

Active Learning for Mathematics

There are any number of teaching strategies that can be employed to actively engage students in the learning process, including group discussions, problem solving, case studies, role plays, journal writing, and structured learning groups. The benefits to using such activities are many. They include improved critical thinking skills, increased retention and transfer of new information, increased motivation, and improved interpersonal skills.

Some Simple Paired Activities

Having students work in pairs on a task is a low-risk strategy which virtually ensures 100 percent participation in classes of any size. Below are a few simple activities which can be adapted to almost any content area.

Homework-checking Pairs

The objective is to engage students with checking their homework during the first of class in order to clarify major and minor points of the homework, and to increase communication in teaching. Each pair is given the task to check the homework answers that are displayed and to help each other with homework errors. Approximate time: two to five minutes.

Think/Pair/Share

The objectives are to engage the class with the material on an individual level, in pairs, and finally as a large group. The activity can help to organize prior knowledge; brainstorm questions; or summarize, apply, or integrate new information. Approximate time: six to eight minutes.

The procedure is as follows: 1) individuals reflect on and write brief notes or solve a problem for one minute in response to a question or problem; 2) students pair up with someone sitting near them and share their answers or method of solution verbally for two to three minutes, or they may choose to work together to create a better answer; 3) the instructor randomly chooses a few pairs to give thirty-second summaries of individual or joint answers.

Question and Answer Pairs

The objective here is to engage individuals and then to pair them to answer particular questions. This helps to increase motivation to participate in the instruction of concepts, to deepen the level of analysis particular problems, and to practice explaining difficult concepts. Instructors may choose to model the kinds of questions that are appropriate to this exercise or somehow indicate the level, content, or scope of appropriate questions. Approximate time: five to ten minutes.

The procedure is as follows: 1) students do the assignment and compose one or two questions about it; 2) in class, the students pair up; A asks a prepared question and B responds; then B asks a prepared question and A responds; 3) the instructor may ask students to turn in their questions and summary answers.

Note-checking Pairs

The objective is to engage students with their notes during class in order to integrate their notes on new material with previous material, to clarify major and minor points, and to increase accuracy in note-taking. Approximate time: two to five minutes.

The procedure is as follows: 1) at the end of a lecture segment (15 minutes is a good length for instruction before formative assessment), students pair up to complete a task with their notes; for example, they could summarize the three major arguments of the lecture, choose the most important idea that will appear on the exam, check the accuracy of some information, or use the notes to solve an example problem; the instructor may generate a question from the group for the pairs to work on; 2) the instructor may ask students to turn in their answers.

General Guidelines for Paired Activities

- Don't use the same techniques too often. Once per week per technique is a reasonable use.
- Vary the accountability by occasionally having students turn in the work. Read a sample then comment specifically on it.
- Have students occasionally pair up with the student behind them, since friends often sit side by side.
- Request students vary their seating arrangements to increase their chances to work with different people.
- Reflect some of the informal activities in the formal evaluations in some way. For example, include a short essay question that was used in a think/pair/share.
- Be candid with the students as to why you are asking them to do these things. Explain attention span, the need for engaging material individually and socially, and that research shows better learning occurs by using active learning.

Planning an Active Learning Activity

- When planning an active learning activity, answering the following questions will help you clarify your goals and structure:
- What are your objectives for the activity?
- Who is interacting? Will students pair up with someone beside them? Or perhaps someone sitting behind/in front of them? Should they pair up with someone with a different background? Someone they don't know yet?
- When does the activity occur during the class? Beginning? Middle? End? How much time are you willing to spend on it?
- Will they write down their answers/ideas/questions or just discuss them?
- Will they turn in the responses or not? If they are asked to turn them in, should they put their names on them?
- Will you give individuals a minute or so to reflect on the answer before discussing it or will they just jump right into a discussion?
- Will you grade their responses or not?
- How will they share the paired work with the whole class? How will you share the feedback and insight you gain from their responses?
- If they are responding to a question you pose, how are you going to ensure that they leave with confidence in their understanding? Often, if the various student answers are not discussed without the instructor explicitly indicating which ones are "right", students become frustrated. Even with a question that has no absolute "right" answer, students want to know what the instructor's stand on the question is.
- What preparation do you need to use the activity? What preparation do the students need in order to participate fully?

Keys to Success

- Start small and be brief
- Develop a plan for an active learning activity, try it out, collect feedback, then modify and try it again.
- Start from the first day of class.
- Always try the question or task yourself first. Whenever possible, also try it on a colleague.
- Be explicit with students about why you are doing this and what you know about the learning process.
- Negotiate a signal to stop talking.
- Randomly call on pairs to share. Have every member of the pair contribute.
- Find a colleague or two to plan with (and perhaps teach with) while you're implementing active learning activities.
- Continue learning through workshops, reading, and practice.

Some Simple Group Activities

Group activities require teachers to develop positive routines, expectations, communication patterns, and student behavior management.

Group Project Assignments

The objective is to engage students with solving a multiple-step problem.. Instructions for cooperative learning activities 3/3/2009

A bag contains clues for students to solve a mathematical situation. There is one clue per card.

Cooperative Learning

The objective is to engage students with solving a multiple-step problem..

A bag contains clues for students to solve a mathematical situation. There is one clue per card.

Rules:

- 1. Each person can ONLY look at his/her own clue cards.**
 - a. No one gets to look at the clues until they are passed out.
 - b. A team member can read his clues to the group.
 - c. A team member can ask another member to read his/her clue.
2. Each person is responsible for his/her clue or information.
3. Each person is expected to contribute to the solution of the problem.
4. Diagrams, pictures, images, descriptions, etc. are encouraged for solving the problem.
5. Work together to answer the question posed.
6. **Solutions** will be discussed in _____ minutes.

Note: Solutions can be discussed as a class or groups can present their solutions.

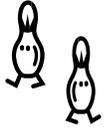
Be sure to answer the following guiding questions.

How did you arrive at your solution?

How do you know it is correct?

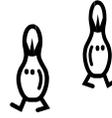
Approximate time: five to ten minutes.

A sample is attached.



Clue #1: A group of 4 friends is going bowling. Three of them have not been bowling before. They would like to go next Wednesday.

Work with your group to determine whether the friends should go bowling at Bowling Bonanza or Super Bowl.



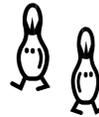
Clue #2: Bowling Bonanza charges \$2.50 for each player to rent shoes.

Work with your group to determine whether the friends should go bowling at Bowling Bonanza or Super Bowl.



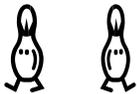
Clue #3: At Bowling Bonanza, there is also a flat-rate charge of \$20 per hour for a group of 4 to bowl.

Work with your group to determine whether the friends should go bowling at Bowling Bonanza or Super Bowl.



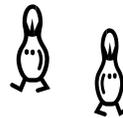
Clue #4: The group of friends has only \$80 to spend on the bowling activity. They can be gone for only 3 hours.

Work with your group to determine whether the friends should go bowling at Bowling Bonanza or Super Bowl.



Clue #5: They find an ad in the newspaper for a new bowling alley, Super Bowl which charges each player \$3.00 per game.

Work with your group to determine whether the friends should go bowling at Bowling Bonanza or Super Bowl.



Clue #6: Super Bowl also provides free bowling shoes on Wednesdays. They figure they can play 2 games per hour.

Work with your group to determine whether the friends should go bowling at Bowling Bonanza or Super Bowl.

Cubing

The objective is to engage students with verbal discourse in explaining concepts. This is an excellent activity for **diverse** learners to bring out prior knowledge and background.

Description: Cubing is a strategy which uses a concrete visual of a cube with its six sides to serve as a starting point for consideration of the multiple dimensions of topics within subject areas. **This activity can also be done using dice with numbers 1-6. Students (usually in groups) can take turns rolling the dice and whatever number it lands on, the student will answer that particular question. It should be done in groups of 3-6 students, with a recap of the responses for the class.**

The students examine and discuss the topic using the prompts from the six sides of the cube.

1. **Describe it**

If applicable, include color, shape, and size.

How would you describe the item/concept/topic?

2. **Compare it**

What it is similar to or different from.

"It's sort of like _____."

3. **Associate it**

What it makes you think of.

How does the topic connect to other concept/subjects?

4. **Analyze it**

Tell how it is made or what it is composed of.

How would you break the problem/concept into smaller parts?

5. **Apply it**

Tell how it can be used.

How does it help you understand other topics/concepts?

6. **Argue for/against it**

Take a stand and support it.

I am for this because _____.

This works because _____.

Compare/Contrast

Comparing concepts is very important in mathematics. Here are a few example forms that help to recognize similarities and differences.

Forms of Linear Equations

Name of form of the linear equation	Slope-intercept	Point-slope	Standard	Point-point	
Equation					
Used when					
Slope					
Point					
y-intercept					
Graph					

Summary:

Forms of equations

Graph/Parent	General form	Standard / Graphing Form/Graph	Characteristics/Vocabulary
Line			
Parabola			
Circle			
Ellipse			
Hyperbola			

To translate graphs:

To stretch/shrink graphs:

Vocabulary (Word Categorization)

This activity can be done as a whole-class activity or as a small-group activity. It can be done quickly as a review, or in more detail as instruction with compare/contrast capabilities.

- The word or phrase is written in the center box
- Examples are written in the bottom left section (Quadrant III)
Each of the quadrants should include examples, illustrations, images as appropriate
Teacher may give some, but students should see the pattern and provide input
- Non-examples are written in the bottom right section (Quadrant IV).
Teacher may give some, but students should see the pattern and provide input
- Characteristics are written in the top right section (Quadrant I).
Teacher may provide some, but students should see the pattern and provide input
- A definition is written in the upper left section (Quadrant II). As students offer definitions, it may be necessary to add more information in the quadrants or to point to information that will focus their definition.

Variations:

- The order of placing information in the quadrants can be changed. For example, on new material, the definition could be given first, then maybe an example or two, then ask students for more examples or non-examples, then characteristics . . .
- The word or phrase can be left blank and then determined by students based on the information in the quadrants.
- Go through the word categorization quickly, then use the back of the paper (or other space) to expand instruction or practice if needed.

Benefits of this activity:

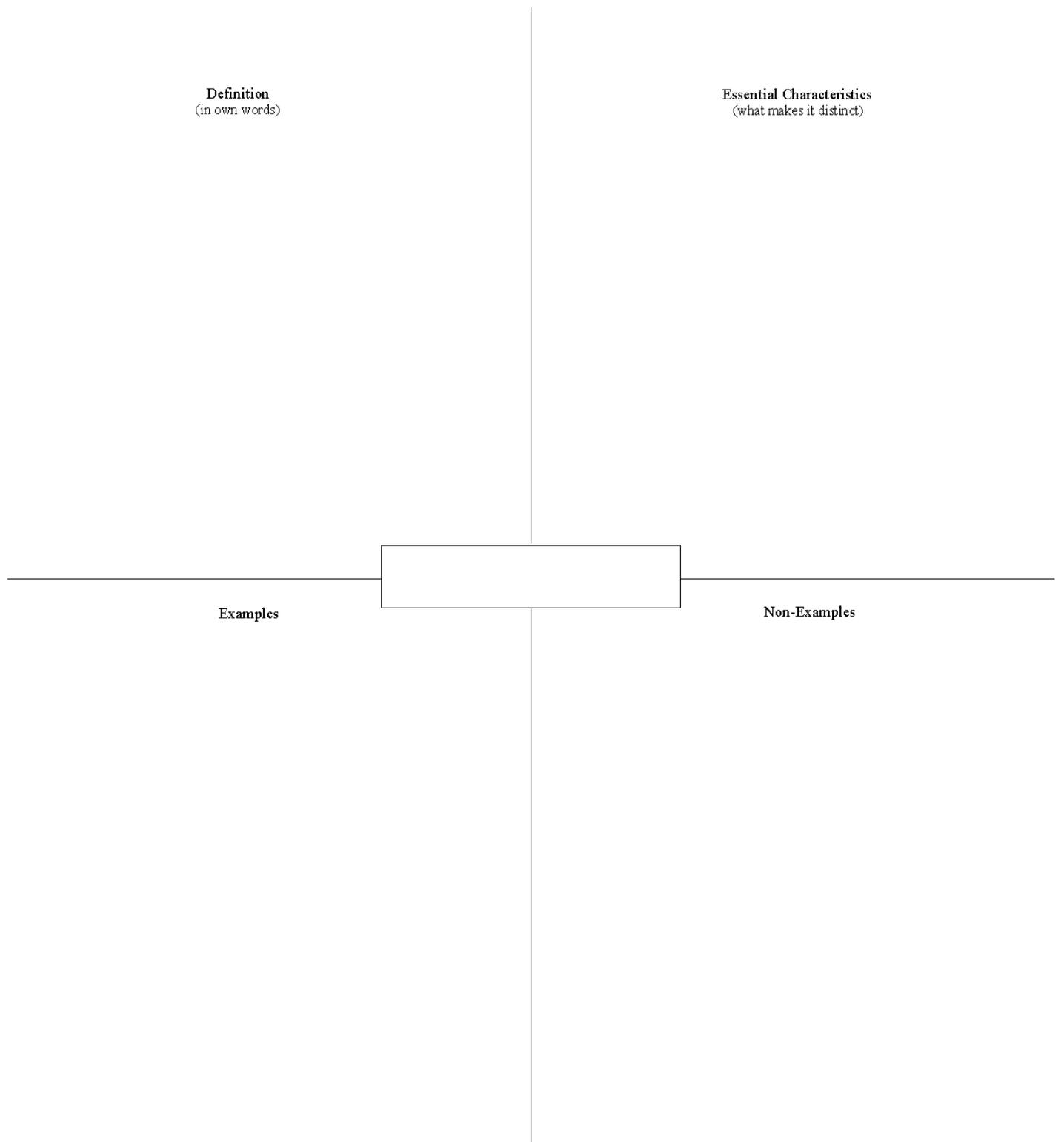
- Efficient compare/contrast
- Recognition of similarities/differences
- Recognition of defining characteristics
- Verbal/written translation to definition
- Discussions and student engagement

Definition
(in own words)

Essential Characteristics
(what makes it distinct)

Examples

Non-Examples



Poster Method

The poster method is another strategy to implement Standards of Mathematical Practice and student engagement. It has students solve a problem, make a poster, and share it with the class. It should be used in the beginning of the year to build student capacity to solve problems. The teacher will not tell or hint how to do the problem. Instead, use effective questions that foster thinking about what they have, what they need to know, if they see a pattern, if their answer seems reasonable, etc.

Steps

1. Students work in small groups (4)
2. There is a brief period (1-2 minutes) of individual work to paraphrase, plan, or make sense of the task.
3. Group discusses and agrees on a solution. (There may be multiple ways to solve the task.)
4. Group finalizes answer and puts it on a poster showing the work, models, etc.
5. Group creates written explanation and writes these on the back of the poster.

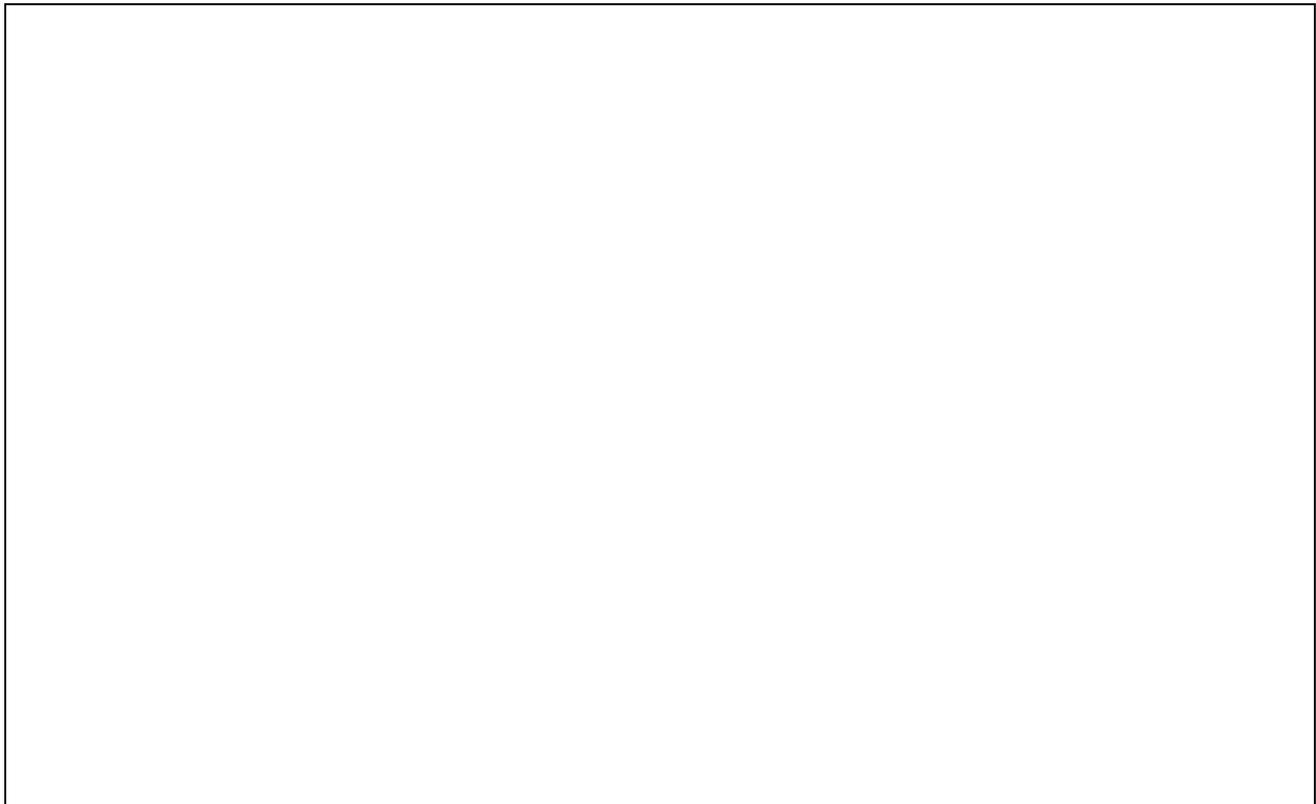
How did your group solve the problem?

How does your group know your solution is correct mathematically?

6. Circle discussion: One student from each group will take the poster and go with the other groups to encircle the class, holding the poster toward the class. That student must explain to the class their group's solution. (Note: Now the written explanation is on the back to help the student.)
7. Class determines solution from those presented.

Consider the 8 Standards for Mathematical Practice. This activity can utilize each of them in various ways.

Data Sheet _____



Examples of activities used for solving problems cooperatively.

Lawn Mowing Chore

At my house, summer vacation means added chores for my three sons. Every Saturday, the lawn must be mowed. Mark starts the mower and completes $\frac{1}{3}$ of the lawn. Sam takes over and mows exactly $\frac{1}{4}$ of the lawn. Josh finishes off the last 700 square feet of the lawn. What is the area of my lawn?

Meeting with Friends

Janet and Sally live 540 miles apart. They decide to drive to meet each other. Janet drives 40 miles per hour and leaves at 1 p.m. Sally drives 60 miles per hour and leaves at 2 p.m. What time will they meet?

Finding balance

A slab of soap on one pan of a scale balances $\frac{3}{4}$ of a slab of soap of equal weight and a $\frac{3}{4}$ pound weight on the other pan. How much does the slab of soap weigh? (Solve the problem both with an algebraic equation and by direct arithmetic reasoning.)

Fuel for Thought

Which of the following would save more fuel?

- Replacing a compact car that gets 34 miles per gallon (mpg) with a hybrid that gets 54 mpg?
- Replacing a sport utility vehicle (SUV) that gets 18 mpg with a sedan that gets 28 mpg?
- Both changes save the same amount of fuel.

Letting it all out

A water tank can be filled by hose in 4 hours. The drain valve at the bottom can drain the tank in 6 hours. If the drain valve were mistakenly left open, how long would it take to fill the tank from empty?

An Age Old Problem

This year your brother Jack will be 2 years from being twice as old as your sister Jan. The sum of Jack's age and three times Jen's age is 66. How old is Jen?

Colorful Fence

Dana needs 300 lights for her colorful picket fence. She wants equal amounts of each of her 4 selected colors. She already has 32 red, 26 green, 9 yellow, and no blue. How many more of each color does Dana need to buy? If the bulbs cost 25 cents and you get 20% off if you purchase 50 or more of the same color and 30% off if you purchase 60 or more of one color, how much does Dana need to spend?

Florida Vacation

Your family is planning a 7-day trip to Florida. You estimate that it will cost \$275 per day in Tampa and \$400 per day in Orlando. Your total budget for the 7 days is \$2300. How many days should you spend in each location.

Sports Equipment

For 2002 through 2011 the sales S (in millions of dollars) of gym shoes and sneakers can be modeled by

$$S = -0.982t^5 + 24.6t^4 - 211t^3 + 66t^2 - 318t + 1520$$

where t is the number of years since 2002. Were there any years in which sales were about \$2 billion?