F-LE, A-REI Population and Food Supply

Alignments to Content Standards: F-LE.A.2 F-LE.A.3 A-REI.D.11

Task

The population of a country is initially 2 million people and is increasing at 4% per year. The country's annual food supply is initially adequate for 4 million people and is increasing at a constant rate adequate for an additional 0.5 million people per year.

a. Based on these assumptions, in approximately what year will this country first experience shortages of food?

b. If the country doubled its initial food supply and maintained a constant rate of increase in the supply adequate for an additional 0.5 million people per year, would shortages still occur? In approximately which year?

c. If the country doubled the rate at which its food supply increases, in addition to doubling its initial food supply, would shortages still occur?

IM Commentary

In this task students construct and compare linear and exponential functions and find where the two functions intersect (F-LE.2, F-LE.3, A-REI.11).

One purpose of this task is to demonstrate that exponential functions grow faster than linear functions even if the linear function has a higher initial value and even if we increase the slope of the line. This task could be used as an introduction to this idea. The steps in this task require students to find linear and exponential functions from verbal descriptions. If they previously learned how to do this, the task can be completed independently. So they can practice previously learned skills and explore the idea of the dominance of exponential over linear functions at the same time.

We note that it may be worth commenting or holding a discussion on the implications of this investigation. The question of population growth versus depletion of human resources is of both historical and current significance, perhaps starting with Thomas Malthus' "An Essay on the Principle of Population," and continuing today in terms of food production, oil production, pollution thresholds, etc.

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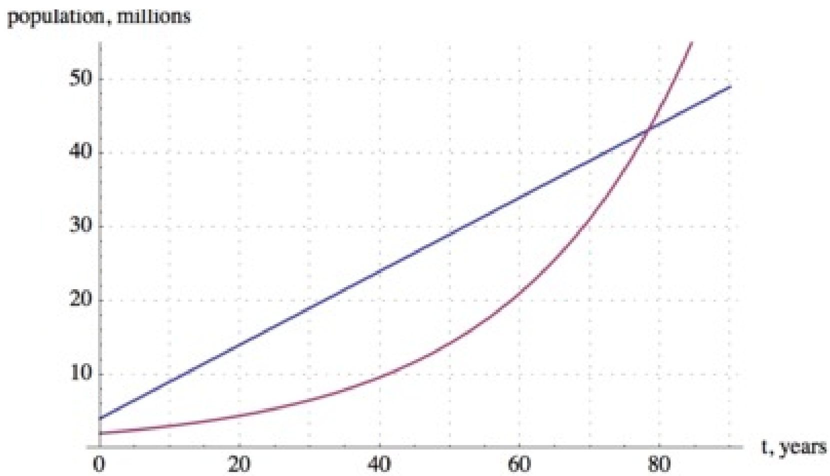
Solution

a. We can first express the country's population, $p(t)$, in millions of people, as a function of the time t, measured in years from the initial time. Since we know the initial population $p(0) = 2$ and the annual growth rate is 4%, then $p(t)$ is an exponential function:

$$p(t) = 2(1.04)^t.$$

We are also given that the food supply grows at a constant rate. So we can express the country's food supply at time t, which we call $f(t)$, as a linear function of t. Again, we know the initial value $f(0) = 4$ and the constant rate of change is 0.5 million people per year, so we have:

$$f(t) = 4 + .5t.$$



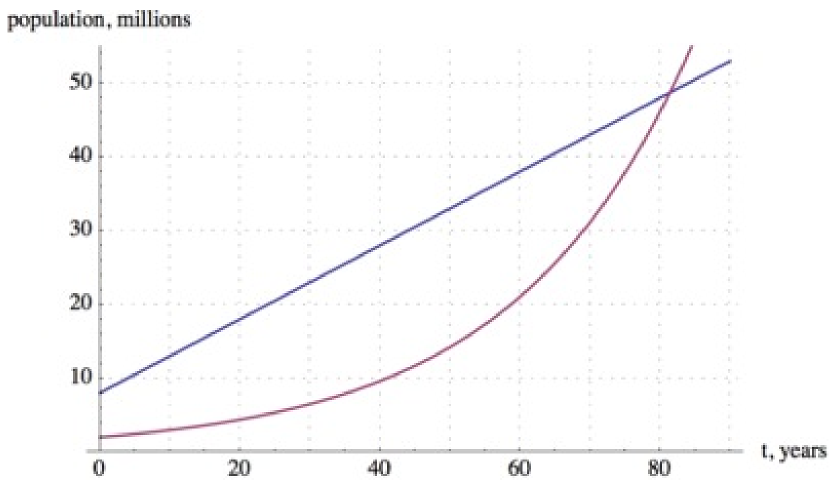
We are looking for the value of t which makes $p(t)$ greater than $f(t)$ for the first time.

We see from the graph that the two functions intersect at around $t = 78$. So after 78 years the food supply is just barely enough for the country's population. After this point, however, we see that $p(t) > f(t)$, so this country will first experience shortages of food after approximately 78 years.

b. If the country doubled its initial food supply, our new function for the food supply would be

$$h(t) = 8 + .5t$$

We would expect food shortages to occur, if at all, later than in part (a).

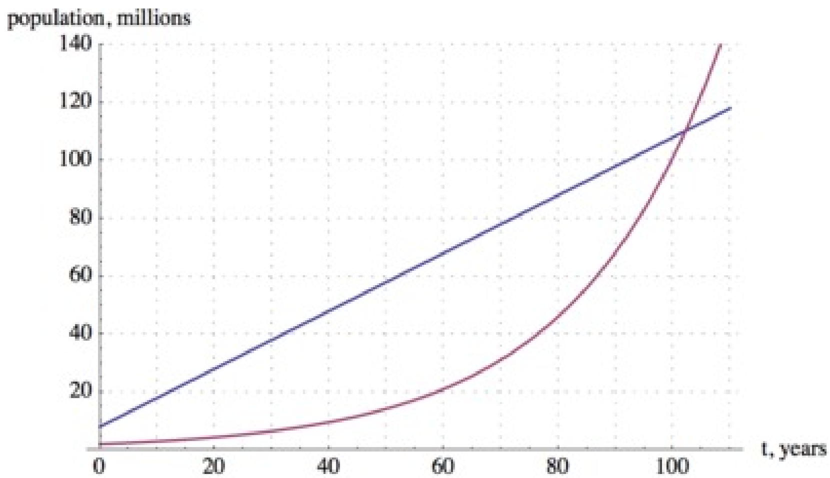


Again looking at the graph, we see that the two functions intersect, and so food shortages would still occur. We find $p(t) = h(t)$ at roughly $t = 81$. So, the country will first experience food shortages after 81 years. So doubling the initial food supply delays the eventual food shortage by only 3 years.

c. If the country doubled the rate at which its food supply increases, in addition to doubling its initial food supply, we have the new food supply function:

$$j(t) = 8 + t$$

We would expect, in this case, for food shortages to occur much later than in part (b), if at all.



Looking at the graph we see that this time the food shortage occurs at $t = 103$, about 25 years later than in part (a).

Examining the behavior of the exponential function more closely we observe, that the slope of the exponential function keeps increasing whereas the slope of any linear function is constant. Even if a linear function has a very large slope, an exponential function will eventually grow even faster and overtake the linear function.