} = (12.1)(6), \text{which is 72.6. However, the lowest number of significant digits is one, so we would round the answer to 70 square inches.} \]

(b) \( 14.2 \div 0.05 \)
\[ 14.2 \div 0.05 = 284. \text{However, the lowest number of significant digits is one, so we would round the answer to 300.} \]

Another concept that has to do with measurement is accuracy. Many people think precision and accuracy are the same thing. They are not! The accuracy of a measurement refers to how close the measured value is to the true or accepted value. For example, if you are in a lab and you obtain a weight measurement of 4.7 kg for an object, but the actual or known weight is 10 kg, then your measurement is not accurate (your measurement is not close to the accepted value). However, if you weigh the object five times, and get 4.7 kg each time, then your measurement is precise—each measurement was the same as the previous. Precision is independent of accuracy. In this case, you were very precise, but inaccurate.

Another way to explain it:

Imagine a basketball player shooting baskets. If the player shoots with precision, his aim will always take the ball to the same location, which may or may not be close to the basket. If the player shoots with accuracy, his aim will always take the ball close to or into the basket. A good player will be both precise and accurate: shoot the ball the same way each time and make the basket.
If you are a soccer player, and you always hit the left goal post (instead of scoring), what can you conclude? You are precise, but not accurate!

A dartboard analogy is often used to help us understand the difference between accuracy and precision. Imagine a person throwing darts, trying to hit the bull’s eye. There are 4 scenarios:

- **Not Precise/Not Accurate**
  It is a random pattern: darts are not clustered and are not near bull’s eye.

- **Precise/Not Accurate**
  Darts are clustered together but did not hit the bull’s eye.

- **Not Precise/Accurate**
  Darts are not clustered together, but their “average” hit the bull’s eye.

- **Precise/Accurate**
  Darts are clustered together and their “average” position hit the bull’s eye.

It is easy to confuse precision and accuracy. The tool that you use affects both the precision and accuracy of your measurement. Measuring with a millimeter tape allows greater precision than measuring with an inch tape. Because the error using the millimeter tape should be less than the inch tape, accuracy also improves.
Suppose that a tape measure is used to measure the diameter of two circles. Let’s suppose you measure the first circle to be 15 cm, and a second circle to be 201 cm. The two measures are equally precise (both measured to the nearest cm). However, their accuracy may be quite different. Let’s further suppose that the accepted values for the measurements are 16 cm and 202 cm. The errors for these measurements are \( \frac{1}{16} = 0.0625 \) or 6.25% and \( \frac{1}{202} = 0.0049504 \) or about 0.5%. The second measurement is more accurate because the error is smaller.

One more way to think of this: accuracy implies that a measurement is basically right, given a margin of error. Precision is the level of detail; or typically the number of digits after a decimal point. For instance, I ask how far it is to the store. You could tell me “about 3 miles”, while a GPS device might tell me “2.85 miles”. About 3 miles is pretty accurate, but 2.85 is both accurate and precise. Now, you could have told me “15.345 miles”, which would make you very precise, but not accurate.

Most of the time in our everyday life, we want accuracy; precision is not as useful. But in science and engineering, both precision and accuracy are important.