Objective

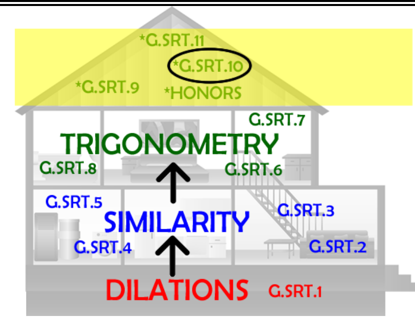
G.SRT.10 **(HONORS ONLY)**

Prove the Laws of Sines and Cosines and use them to solve problems.

Interpretation of Objective

No interpretation needed - prove the Law of Sines and the Law of Cosines and then solve problems using them. Of course hidden inside this very short objective is the ambiguous case for the Law of Sines. That can be a difficult concept for students to see and understand.

Connections



This is about extending right triangle trigonometry to oblique triangles.

Student Outcomes

(1) The student will be able to solve a triangle using the Laws of Sines and/or the Law of Cosines.

(2) The student will be able to explain the cases of AS_1S_2.

(3) The student will be able to determine which relationship (Law of Sines/Law of Cosines) is required for solving the triangle.

Emphasis

There is a lot inside of this objective. The Law of Sines requires an understanding of congruence criteria, obtuse angle trigonometry, and different cases. These items traditionally are all difficult for students.

Tips

1 – The Ambiguous Case – This is so much easier to understand if the AS_1S_2 case was discussed during the investigation of congruence in triangles. If students already know that in a particular case more than one answer is possible, the concept is handled MUCH easier.

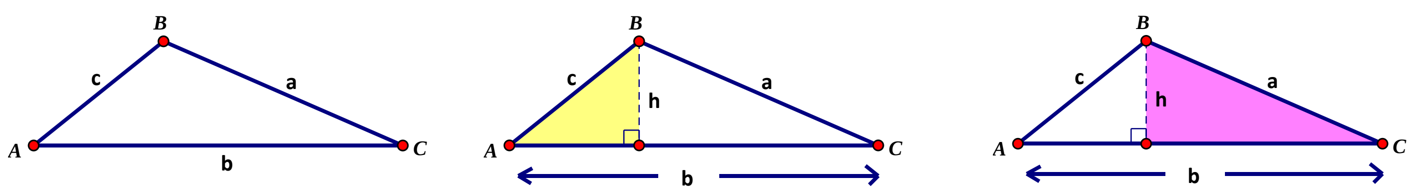
2 – Provide Plenty of Time – The different cases each have some big concepts and issues and students need to be provided enough time to learn them.

CONCEPT 1 – Prove the Laws of Sines.

As mentioned in the previous objective, the previous focus has been right triangles, but now are starting to expand the use of trigonometry to all triangles, even oblique triangles. In this objective show that if any three of the six measures of triangle are given (provided at least one measure is a side), then the other three measures can be found.

One thing to consider when doing this is if the three pieces of information force congruence or not. So for example, if given SAS, ASA, AAS/SSA, SSS, and some cases of AS_1S_2 the information is enough to guarantee congruence in the triangle.

The first new relationship in this objective that should be derived is called the **Law of Sines**. Follow similar logic as that used to derive the new area formula.



$$\sin A = \frac{h}{c}$$
$$h = (\sin A)(c)$$

$$\sin C = \frac{h}{a}$$
$$h = (\sin C)(a)$$

The two heights are equal so set these values equal to each other.

$$(\sin A)(c) = (\sin C)(a)$$
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

In a similar way, by constructing the perpendiculars from other vertices, it can be shown that:

$$\frac{\sin A}{a} = \frac{\sin B}{b} \qquad \frac{\sin B}{b} = \frac{\sin C}{c}$$

Thus the Law of Sines is:

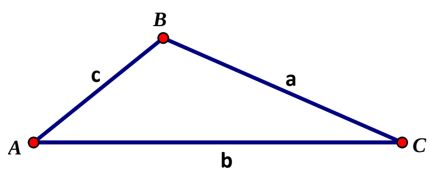
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

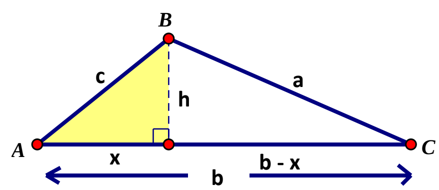
CONCEPT 2 – Prove the Laws of Cosines.

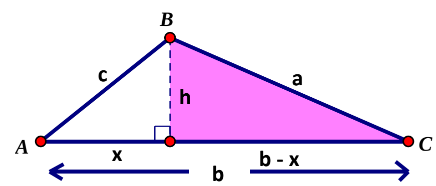
In the previous concept two cases did not work for the Law of Sines. Review the following chart…

ASA	AAS/SAA	AS_1S_2	SSS	SAS
$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$	$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$	$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$	$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$	$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$
If given 2 ∠'s the 3rd can be found.	If given 2 ∠'s the 3rd can be found.	S will always be opposite the ∠.	No ∠ info.	No ∠ or side opposite each other.
Law of Sines will work (ratio present).	Law of Sines will work (ratio present).	Law of Sines will work (ratio present).	Law of Sines will NOT work (no ratio).	Law of Sines will NOT work (no ratio).

It is those last two relationships, SSS and SAS that must be addressed using a different law, **the Law of Cosines**.







$$x^2 + h^2 = c^2$$
$$x^2 = c^2 - h^2$$

$$\cos A = \frac{x}{c}$$
$$x = (\cos A)c$$

$$a^2 = (b-x)^2 + h^2$$
$$a^2 = b^2 - 2bx + x^2 + h^2$$

Substitute x^2 from the first relationship into the second.

$$a^2 = b^2 - 2bx + x^2 + h^2$$
$$a^2 = b^2 - 2bx + c^2 - h^2 + h^2$$
$$a^2 = b^2 + c^2 - 2bx$$

Finally, eliminate the *x* by making one more substitution.

$$a^2 = b^2 + c^2 - 2bx$$
$$a^2 = b^2 + c^2 - 2b(\cos A)c$$
$$a^2 = b^2 + c^2 - 2bc(\cos A)$$

Notice now after the second substitution only sides and angles of the original triangle are in the newly found relationship.

Subsequent cases would result in the same pattern.

Thus the Law of Cosines is:

$$a^2 = b^2 + c^2 - 2bc(\cos A)$$
$$b^2 = a^2 + c^2 - 2ac(\cos B)$$
$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

Notice the Pythagorean Theorem in the Law of Cosines?

1. When $m\angle B = 39°$, $b = 25$ cm, $a = 24$ cm are used to form ΔABC, the following will result in:

A) no triangle B) 1 triangle C) 2 triangles D) depends on diagram

2. When $m\angle A = 39°$, $b = 25$ cm, $a = 24$ cm are used to form ΔABC, the following will result in:

A) no triangle B) 1 triangle C) 2 triangles D) depends on diagram

3. Which group of three pieces of information about a triangle could NOT be used in the Law of Sines?

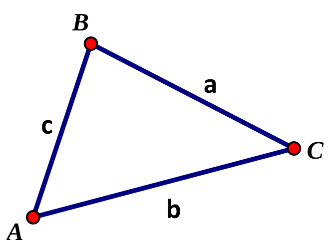
A) ASA B) AAS C) SSA D) SSS

4. Which of the following is the Law of Cosines?

A) $a^2 = b^2 - c^2 + 2bc(CosA)$ B) $a^2 = b^2 + c^2 - 2bc(CosB)$

C) $a^2 = b^2 + c^2 - bc(CosA)$ D) $a^2 = b^2 + c^2 - 2bc(CosA)$

5. Which of the following three pieces of information work with the Law of Sines?



a) Yes or No

$m\angle A = 33°$
$b = 17$ cm
$c = 24$ cm

b) Yes or No

$m\angle B = 53°$
$b = 11$ cm
$a = 7$ cm

c) Yes or No

$m\angle A = 24°$
$m\angle B = 65°$
$m\angle C = 91°$

d) Yes or No

$m\angle A = 75°$
$m\angle C = 38°$
$b = 9$ cm

e) Yes or No

$m\angle C = 66°$
$b = 15$ cm
$c = 15$ cm

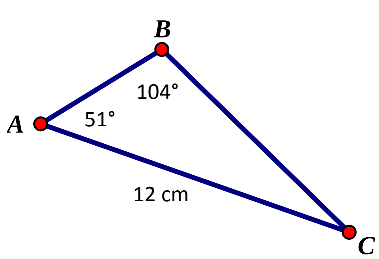
f) Yes or No

$c = 13$ cm
$b = 14$ cm
$a = 24$ cm

6. If given the following information about ΔABC, $m\angle A = 34°$, $m\angle B = 100°$ and $c = 15$ cm, can the Law of Sines be used? Explain.

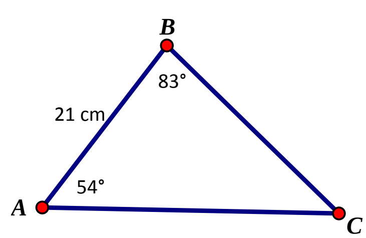
7. Solve for all the sides and angles of ΔABC using the Law of Sines. (Round answers to the hundredths)

a)



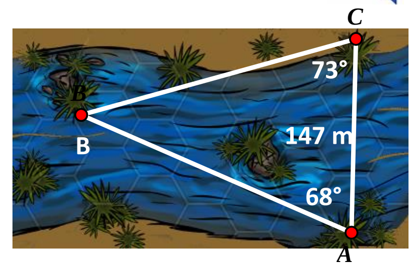
$m\angle A = 51°$
$m\angle B = 104°$
$m\angle C =$ ________
$a \approx$ ________
$b = 12$ cm
$c \approx$ ________

b)



$m\angle A = 54°$
$m\angle B = 83°$
$m\angle C =$ ________
$a \approx$ ________
$b \approx$ ________
$c = 21$ cm

8. On a rafting trip, the raft hit a rock (point *B*) and got stuck there. The two rafters Calvin and Alvin were thrown from the raft and ended up getting out of the river further downstream at points *A* (Alvin) & *C* (Calvin). If the river is 147 m wide and Calvin sees the raft at 73° while Alvin sees it at 68°. Who is closer to the raft, and how much closer are they? (Round to the nearest meter)



9. Solve the triangle for all angles and sides. ***m∠A* = 51°, *a* = 21 cm, *c* = 25 cm**

(Round answers to the hundredths place)

Draw a Diagram

***m∠A* = 51°**

***m∠B* ≈ _________**

***m∠C* ≈ _________**

***a* = 21 cm**

***b* ≈ ___________ cm**

***c* = 25 cm**

(If needed)

***m∠A* = 51°**

***m∠B* ≈ _________**

***m∠C* ≈ _________**

***a* = 21 cm**

***b* ≈ ___________ cm**

***c* = 25 cm**

Answers:

1) B

2) C

3) D

4) D

5) **a)** No

b) Yes

c) No

d) Yes

e) Yes

f) No

6) Yes it can – given two angles the 3rd angle is found using 180 - $m\angle A$ - $m\angle B$ = $m\angle C$. Once $\angle C$ is found then the 'pairing' of $\angle C$ and c occurs. Thus the Law of Sines can be used.

7)

$$\frac{\sin 51}{a} = \frac{\sin 104}{12}$$
$$(\sin 104)a = 12(\sin 51)$$
$$a = \frac{12(\sin 51)}{(\sin 104)}$$
$$a = 9.61\ cm$$

$$\frac{\sin 104}{12} = \frac{\sin 25}{c}$$
$$(\sin 104)c = 12(\sin 25)$$
$$c = \frac{12(\sin 25)}{(\sin 104)}$$
$$c = 5.23\ cm$$

$$\frac{\sin 43}{21} = \frac{\sin 83}{b}$$
$$(\sin 43)b = 21(\sin 83)$$
$$b = \frac{21(\sin 83)}{(\sin 43)}$$
$$b = 30.56\ cm$$

$$\frac{\sin 43}{21} = \frac{\sin 54}{a}$$
$$(\sin 43)a = 21(\sin 54)$$
$$a = \frac{21(\sin 54)}{(\sin 43)}$$
$$a = 24.91\ cm$$

180 – 104 – 51 =
25° = $m\angle C$

180 – 83 – 54 =
43° = $m\angle C$

8)

$$\frac{\sin 39}{147} = \frac{\sin 73}{c}$$
$$(\sin 39)c = 147(\sin 73)$$
$$c = \frac{147(\sin 73)}{(\sin 39)}$$
$$c = 223.38\ m$$

180 – 73 – 68 = 39° = $m\angle B$

$$\frac{\sin 39}{147} = \frac{\sin 68}{a}$$
$$(\sin 39)a = 147(\sin 68)$$
$$a = \frac{147(\sin 68)}{(\sin 39)}$$
$$a = 216.58\ m$$

Alvin is 223.38 m away.

Calvin is 216.58 m away.

Calvin is closer by 6.8 m or 7 m.

9) **(2 solutions)**

$$\frac{\sin 51}{21} = \frac{\sin C}{25}$$

$$(\sin C)21 = 25(\sin 51)$$

$$\sin C = \frac{25(\sin 51)}{21}$$

$$\sin C = 0.9252$$

$$\sin^{-1}(0.9252) = m\angle C$$

$$67.69° \approx m\angle C$$

180 – 51 – 67.69 = 61.31° $\approx m\angle B$

$$\frac{\sin 51}{21} = \frac{\sin 61.31}{b}$$

$$(\sin 61.31)21 = b(\sin 51)$$

$$b = \frac{(\sin 61.31)21}{(\sin 51)}$$

$$b \approx 23.70\,cm$$

***m∠A* = 51°**

***m∠B* ≈ 61.31°**

***m∠C* ≈ 67.69°**

***a* = 21 cm**

***b* ≈ 23.70 cm**

***c* = 25 cm**

A second triangle is possible so use and solve for the supplement of $\angle C$.

180 – 67.69° = 112.31° $\approx m\angle C$

180 – 51 – 112.31 = 16.69° $\approx m\angle B$

$$\frac{\sin 51}{21} = \frac{\sin 16.69}{b}$$

$$(\sin 16.69)21 = b(\sin 51)$$

$$b = \frac{(\sin 16.69)21}{(\sin 51)}$$

$$b = 7.76\,cm$$

***m∠A* = 51°**

***m∠B* ≈ 16.69°**

***m∠C* ≈ 112.31°**

***a* = 21 cm**

***b* ≈ 7.76 cm**

***c* = 25 cm**