F-LE Finding Linear and Exponential Models

Alignments to Content Standards: F-LE.A.1.b F-LE.A.1.c

Task

For each or the scenarios below, decide whether the situation can be modeled by a linear function, an exponential function, or neither. For those with a linear or exponential model, create a function which accurately describes the situation.

a. From 1910 until 2010 the growth rate of the United States has been steady at about 1.5% per year. The population in 1910 was about 92,000,000.

b. The circumference of a circle as a function of the radius.

c. According to an old legend, an Indian King played a game of chess with a traveling sage on a beautiful, hand-made chessboard. The sage requested, as reward for winning the game, one grain of rice for the first square, two grains for the second, four grains for the third, and so on for the whole chess board. How many grains of rice would the sage win for the n^{th} square?

d. The volume of a cube as a function of its side length.

IM Commentary

The goal of this task is to present students with real world and mathematical situations which can be modeled with linear, exponential, or other familiar functions. In each case, the scenario is presented and students must decide which model is appropriate. The question can be taken much farther as, for example, students could be asked what

the growth model for question (a) would predict for 2010, 2020, 2050, and so on. Similarly for part (c), a good question would be which square on the chessboard is the first worth more than 1000 grains of rice, or 1,000,000 or 1,000,000,000? For some problems with this second level of questions,
see https://www.illustrativemathematics.org/tasks/1911.

Parts (a) and (c) have been adapted from
http://en.wikipedia.org/wiki/Exponential_growth which presents much more useful information and examples of exponential models.

Edit this solution

Solution

a. We will write $P(t)$ for the United States population t years after 1910. We have $P(0) = 92,000,000$ since the population in 1910 was given as 92,000,000. Since the annual growth rate is 1.5%, the population increases by a factor 0.015 each year. So an equation modeling the U.S. population between 1910 and 2010 is $P(t) = 92,000,000(1.015)^t$. This is an exponential function of t: the base of the exponential expression is 1.015, indicating that each year the population increases by 1.5% compared to the previous year).

b. The circumference of a circle is πd where d denotes the diameter of the circle. The diameter is twice the radius so the circumference of a circle of radius r is $2\pi r$. So if $C(r)$ is the circumference of a circle of radius r then $C(r) = 2\pi r$. This is a linear function of the radius r.

c. For the first square on the chess board there is one grain of rice which we can also write at 2^0 grains of rice. For the second square there are two grains of rice or 2^1. For each new square we double the previous number which means adding one more factor of 2. The number of factors of 2 is one less than the number of the square. So if $f(n)$ denotes the number of grains of rice for the n^{th} square we have $f(n) = 2^{n-1}$. This is an exponential function of n.

d. If s is the side length of the cube, then its volume $V(s)$ is found by taking the product of length, width, and height. Since each of these is s we find $V(s) = s^3$. This is neither linear nor exponential but rather a polynomial relationship.