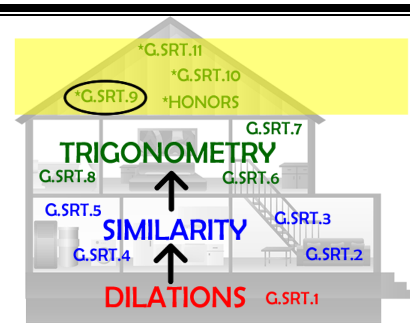
Objective	Interpretation of Objective	Connections
G.SRT.9 **(HONORS ONLY)** Derive the formula *A* = 1/2 *ab* sin(*C*) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	Not much to interpret here, derive the formula and then use it.	 This is preparatory concept to the Law of Sines and working in oblique triangles.

Student Outcomes	Emphasis	Tips
(1) The student will be able to derive the area formula for a triangle using the sine ratio. (2) The student will be able to determine the area of a triangle given a variety of information.	Being able to solve an oblique triangle using this area formula is not a very important skill; this objective is designed to teach how to calculate the height of an oblique triangle using trigonometry providing linkage so that the Law of Sines can be derived in the next objective.	1 – Derive… have students trouble shoot the issue of determining the height. It isn't a big leap but it will certainly help them get ready for deriving the Law of Sines.

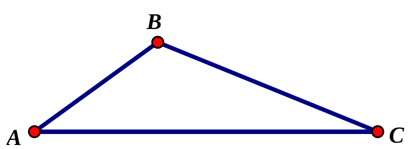
CONCEPT 1 – Derive the formula $A = \frac{1}{2}ab\sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

This objective is quite direct. Most objectives thus far leave something to be interpreted but here it is simply stated – Derive the formula for area using sine.

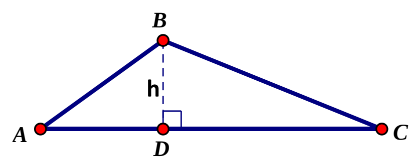
So far all trigonometry work has been in right triangles but the next three objectives introduce the use of trigonometry to solve for sides, angles and areas of all triangles, including the OBLIQUE TRIANGLES. An oblique triangle is a non-right triangle.

So, how is the sine ratio used to solve for the area of a triangle?

Oblique ΔABC



Drop an altitude to a side



Standard Area Formula

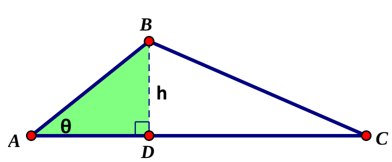
$$Area = \frac{1}{2}bh$$

$b = base\ length$

$h = height$

To solve for the area of this triangle multiply ½ (base)(height), in this case it would be ½ (*AC*)(*BD*) but the height is usually not provided when giving information about a triangle. So a formula must be created for area that uses only side lengths of the triangle. By dropping an altitude (the height) to a side of the triangle, a right triangle is formed and this allows the use of the trigonometry ratios. Isn't that tricky – since the right triangle trigonometry ratios have been established, turn the oblique triangle into right triangles by dropping an altitude and use those ratios to find the height.

Use the right triangle with $\angle A$ and the sine ratio will solve for height (*h*).



$$\sin A = \frac{h}{AB}$$

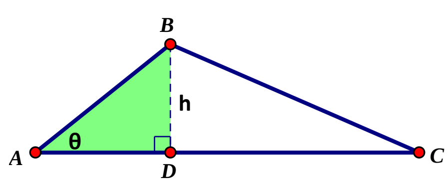
$$h = (\sin A)(AB)$$

$$Area = \frac{1}{2}bh$$

Now substitute the values for *b* and *h*.

$$Area = \frac{1}{2}(AC)(\sin A)(AB)$$

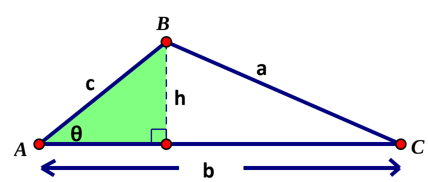
$$Area = \frac{1}{2}(AC)(AB)(\sin A)$$



$$Area = \frac{1}{2}(AC)(AB)(\sin A)$$

Notice that this formula requires no provided value for the height, the height is being calculated using the sine ratio. This is a very handy formula for area.

Calculating area then requires two sides of a triangle and the INCLUDED ANGLE (SAS).



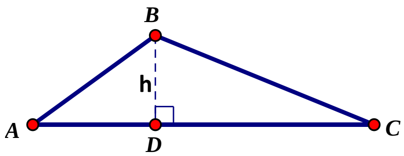
$$Area = \frac{1}{2}bc(\sin A)$$

"SPECIAL LABELING"

Sometimes to simplify a formula the side opposite of an angle is referred to as the lowercase letter of the vertex that is opposite it. So *c* is the opposite side of $\angle C$, *b* is the opposite side of $\angle B$ and of course *a* is the opposite side of $\angle A$.

NOTES Derive & Use the Area Formula in Terms of Sine G.SRT.9-HONORS

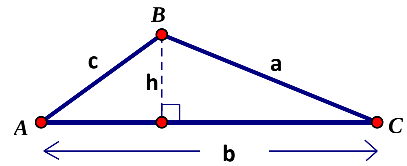
This was a bit of an aside topic to help illustrate that the following relationships work for all triangles and all angles in those triangles.



$$Area = \frac{1}{2}(AC)(AB)(\sin A)$$

$$Area = \frac{1}{2}(AC)(BC)(\sin C)$$

$$Area = \frac{1}{2}(BA)(BC)(\sin B)$$



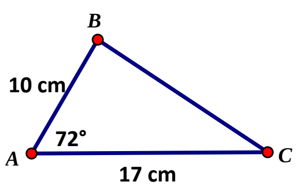
$$Area = \frac{1}{2}bc(\sin A)$$

$$Area = \frac{1}{2}ab(\sin C)$$

$$Area = \frac{1}{2}ac(\sin B)$$

Here are a few examples of how to use this formula…. Determine the area of the following triangles.

a)

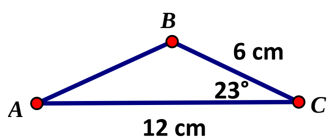


$$Area = \frac{1}{2}bc(Sin\,A)$$

$$Area = \frac{1}{2}(17)(10)(Sin\,72°)$$

$$Area = 80.84\,cm^2$$

b)

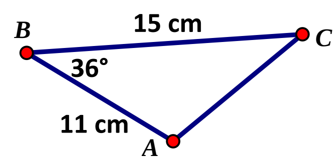


$$Area = \frac{1}{2}ab(Sin\,C)$$

$$Area = \frac{1}{2}(6)(12)(Sin\,23°)$$

$$Area = 14.07\,cm^2$$

c)

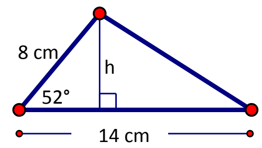


$$Area = \frac{1}{2}ac(Sin\,B)$$

$$Area = \frac{1}{2}(15)(11)(Sin\,36°)$$

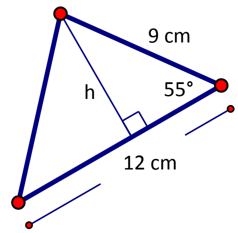
$$Area = 48.49\,cm^2$$

1. What is the height of this triangle?



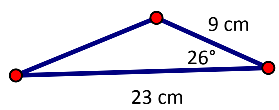
A) 4.92 cm B) 6.30 cm C) 10.24 cm D) 11.03 cm

2. What is the area of the triangle?



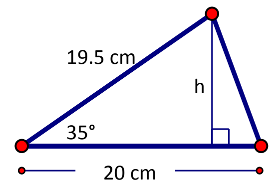
A) 7.37 cm^2 B) 30.97 cm^2 C) 44.23 cm^2 D) 88.47 cm^2

3. What is the area of the triangle?



A) 45.37 cm^2 B) 50.48 cm^2 C) 90.37 cm^2 D) 103.50 cm^2

4. What is the height of this triangle?

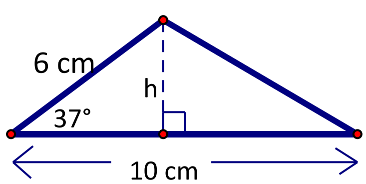


A) $19.5(\sin 35°) = h$ B) $20(\sin 35°) = h$

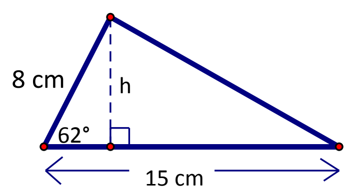
C) $19.5(\cos 35°) = h$ D) $19.5(\tan 35°) = h$

5. Determine the heights of the given triangles. (Round to the hundredths place when necessary)

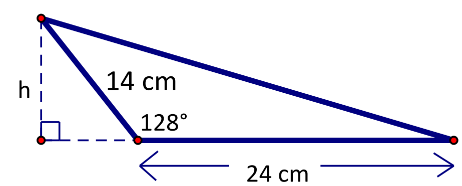
a)



b)

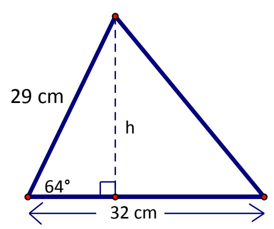


c)

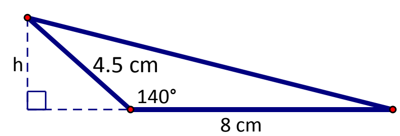


6. Determine the area of the given triangles. (Round to the hundredths place when necessary)

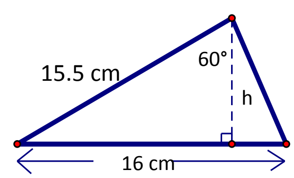
a)



b)



c)



Answers:

1) B
2) C
3) A
4) A
5) a) $\sin 37° = \frac{h}{6},\ h \approx 3.61\ cm$

   b) $\sin 62° = \frac{h}{8},\ h \approx 7.06\ cm$

   c) $\sin 128° = \frac{h}{14}\ \ or\ \sin 52° = \frac{h}{14},\ \ h \approx 11.03\ cm$

6) a) $Area = \frac{1}{2}(29)(32)(\sin 64°) = 417.04\ cm^2$

   b) $Area = \frac{1}{2}(8)(4.5)(\sin 140°) = 11.57\ cm^2$

   c) $Area = \frac{1}{2}(16)(15.5)(\sin 30°) = 62\ cm^2$