Lesson 2: How Many Solutions?

Brief Overview of Lesson:

Students will categorize linear equations in one variable into one of the following three categories:

1. One solution: the equation is sometimes true; it is true for one specific value of x.
2. No solution: the equation can never be true; there are no values of x that will make the equation true.
3. Infinite solutions: the equation is always true; no matter what value is assigned to x, the equation will be true.

As you plan, consider the variability of learners in your class and make adaptations as necessary.

Prior Knowledge Required:

8.EE.7 Solve linear equations in one variable.

8.EE.7b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Estimated Time (minutes): Two 50 minute lessons

Resources for Lesson

Formative Assessment: *Shopping Scenarios*
Directions for *Always, Sometimes, Never Activity*
Solution for Categorizing Equations as Always True, Sometimes True, Never True
Ticket to Leave
Homework: Working with Linear Equations
Chart paper, sticky notes, internet access and projector with speakers
www.teachingchannel.org "Sorting and Classifying Equations Overview" video. (9:50)
www.teachingchannel.org "Sorting and Classifying Equations: Class Discussion" video. (8:24)

Unit: Analyzing and Solving Linear Equations and Pairs of Simultaneous Linear Equations
Content Area/Course: Grade 8 Mathematics
Lesson # 2: How Many Solutions?
Time (minutes): Two 50 minute lessons

By the end of this lesson students will know and be able to:

- Identify the different forms of solutions and connect the form to the number of solutions (one solution: x=a; infinite solution: a=a; no solution: a = b)
- Produce examples of linear equations that represent one solution, infinite solutions, and no solutions.
- Estimate solutions to simple cases by inspection of equations (and/or graphs).

Essential Question(s) addressed in this lesson:

- How can equations be used to represent real-world and mathematical situations?

Targeted Academic Language: solution, identity, infinite

Standard(s)/Unit Goal(s) to be addressed in this lesson:

8.EE.7a Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x=a, a=a, or the given results (where a and b are different numbers).
SMP.3 Construct viable arguments and critique the reasoning of others.
SMP.7 Look for and make use of structure.

Teacher Content and Pedagogy

After working in pairs or small groups to make decisions regarding assigned equations, students will be asked to critique the work of others (SMP.3 *Construct viable arguments and critique the reasoning of others*).
Collaboration and opportunities to share work and explanations are an important part of this lesson.
Students should try to justify their decisions mathematically rather than “I tried some other numbers and they didn’t work”.

Use this “Math Talk” protocol during problem-solving activities:

- Think about the problem – what strategies could you use?
- How do the problems and strategies compare to work you have done previously?
- Share your ideas with a partner.
- Work on your own to rethink the problem and arrive at an answer.
- Share your solution with a partner.
- Share with entire class.

Notes on One, Infinite, or No Solutions

Linear equations in one variable can have one solution, infinite solutions, or no solutions.

When the equation has **one solution**, the variable has one value that makes the equation true, and the equation is **sometimes true**.
Example: 10 – 3x = 4.
The only value for x that makes this equation true is 2.

When the equation has **infinite solutions**, the equation is true for all real numbers, and the equation is **always true**.
Example : 3x + 6 = 3 (x + 2).
When the distributive property is applied correctly, the expressions on each side of the equal sign are equivalent; therefore, the value for the two sides of the equation will be the same regardless of which value is chosen for x.
Example: 2x + 3*x* + 9 = 5x + 9
When like terms are combined, the expressions on each side of the equal sign will be equivalent.

When an equation has **no solution**, there is no value that could possibly make the equation true. The equation is never true.
Example: 2x + 5 = 2 (x + 5)

The two expressions are not equivalent. The equation simplifies to 5 = 10.Because all of these equations are linear, there will only be ONE solution when the equation is sometimes true. *It is important for students to explore this*. As equations become more complex (exponents), equations that are sometimes true will sometimes have more than one solution.

Key mathematical terms can be reinforced during this discussion:

solution identity infinite

Notes for Always, Sometimes, or Never True Activity

The use of different color markers on the poster (a different color for each member of the group) will allow the teacher to monitor who is completing the work.

As students are creating their posters in groups, the teacher observes and encourages student thinking via effective questioning:

- How do you know for sure?
- What are you thinking?
- Sam put the equation in this column. Nick, do you think it should go there? Why/why not? (SMP 3)
- Can you convince me that it would be true for EVERY number? (SMP 3)
- How could you change this equation, so it would belong in a different column?
- Can you create a new equation for each column?
- What do you notice about the equations that are in this column?

Anticipated Student Preconceptions/Misconceptions

- Errors with application of properties. Example: assuming that subtraction is commutative (5 – x = x - 5)
- Incorrect use of the equal sign. Example: student writes 5 – x = 6 = x = –1.
- Finding one solution and missing other solutions; not continuing to find other values or not finding a value that does not work.
- Belief that an equation is not true because the expression on each side of the equal signs looks different.
- Failure to fully simplify expressions.

Lesson Sequence

DAY 1

- Opening: Explicitly connect the lesson to the previous lesson. Remind students that equations can be solved in a variety of ways. Reference the chart that the class made the day before**.**

- Activator: Present students with the three shopping scenarios in the **Shopping Scenarios** handout.
  Students work with a partner to interpret the equations provided to determine how many boxes of cookies each person bought.

  Do a quick share-out following this introductory activity. What do students notice? (The Shopping Scenarios problems have one solution, no solution, or infinite solutions. Students may be confused by equations that are not one-solution equations.) Students begin to look at *the structure of equations* to draw conclusions about the number of solutions. (SMP.7)

- Display the following equation: $3x + 2 = 11$

  Ask: Can you think of a value for *x* that makes this equation FALSE?

- Display the equation again: $3x + 2 = 11$

  Ask: Can you think of a value for x that makes this equation TRUE?"

  When a value is found, challenge students to try to find other values for *x* that will make the equation true.

  Ask: Would we describe this equation as **always true**, **never true**, or **sometimes true**?
  When is it true? ($x = 3$)
  Are there any other values for *x* that make it true? How do you know?

- Repeat the same process with the following equations:

$x + 7 = 7 + x$ (always true for all values of x …)

$5 – x = 2x + 20$ (sometimes true; x = -5)

$x – 3 = x + 2$ (never true)

Students are encouraged to look at the *structure of the equations* in order to make generalizations about the solutions (SMP.7). Include the correct mathematical “label”:

- sometimes true = one solution
- never true = no solution
- always true = infinite solutions

Post each equation with its solution.

Encourage multiple students to contribute. Elicit a variety of answers, encouraging students to “think outside the box”. Discuss the reasons for their choices.

Record exemplary responses. Post these, so students can reference them at a later date. IF necessary, have students support their answers with calculations.

- **Always, Sometimes, or Never True** small group activity:
    - Watch video: “Sorting and Classifying Equations Overview” video (9:50) at www.teachingchannel.org. (Enter the title in the search field.) Explain to students that they will be completing a similar activity.
    - Divide students into groups of three.
    - Distribute the set of equations, chart paper, markers (a different color for each student in the group), and directions.
    - Review the three possible outcomes.

ALWAYS TRUE or INFINITE SOLUTIONS	means	The equation is true for any value of *x*.
NEVER TRUE or NO SOLUTION	means	There are no values of *x* that make the equation true.
SOMETIMES TRUE or ONE SOLUTION	means	There is at least one value of *x* that makes the equation true.

- To prove that an equation is SOMETIMES TRUE, you need two examples: one value for *x* that makes the equation true, and one that makes it false.

- Review directions: **Directions for Always, Sometimes, Never True.**

- Students spend the remainder of the class period completing the poster.
- Lesson Closing- have students reflect upon the lesson and share out either with a partner or in a whole class discussion.

DAY 2
Introduce the lesson and explicitly connect it to the previous lesson.

- Share posters … Conduct a gallery walk, so students can critique each others' work.
  - Display charts in different areas of the room (hang on walls or lay on tables/desks).
  - Before beginning, distribute sticky notes to each student.
  - Provide the following directions:

    If you disagree with where an equation has been placed, write three things on a sticky note:

    1) Why you disagree.
    2) Which column the equation belongs in.

3) Why you think the equation belongs there. Place the sticky note on the chart.

This task requires students to critique the reasoning of others (SMP.3).

- If you see a comment that has already been written and you agree, put a check mark on the sticky note.
- If you see an explanation that you particularly like or something you didn't think of, leave a note telling the group what you liked about it. (SMP.3)
- Students walk silently around the room and review the work of others. Set a limit regarding the number of students looking at each poster. Or, students can be divided equally among the charts and given a time limit before moving on to the next poster. It is not necessary to have all students critique all posters; two are sufficient.

- Closing/Summarizer
    - Conduct a whole class discussion.
    - Highlight exemplary explanations.
    - Give me an equation that is… (ask for one of each type)
    - Why did you put that equation in that column?
    - Can anyone add to this explanation?
    - What did you learn by looking at other students' work?
    - Which equations were the most difficult to categorize? Why?
    - Is there an easy way to tell which column an equation belongs in just by looking at it?
- Watch "Sorting and Classifying Equations: Class Discussion" www.teachingchannel.org (8:24)
- **Ticket to Leave**: Students use their understanding of the structure of equations to classify equations as always, sometimes, or never true, and create an example of each type of equation on their own. (SMP.7)

- **Homework: Working with Linear Equations**

Formative Assessment:
Three **Shopping Scenarios** check student understanding of the meaning of the terms and variables in linear equations.

Preview outcomes for the next lesson:
Students will be able to solve simultaneous linear equations through the use of graphing.

Lesson 2 *Shopping Scenarios* Formative Assessment: *What Does the Equation Tell Me*?

Name: ___________________________________ ____ Class: __________ Date:____________________

Read each shopping scenario.
Review the equations that have been written to represent them.
What can you say about the number of packages of cookies each person bought?

	Description	**Equation**	**What can you say about the number of packages of cookies each person bought?**
A.	Noelle went shopping and spent $19. She bought one container of ice cream and some packages of cookies.	5c + 4 = 19	
B.	Last week, on Monday, Anna bought 3 packages of cookies and spent $5 more on other items On Tuesday, she bought the exact same thing. This week, Anna bought 6 packages of cookies. She also spent $10 more on other items. She said she spent the **same** amount of money last week as this week.	2(3c+ 5) = 6c + 10	
C.	Last week, Jared bought 8 packages of cookies and spent $8 more on some cheese spread. This week, on Monday, Tuesday, Wednesday, and Thursday, Jared bought two packages of cookies each day.	8c + 8 = 4(2c)	

	He said he spent the **same amount** of money last week as this week.		

Name: ___________________________________ ____ Class: __________ Date:____________________

Lesson 2: Directions for *Always, Sometimes, or Never True Group Activity*

1. Each person in your group selects a marker of a different color. This is the only color that you may write with.
2. Divide your chart paper into three equal columns.
3. Label the columns:

Always True – Infinite Solutions	Sometimes True – One Solution	Never True – No Solution

4. Select an equation. Try out different values for *x*. Discuss with your group. Record the equation in the column where it belongs.
5. In writing, next to each equation, explain why you chose to place the equation where you did.
    - If you think the equation is **sometimes** true, give values of *x* for which it is true and for which it is false.
    - If you think the equation is **always** true or **never** true, explain how you can be sure this is the case.

Guidelines:

1. Take turns recording the equation and writing the explanation.
2. Be sure you agree with your partners. If you are not sure, challenge the explanation.
3. Describe your thinking in your own words.

Lesson 2: Equations for Always, Sometimes, Never Activity

Name: ___________________________________ ____ Class: __________ Date:____________________

Decide whether each equation is always true, sometimes true, or never true. Record each equation in your ALWAYS TRUE, SOMETIMES TRUE, NEVER TRUE chart. Remember to record your reasoning and some examples next to each equation on the chart.

1. $2 - x = x - 2$
2. $3 + x = x + 3$
3. $x + 5 = x - 3$
4. $3x - 5 = 2x$
5. $\frac{x}{2} = x$
6. $2(x + 1) = 2x + 1$
7. $6x = x$
8. $7x + 14 = 7(x + 2)$
9. $\frac{10}{2x} = 5$
10. $\frac{2x+4}{2} = x + 2$
11. $5x - 5 = 5(x + 1)$
12. $4x = 4$
13. $\frac{1}{2}x - 5 + \frac{3}{2}x = x + x - 5$
14. $\frac{3}{4}x + 8 = \frac{1}{4}(3x + 16)$

Lesson 2: Equations for Always, Sometimes, Never Activity ANSWER KEY

Name: ____________________________________ ____ Class: ___________ Date:____________________

ANSWER KEY

Always True – **Infinite Solutions**	**Sometimes True –** **One Solution**	**Never True –** **No Solution**
$3 + x = x + 3$	$2 – x = x – 2$ (Solution: $x = 2$)	$x + 5 = x – 3$
$7x + 14 = 7(x + 2)$	$3x – 5 = 2x$ (Solution: $x = 5$)	$2(x + 1) = 2x + 1$
$\frac{2x+4}{2} = x + 2$	$\frac{x}{2} = x$ (Solution: $x = 0$)	$5x – 5 = 5(x + 1)$
$\frac{1}{2}x – 5 + \frac{3}{2}x = x + x – 5$	$6x = x$ (Solution: $x = 0$)	$\frac{3}{4}x + 8 = \frac{1}{4}(3x + 16)$
	$\frac{10}{2x} = 5$ (Solution: $x = 1$)	
	$4x = 4$ (Solution: $x = 1$)	

Lesson 2: Ticket to Leave

Name: ____________________________________ ____ Class: ___________ Date:____________________

For which values are the following equations true?

	Equation	For what values of *x* is the equation true?	Number of Solutions (one, none, infinite)
Ex.	**$6x + 3 = 15$**	**Only true when x = 2.**	**[✓ one] none infinite**
1.	$12 - x = 15$		one none infinite
2.	$x - 3 = 3 - x$		one none infinite
3.	$3 (x + 4) = 3x + 4$		one none infinite
4.	$\frac{x}{2} = 6$		one none infinite
5.	$2 (x + 3) = 2x + 6$		one none infinite

Create three examples of your own:			
6.			One solution
7.			None
8.			Infinite solutions

Lesson 2: Ticket to Leave ANSWER KEY

Name: ___________________________________ ____ Class: __________ Date:____________________

For which values are the following equations true?

	Equation	For what values of *x* is the equation true?	Number of Solutions (one, none, infinite)
Ex.	**6*x* + 3 =15**	**Only true when x = 2.**	**[✓ one] none infinite**
1.	12 - *x* = 15	**Only true when x = -3.**	[✓ one] none infinite
2.	*x* - 3 = 3 - *x*	**Only true when x = 0.**	[✓ one] none infinite
3.	3 (x + 4) = 3x + 4	**Never true 12 ≠ 4.**	one [✓ none] infinite
4.	$\frac{x}{2} = 6$	**Only true when x = 12.**	**[✓ one]** none infinite
5.	2 (x + 3) = 2x + 6	**True for all real numbers.**	one none [✓ infinite]

Create three examples of your own:			
6.			One solution
7.			None
8.			Infinite solutions

Lesson 2 Homework: Working with Linear Equations

Name: ___________________________________ ____ Class: __________ Date:____________________

x	-3	2	3
y	-3	7	9

A

x	0	2	4
y	5	7	9

B

x	-1	0	2
y	5	1	7

C

x	-1	0	2
y	1	3	7

D

1. Which of these tables of values satisfy the equation $y = 2x + 3$?

   ____

2. Explain how you know.

   ____

   ____

3. Complete the tables below for the lines $y = 2x + 3$ and $x = 1 - 2y$.

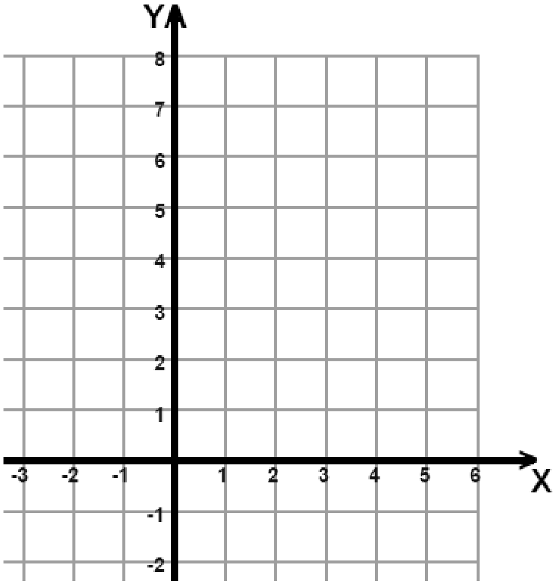
   Then use the values to graph the lines.

$y = 2x + 3$

x	-2	0	
y			5

$x = 1 - 2y$

x	0		5
y		0	



4. Do the equations $y = 2x + 3$ and $x = 1 - 2y$ have one common solution, no common solutions, or infinitely many common solutions? Explain how you know.

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Lesson 2 Homework: Working with Linear Equations

5. Draw a straight line on the graph that has **no common solutions** with the line $y = 2x + 3$.

What is the equation of your new line? Explain your answer.

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Lesson 2 Homework: Working with Linear Equations ANSWER KEY

Name: ______________________________________ ____ Class: ___________ Date:_____________________

x	-3	2	3
y	-3	7	9

A

x	0	2	4
y	5	7	9

B

x	-1	0	2
y	5	1	7

C

x	-1	0	2
y	1	3	7

D

1. Which of these tables of values satisfy the equation $y = 2x + 3$?

   TABLE D

2. Explain how you know.
   Answers may vary. Ex: The y-intercept in the table is (0,3) and the slope of the table is 2/1 because the x-values add one each time and the y-values add two each time. I added the point (1,5) to my table to clearly see the constant rate of change.

3. Complete the tables below for the lines $y = 2x + 3$ and $x = 1 - 2y$.

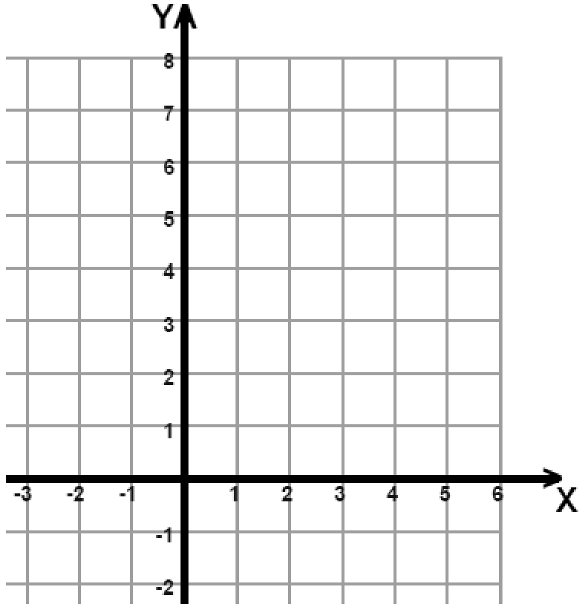
   Then use the values to graph the lines.

$y = 2x + 3$

x	-2	0	**1**
y	**-1**	**3**	5

$x = 1 - 2y$

x	0	**1**	5
y	**-0.5**	0	**-2**



Check students' graphs.

4. Do the equations $y = 2x + 3$ and $x = 1 - 2y$ have one common solution, no common solutions, or infinitely many common solutions? Explain how you know.

   One Solution (-1,1) Students explanations will vary. Ex: Point of Intersection.

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5. Draw a straight line on the graph that has **no common solutions** with the line $y = 2x + 3.$ What is the equation of your new line? Explain your answer.

   Student answers will vary. Ex: y = 2x + 5 will have no common solutions with the line y = 2x + 3.