



In Algebra I, a review of properties of integer exponents may be required. Students begin their exploration of *power* under the Common Core in Grade 6 by writing and evaluating expressions with exponents. In Grade 8, they extend their knowledge of power to properties of exponents. Specifically, students should explore, learn, and apply eight properties of integer exponents in the real numbers. Rational exponents will be addressed later in the course.

8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions.

Product of powers: $a^m \cdot a^n = a^{m+n}$

Negative powers: $a^{-m} = \frac{1}{a^m}$

Quotient of powers: $\frac{a^m}{a^n} = a^{m-n}$

Power of a power: $(a^m)^n = a^{m \cdot n}$

First power: $a^1 = a$.

Power of a product: $(ab)^m = a^m \cdot b^m$

Zeroth power: $a^0 = 1$, for all $a \neq 0$.

Power of a quotient: $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

Knowledge Targets

- Students must know properties of exponents including: zero exponent, negative exponent, product/quotient of powers, power of power, power of product/quotient.
- The student will determine the value of exponential expressions using a variety of methods.
- The student will simplify algebraic expressions by applying the properties of exponents.

Vocabulary

- base
- exponent
- exponential form
- power

Mathematical Practices

2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Essential Questions / Big Ideas

How can we rewrite and/or simplify expressions in exponential form? **Knowing the properties of integer exponents allows us to rewrite expressions in equivalent form.**

Exponential Expression: consists of a base and an exponent. The exponent tells you how many times to use the base as a factor.

$$a^5 = a \cdot a \cdot a \cdot a \cdot a \quad (a \text{ is used as a factor 5 times})$$

a is the BASE, 5 is the EXPONENT

a^5 is the 5th POWER of a .

Caution: There are some exponential expressions that can be tricky.

Ex: Power of a Quantity: $(a+b)^3 = \boxed{(a+b)(a+b)(a+b)}$ $(a+b)^3 \neq a^3 + b^3$!!!

Ex: Power of a Negative Number: $(-3)^4 = (-3)(-3)(-3)(-3) = \boxed{81}$



Ex: Negative Power of a Number: $-3^4 = -(3)(3)(3)(3) = \boxed{-81}$

Exploration:

Ex: Simplify the expressions by expanding them first using the definition of an exponent.

$$1. \quad b^2 \cdot b^3 \qquad b^2 \cdot b^3 = (b \cdot b)(b \cdot b \cdot b) = b \cdot b \cdot b \cdot b \cdot b = \boxed{b^5}$$

$$2. \quad x^4 \cdot x^2 \qquad x^4 \cdot x^2 = (x \cdot x \cdot x \cdot x)(x \cdot x) = x \cdot x \cdot x \cdot x \cdot x \cdot x = \boxed{x^6}$$

$$3. \quad 8^2 \cdot 8^7 \qquad 8^2 \cdot 8^7 = (8 \cdot 8)(8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8) = 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 = \boxed{8^9}$$

$$4. \quad a^3 \cdot a^3 \qquad a^3 \cdot a^3 = (a \cdot a \cdot a)(a \cdot a \cdot a) = a \cdot a \cdot a \cdot a \cdot a \cdot a = \boxed{a^6}$$

Using the pattern that emerged from the examples above, have students write a rule for multiplying exponential expressions.

Product of Powers Property: To multiply powers with the *same base*, **add** the exponents.

Ex: Simplify the expressions using the product of powers property.

$$1. \quad b^2 \cdot b^3 \qquad b^2 \cdot b^3 = b^{2+3} = \boxed{b^5}$$

$$2. \quad x^4 \cdot x^2 \qquad x^4 \cdot x^2 = x^{4+2} = \boxed{x^6}$$

$$3. \quad 8^2 \cdot 8^7 \qquad 8^2 \cdot 8^7 = 8^{2+7} = \boxed{8^9}$$

$$4. \quad a^3 \cdot a^3 \qquad a^3 \cdot a^3 = a^{3+3} = \boxed{a^6}$$

Exploration:

Ex: Simplify the expressions by expanding them first using the definition of an exponent.

$$1. \quad (y^2)^3 \qquad (y^2)^3 = (y^2)(y^2)(y^2) = y^{2+2+2} = \boxed{y^6}$$

$$2. \quad (4^5)^2 \qquad (4^5)^2 = (4^5)(4^5) = 4^{5+5} = \boxed{4^{10}}$$

$$3. \quad (x^3)^4 \qquad (x^3)^4 = (x^3)(x^3)(x^3)(x^3) = x^{3+3+3+3} = \boxed{x^{12}}$$

Using the pattern that emerged from the examples above, have students write a rule for raising an exponential expression to a power.



Power of a Power Property: To find a power of a power, **multiply** the exponents.

Ex: Simplify the expressions using the power of a power property.

$$1. (y^2)^3 \quad (y^2)^3 = y^{2 \cdot 3} = \boxed{y^6}$$

$$2. (4^5)^2 \quad (4^5)^2 = 4^{5 \cdot 2} = \boxed{4^{10}}$$

$$3. (x^3)^4 \quad (x^3)^4 = x^{3 \cdot 4} = \boxed{x^{12}}$$

Exploration:

Ex: Simplify the expressions by expanding them first using the definition of an exponent.

$$1. (xy)^4 \quad (xy)^4 = (xy)(xy)(xy)(xy) = (x \cdot x \cdot x \cdot x)(y \cdot y \cdot y \cdot y) = \boxed{x^4 y^4}$$

$$2. (3ab)^2 \quad (3ab)^2 = (3ab)(3ab) = 3 \cdot 3 \cdot a \cdot a \cdot b \cdot b = 3^2 a^2 b^2 = \boxed{9a^2 b^2}$$

$$3. (-2x)^4 \quad (-2x)^4 = (-2x)(-2x)(-2x)(-2x) = (-2)^4 x^4 = \boxed{16x^4}$$

Using the pattern that emerged from the examples above, have students write a rule for raising a product to a power.

Power of a Product Property: To find a power of a product, find the **power of each factor** and multiply.

Ex: Simplify the expressions using the power of a product property.

$$1. (xy)^4 \quad (xy)^4 = (x)^4 (y)^4 = \boxed{x^4 y^4}$$

$$2. (3ab)^2 \quad (3ab)^2 = (3)^2 (a)^2 (b)^2 = \boxed{9a^2 b^2}$$

$$3. (-2x)^4 \quad (-2x)^4 = (-2)^4 (x)^4 = \boxed{16x^4}$$

Combining the Properties

Ex: Simplify the expression $(-3x^2y)^3 \cdot y^4$.

Step One: Power of a product property $(-3)^3 (x^2)^3 (y)^3 \cdot y^4$

Step Two: Power of a power property $-27x^{2 \cdot 3} y^3 y^4 = -27x^6 y^3 y^4$



Step Three: Product of powers property

$$-27x^6y^{3+4} = \boxed{-27x^6y^7}$$

Ex: Simplify the expression $-a^2c(4b^3c^2)^2$.

Step One: Power of a product property

$$-a^2c(4)^2(b^3)^2(c^2)^2$$

Step Two: Power of a power property

$$-a^2c(16)(b^{3 \cdot 2})(c^{2 \cdot 2}) = -a^2c(16)(b^6)(c^4)$$

Step Three: Product of powers property

$$-a^2c^1(16)(b^6)(c^4) = -a^2c^{1+4}(16)(b^6) = \boxed{-16a^2b^6c^5}$$

You Try: Simplify the expression using the properties of exponents. $(-4rs^3t^5)^2(2r^5s^2)$

QOD: Describe the difference between $-a^b$ and $(-a)^b$, where a and b are whole numbers.

Sample Practice Question(s):

1. Evaluate $(x^3)^2$ when $x = 2$.

- A. 12
- B. 16
- C. 36
- D. 64

2. Determine the value of $3^2 \cdot 3$.

- A. 9
- B. 18
- C. 27
- D. 81

3. Which expression is equivalent to $(x^5y^3z)(5x^4y^3)^3$?

- A. $5x^{12}y^9z$
- B. $5x^{17}y^{12}z$
- C. $125x^{20}y^9z$
- D. $125x^{17}y^{12}z$



Additional Information for Teachers

Determine the value of exponential expressions using a variety of methods.

Simplify algebraic expressions by applying the properties of exponents.

Simplify expressions containing negative and zero exponents.

Exploration: Have students use patterns to complete the tables. (Answers are in red.)

2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}
16	8	4	2	1	1/2	1/4	1/8	1/16

Note: The pattern is to divide by 2 to move from left to right.

3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}
81	27	9	3	1	1/3	1/9	1/27	1/81

Note: The pattern is to divide by 3 to move from left to right.

Have students write a conjecture for the value of a base raised to the zero or negative power.

Zero Exponent: Any nonzero number raised to the zero power is equal to 1. $a^0 = 1, a \neq 0$

Negative Exponent: For any nonzero base, a^{-n} is the reciprocal of a^n . $a^{-n} = \frac{1}{a^n}, a \neq 0$

Evaluating Powers with Zero and Negative Exponents

Ex: Evaluate the following.

1. $(-3)^0$
2. 4^{-2}
3. $\left(\frac{3}{2}\right)^{-3}$
4. $\frac{1}{5^{-2}}$
5. $4^0 \cdot 6^{-1}$
6. $5^{-6} \cdot 5^6$
7. $(2^{-2})^3$
8. $\frac{4}{4^{-2}}$

Simplifying Exponential Expressions

Ex: Simplify the expression. Write your answer with positive exponents.

1. $3x^{-2}y^4$
2. $5a^{-3}b^0(2a^2b^{-6})$
3. $\frac{(-3r^2s^{-3})^3}{9s}$
4. $(2ab)^{-3}$
5. $\left(-\frac{1}{a}\right)^{-2}$
6. $\left(\frac{(a^3b^{-2}c)^2}{a^{-5}b^4}\right)^{-3}$



You Try: Simplify the expression $(3b^2)^{-2} \cdot 4a^{-1}b^5$. Write your answer with positive exponents.

QOD: Can you evaluate 0^{-1} ? Explain.

Sample Practice Questions:

1. Evaluate the expression $2^0(2^{-5})$.

A. -32

B. 0

C. $\frac{1}{32}$

D. $\frac{1}{16}$

2. Evaluate the expression $5^{-3} \cdot 5^4 \cdot 5$.

A. 1

B. 5

C. 10

D. 25



Skills:

- Determine the value of exponential expressions using a variety of methods.
- Simplify algebraic expressions by applying the properties of exponents.

Exploration:

Ex: Simplify the expressions by expanding them first using the definition of an exponent.

$$1. \quad \frac{5^8}{5^2} = \frac{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}{5 \cdot 5} = 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = \boxed{5^6}$$

$$2. \quad \frac{x^5}{x^3} = \frac{x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x} = x \cdot x = \boxed{x^2}$$

$$3. \quad \frac{b^9}{b^2} = \frac{b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{b \cdot b} = b \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b = \boxed{b^7}$$

Using the pattern that emerged from the examples above, have students write a rule for dividing powers that have the same base.

Quotient of Powers Property: To divide powers having the same base, **subtract** the exponents.

Ex: Simplify the expressions using the quotient of powers property.

$$1. \quad \frac{5^8}{5^2} = 5^{8-2} = \boxed{5^6}$$

$$2. \quad \frac{x^5}{x^3} = x^{5-3} = \boxed{x^2}$$

$$3. \quad \frac{b^9}{b^2} = b^{9-2} = \boxed{b^7}$$

Exploration:

Ex: Simplify the expressions by expanding them first using the definition of an exponent.

$$1. \quad \left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3} = \boxed{\frac{2^3}{3^3}}$$

$$2. \quad \left(\frac{a}{b}\right)^5 = \left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right)\left(\frac{a}{b}\right) = \frac{a \cdot a \cdot a \cdot a \cdot a}{b \cdot b \cdot b \cdot b \cdot b} = \boxed{\frac{a^5}{b^5}}$$



Using the pattern that emerged from the examples above, have students write a rule for dividing powers that have the same base.

Power of a Quotient Property: To find a power of a quotient, find the power of the numerator and the power of the denominator.

Ex: Simplify the expressions using the power of a quotient property.

$$1. \left(\frac{2}{3}\right)^3 \quad \left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3}$$

$$2. \left(\frac{a}{b}\right)^5 \quad \left(\frac{a}{b}\right)^5 = \frac{a^5}{b^5}$$

Evaluating Powers Using the Division Properties of Exponents

Ex: Evaluate the following expressions.

$$1. \frac{8^9}{8^7} \quad 2. \frac{4^3}{4^6} \quad 3. \frac{3^2 \cdot 3^{-3}}{3^4} \quad 4. \left(-\frac{5}{3}\right)^3 \quad 5. \left(\frac{1}{3}\right)^{-4} \cdot 3^{-2}$$

Simplifying Expressions with the Division Properties of Exponents

Ex: Simplify the expressions. Write your answers with positive exponents.

$$1. \frac{-4x^7y}{12x^3y^5} \quad 2. \frac{2a^2b}{3a} \cdot \frac{9ab^2}{b^4} \quad 3. \left(\frac{3x^{-1}}{y^2}\right)^4 \div \frac{9}{x^2}$$

You Try: Simplify the expression $\left(\frac{-2a^{-1}b^3}{a^2b}\right)^4$.

QOD: Using the division property of exponents, show algebraically why $a^0 = 1$, when a is a real number and $a \neq 0$.

Sample Practice Question:

If $\frac{y^t}{y^4} = y^{12}$, what is the value of t ?

- A. 3
- B. 8
- C. 16
- D. 48



Express numbers using scientific notation in mathematical and practical situations.

Exploration: Find the product of the following expressions.

$$1. \quad 4.3 \times 10^5 \quad 4.3 \times 10^5 = 4.3 \times 100000 = \boxed{430,000}$$

$$2. \quad 6.258 \times 10^9 \quad 6.258 \times 10^9 = 6.258 \times 1000000000 = \boxed{6,258,000,000}$$

$$3. \quad 3.2 \times 10^{-5} \quad 3.2 \times 10^{-5} = 3.2 \times 0.00001 = \boxed{0.000032}$$

$$4. \quad 1.452 \times 10^{-3} \quad 1.452 \times 10^{-3} = 1.452 \times 0.001 = \boxed{0.001452}$$

The numbers used in the exploration were written in **scientific notation**. Write in your own words how to convert numbers from scientific notation to decimal form.

Converting from Scientific Notation to Decimal (Standard) Form: To convert from scientific notation with a **positive** power of 10, move the decimal point to the **right**. To convert from scientific notation with a **negative** power of 10, move the decimal point to the **left**.

Note: Scientific notation is a short way to write long numbers with many digits, whether they are very large or very small. A very large number will have a positive power of 10, and a very small number will have a negative power of 10. This should help you remember which way to move the decimal.

Ex: Convert the following to decimal form.

$$1. \quad 4.3 \times 10^5 \quad \text{move decimal 5 places to the right: } 4.3 \times 10^5 = \boxed{430,000}$$

$$2. \quad 6.258 \times 10^9 \quad \text{move decimal 9 places to the right: } 6.258 \times 10^9 = \boxed{6,258,000,000}$$

$$3. \quad 3.2 \times 10^{-5} \quad \text{move decimal 5 places to the left: } 3.2 \times 10^{-5} = \boxed{0.000032}$$

$$4. \quad 1.452 \times 10^{-3} \quad \text{move decimal 3 places to the left: } 1.452 \times 10^{-3} = \boxed{0.001452}$$

Caution: Remember that the exponent determines how many places to move the decimal point, NOT how many zeros are in the decimal form of the answer!

Converting from Decimal Form to Scientific Notation: To convert to scientific notation, always move the decimal so that there is only one digit to the left of the decimal. Do not write any zeros, and use the number of times you moved the decimal point as the power of 10. If it was a **“large”** number (greater than 1), use a **positive** power of 10, and if it was a **“small”** number (smaller than 1), use a **negative** power of 10.



Ex: Convert the following to scientific notation.

1. 5,430,000,000

Step One: Move the decimal so that it is after the 5 and drop the zeros. (Note: Right now, the decimal point is at the end of the number.) 5.43

Step Two: How many places did you need to move the decimal? 9

Step Three: Write your answer in scientific notation using the number found in Step Two as the power of 10. This was a “large” number, so we will use **positive** 9 as the exponent.

$$\boxed{5.43 \times 10^9}$$

2. 0.0046

Step One: Move the decimal so that it is after the 4 and drop the zeros. 4.6

Step Two: How many places did you need to move the decimal? 3

Step Three: Write your answer in scientific notation using the number found in Step Two as the power of 10. This was a “small” number, so we will use **negative** 3 as the exponent.

$$\boxed{4.6 \times 10^{-3}}$$

3. 300

Step One: Move the decimal so that it is after the 3 and drop the zeros. (Note: Right now, the decimal point is at the end of the number.) 3

Step Two: How many places did you need to move the decimal? 2

Step Three: Write your answer in scientific notation using the number found in Step Two as the power of 10. This was a “large” number, so we will use **positive** 2 as the exponent.

$$\boxed{3 \times 10^2}$$

4. 52.1×10^4

Caution: This may appear to already be in scientific notation. However, because the decimal point is not to the right of just **one** digit, it is not.

Step One: Write 52.1 in scientific notation. Move the decimal so that it is after the 5. 5.21

Step Two: How many places did you need to move the decimal? 1

Step Three: Write your answer in scientific notation using the number found in Step Two as the power of 10. This was a “large” number, so we will use **positive** 1 as the exponent. 5.21×10^1



Step Four: Use the product property of exponents to simplify.

$$5.21 \times 10^1 \times 10^4 = 5.21 \times 10^{1+4} = \boxed{5.21 \times 10^5}$$

Multiplying and Dividing in Scientific Notation: Use the product and quotient properties of exponents to multiply and divide powers of 10. Be sure the final answer is in scientific notation.

Ex: Find the product. $(1.2 \times 10^3)(4 \times 10^8)$

Step One: Use the associative property of multiplication. $(1.2 \cdot 4)(10^3 \cdot 10^8)$

Step Two: Simplify. Use the product of powers property of exponents. $4.8 \times 10^{3+8} = \boxed{4.8 \times 10^{11}}$

Ex: Evaluate the expression. $\frac{1.2 \times 10^{-6}}{6 \times 10^{-4}}$

Step One: Rewrite as a product. $\frac{1.2}{6} \cdot \frac{10^{-6}}{10^{-4}}$

Step Two: Simplify. Use the quotient of powers property of exponents.

$$\frac{1.2}{6} \cdot \frac{10^{-6}}{10^{-4}} = 0.2 \times 10^{-6-(-4)} = 0.2 \times 10^{-2}$$

Step Three: Write the answer in scientific notation. $(0.2) \times 10^{-2} = (2 \times 10^{-1}) \times 10^{-2} = \boxed{2 \times 10^{-3}}$

Finding Powers of Numbers in Scientific Notation: Use the power of a product property and power of a power property of exponents to simplify. Write your final answer in scientific notation.

Ex: Evaluate the expression. $(4.0 \times 10^{-5})^3$

Step One: Use the power of a product property. $(4.0 \times 10^{-5})^3 = (4.0)^3 \times (10^{-5})^3$

Step Two: Simplify. Use the power of a power property of exponents. $64 \times 10^{(-5)(3)} = 64 \times 10^{-15}$

Step Three: Write the answer in scientific notation.

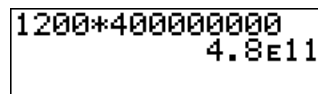
$$(64) \times 10^{-15} = (6.4 \times 10^1) \times 10^{-15} = 6.4 \times 10^{1+(-15)} = \boxed{6.4 \times 10^{-14}}$$



Scientific Notation on the Graphing Calculator

Ex: Find the product of 1,200 and 400,000,000.

The "E" on the calculator screen represents scientific notation. The number after the E is the power of 10.



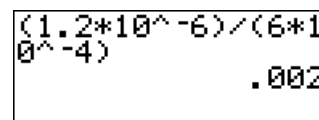
notation.

Answer: 4.8×10^{11}

Note: The calculator uses scientific notation because the number of digits in the answer would not fit on the screen.

Ex: Find the quotient: $\frac{1.2 \times 10^{-6}}{6 \times 10^{-4}}$

This time the calculator did not put the answer in scientific notation. This is because it had few enough digits to fit on screen. We will have to write the answer in scientific notation.



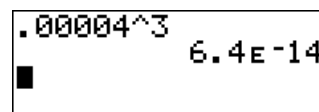
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$.002 = 2 \times 10^{-3}$

Ex: Evaluate the power. $(0.00004)^3$

Write the answer in scientific notation.

6.4×10^{-14}



Note: These are the same examples we calculated by hand. Do you think it is easier to find products, quotients, and powers by hand in scientific notation or in decimal form?

You Try: Evaluate the expression without a calculator. Write the result in scientific notation and in

decimal form. $\frac{(3 \times 10^{-6})^4}{6 \times 10^5}$

QOD: Explain when and why scientific notation is used instead of the decimal form of a number.

Sample Practice Question(s):

1. What is 75,200,000 in scientific notation?

A. 7.52×10^5

B. 75.2×10^5

C. 75.2×10^6

D. 7.52×10^7

2. Multiply: $(2.0 \times 10^{-6})(3.3 \times 10^2)$. What is the product in scientific notation?

A. 66×10^{-5}

B. 6.6×10^{-4}

C. 0.66×10^{-3}

D. 6.6×10^{-12}