



Unit 9 – Polynomials and Factoring

PREREQUISITE SKILLS:

- students should be able to recognize coefficients, factors, terms and constants of an expression
- students should be able to work with exponents
- students should be able to use the distributive property
- students should be able to combine like terms
- students should be able to recognize linear, exponential and quadratic functions

VOCABULARY:

- **Binomial** – An algebraic expression of the sum or the difference of two terms.
- **Difference of Two Squares** – A binomial that is a squared number subtracted from another squared number.
- **Distributive Property** – It is a law that lets you multiply a sum by multiplying each addend separately and then add the products.
- **Factors** – Terms that we can multiply together to get another term.
- **Greatest Common Factor** – The highest number that divides exactly into two or more numbers.
Abbreviated: GCF
- **Leading Coefficient** – The number written in front of the variable with the largest exponent in an expression.
- **Monomial** – An algebraic expression consisting of one term.
- **Perfect Square Trinomial** – It is a three term expression that factors into two identical factors.
- **Polynomial** – An expression of more than two terms.
- **Quadratic expression** – An expression where the highest exponent of the variable is a square (2).
- **Trinomial** – An algebraic expression consisting of three terms.

SKILLS:

- Rewrite polynomial expressions through multiplying, addition and subtraction and factoring
- Write polynomials as products
- Factor polynomials

STANDARDS:

A.SSE.A.1 Interpret expressions that represent a quantity in terms of its context. *(Modeling Standard)

A.SSE.A.2-1 Use the structure of a linear, exponential, or quadratic expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.



A.APR.A.1-1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add and subtract linear and quadratic polynomials, and multiply binomials.

A.CED.A.1-1 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, and exponential functions. Limit exponentials to have integer inputs only. *(Modeling Standard)

LEARNING TARGETS:

- **9.1** To classify polynomials and write polynomials in standard form
- **9.2** To add and subtract polynomial expressions
- **9.3** To multiply polynomial expressions
- **9.4** To factor polynomials

ESSENTIAL QUESTIONS:

- Why is it helpful to be able to change the forms of quadratic expressions?
- How is the arithmetic of polynomials similar to the arithmetic of integers or real numbers?
- What are some of the identifying characteristics of polynomials?

BIG IDEAS:

Polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction and multiplication.

Rewriting algebraic expressions in different forms can reveal previously-hidden characteristics of the expression.



Notes, Examples and Exam Questions

Unit 9.1 To classify polynomials and write polynomials in standard form.

Teacher Note: A nice way to illustrate operations with polynomials is by using algebra tiles. If you have them available, use them with your students throughout this unit.

Polynomial: an expression which is the sum or difference of terms of the form ax^k , where k is a nonnegative integer and a is a real number

Degree of a Term: the value of the exponent of the variable

Degree of a Polynomial: the largest degree of its terms

Standard Form of a Polynomial: the terms of the polynomial are written in descending order, from largest degree to smallest degree

Leading Coefficient: the coefficient of the first term of a polynomial when it is written in standard form

Classifying Polynomials

By Degree:

Degree	0	1	2	3	4
Name	Constant	Linear	Quadratic	Cubic	Quartic

By Number of Terms:

# of Terms	1	2	3	4 and up
Name	Monomial	Binomial	Trinomial	Polynomial

Ex 1: Write the polynomial in standard form. State its degree and leading coefficient. Then, classify by the degree and number of terms. $7 + 4x^3 - x^2$

Standard Form: Write the terms in descending order of the exponents.

$$4x^3 - x^2 + 7$$

Degree of the Polynomial: The exponent of the first term.

$$\text{Degree} = 3$$

Leading Coefficient: The coefficient of the first term.

$$\text{Leading Coefficient} = 4$$

Classify: By degree, it is a cubic. By the number of terms, it is a trinomial.

Cubic Trinomial



Ex 2: Write the polynomial in standard form. State its degree and leading coefficient. Then, classify by the degree and number of terms. $-5 + 2x$

Standard Form: Write the terms in descending order of the exponents. $2x - 5$

Degree of the Polynomial: The exponent of the first term. Degree = 1

Leading Coefficient: The coefficient of the first term. Leading Coefficient = 2

Classify: By degree, it is a cubic. By the number of terms, it is a trinomial. Linear Binomial

On Your Own: Have students write their own examples of various classifications, i.e. quadratic monomial, linear binomial, etc.

Sample Exam Questions

1. Which expression is equivalent to $6x^2 - 4x + 3 - 5 - 8x^2 + 7x$?

A. $-2x^2 + 3x - 2$ **B.** $-2x^2 + 11x - 2$

C. $14x^2 + 3x + 8$ **D.** $14x^2 + 11x + 8$

Ans: A

2. Name this polynomial based on its degree and number of terms: $-6x^3 - 4x + 7$

A. quadratic binomial **B.** cubic binomial

C. quadratic trinomial **D.** cubic trinomial

Ans: D

3. What is the degree of $x^2 - x^3 + 4$?

A. 1 **B.** 2 **C.** 3 **D.** 4

Ans: C



Unit 9.2 To add and subtract polynomial expressions.

Operations with Polynomials

Recall: Expanded notation of a number.

$$453 = 4(100) + 5(10) + 3(1) = 4(10^2) + 5(10) + 3(1)$$

$$312 = 3(100) + 1(10) + 2(1) = 3(10^2) + 1(10) + 2(1)$$

As polynomials, we can write these as

$$\begin{array}{r} 4x^2 + 5x + 3 \\ 3x^2 + 1x + 2 \end{array}$$

When we add $453 + 312$, we must line up the place values. This is just like adding like terms.

$$\begin{array}{r} 453 = 4(10^2) + 5(10) + 3(1) \\ + 312 = 3(10^2) + 1(10) + 2(1) \\ \hline 765 = 7(10^2) + 6(10) + 5(1) \end{array} \quad \Rightarrow \quad \begin{array}{r} 4x^2 + 5x + 3 \\ + 3x^2 + 1x + 2 \\ \hline 7x^2 + 6x + 5 \end{array}$$

Ex: Add in your head $341 + 214 + 132$.

You have $300 + 200 + 100$, that's 600, adding the tens, we have $40 + 10 + 30$ which is 80, and finally adding $1 + 4 + 2$ or 7, the sum is 687.

Addition: to add polynomials, add like terms (terms with the same variable part)

Ex 3: Find the sum. Write the answer in standard form. $(4x^3 - x + 2x^2 + 8) + (6 - 3x^3 + x)$

Method 1 (Vertical Method): Write the polynomials in standard form and line up like terms.

$$\begin{array}{r} 4x^3 + 2x^2 - x + 8 \\ -3x^3 + x + 6 \\ \hline x^3 + 2x^2 + 14 \end{array} \quad \boxed{x^3 + 2x^2 + 14} \quad \text{Note: This is a cubic trinomial.}$$

Method 2 (Horizontal Method): Find the like terms and add them.

$$\left(\boxed{4x^3} - x + 2x^2 + \boxed{8} \right) + \left(\boxed{6} - \boxed{3x^3} + x \right) = (4x^3 - 3x^3) + (-x + x) + (2x^2) + (8 + 6) = \boxed{x^3 + 2x^2 + 14}$$



Subtraction: to subtract polynomials, *add the opposite*

Ex 4: Find the difference. Write the answer in standard form. $(x^2 + 3x - 5) - (6 - 5x + 4x^2)$

Method 1 (Vertical Method): Write the polynomials in standard form and line up like terms.

$$\begin{array}{r} x^2 + 3x - 5 \\ -(4x^2 - 5x + 6) \\ \hline \end{array} \quad \text{add the opposite: } \begin{array}{r} x^2 + 3x - 5 \\ -4x^2 + 5x - 6 \\ \hline -3x^2 + 8x - 11 \end{array} \quad \text{Note: This is a quadratic trinomial}$$

Method 2 (Horizontal Method): Find the like terms and add them. Add the opposite.

$$(x^2 + 3x - 5) + (-6 + 5x - 4x^2) = (x^2 - 4x^2) + (3x + 5x) + (-5 - 6) = -3x^2 + 8x - 11$$

Addition and Subtraction:

Ex 5: Simplify the expression. Classify the remaining polynomial. $(3x^2 - x^3 + 1) + (x^3 + 2) - (8 + 3x^2)$

Use the horizontal method. Add the opposite: $(3x^2 - x^3 + 1) + (x^3 + 2) + (-8 - 3x^2)$

Combine like terms: $(3x^2 - 3x^2) + (-x^3 + x^3) + (1 + 2 - 8) = -5$

Classify: This is a **constant monomial**.

Modeling with Polynomials:

Ex 6: Projected from 1950 through 2010, the female population F and the male population M of the United States (in thousands) can be modeled by the following equations, where t is the number of years since 1950. Find a model that represents the total population P of the U.S. from 1950 through 2010. Use the model to estimate the total population of the U.S. in 2009.

$$F = 1223t + 79,589 \quad M = 1164t + 75,622$$

To find the total population, we need to add the two polynomials above.

$$P = (1223t + 79589) + (1164t + 75622)$$

$$P = 2387t + 155211$$

To estimate the population in 2009, we will use $t = 59$, since 2009 is 59 years after 1950.

$$P = 2387(59) + 155211 = 296,044 \text{ thousand, or } 296,044,000 \text{ people}$$



You Try: Add or subtract the polynomials. Write your answer in standard form and classify the polynomial.

$$(3a^3 - 4a^2 + 3) - (a^4 - a^3) + (1.2a^2 + 2a - 9)$$

QOD: Is it possible for a polynomial to be classified as a linear trinomial? Explain why or why not. Name two classifications for polynomials that are impossible.

Sample Exam Questions

1. What expression must the center cell of the table contain so that the sums of each row, each column, and each diagonal are equivalent?

$5x^2 + x - 9$	$-x^2 - x - 4$	$2x^2 + 3x - 2$
$-x^2 + 3x + 2$		$5x^2 - x - 12$
$2x^2 - x - 8$	$5x^2 + 3x - 6$	$-x^2 + x - 1$

- A. $2x^2 + x - 5$
 B. $4x^2 + 2x - 10$
 C. $6x^2 + 3x - 15$

Ans: A

2. Subtract: $(9y^2 - 5y + 6) - (3y^2 + y - 4)$

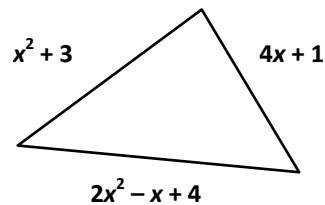
- A. $6y^2 - 4y + 2$
 B. $6y^2 - 4y + 10$
 C. $6y^2 + 6y + 2$
 D. $6y^2 - 6y + 10$

Ans: D



3.

Which expression represents the perimeter of the triangle shown below?



- A. $x^2 - 5x$
 B. $x^2 - 3x + 3$
 C. $3x^2 - 5x + 6$
 D. $3x^2 + 3x + 8$

Ans: D

4. Subtract the following polynomials:

$$(4y^2 + 7y - 5) - (2y^2 - 5y + 3)$$

- A. $2y^2 + 2y - 2$
 B. $2y^2 + 12y - 8$
 C. $6y^2 + 2y - 2$
 D. $6y^2 + 12y - 8$

Ans: B

Unit 9.3 To multiply polynomial expressions.

Recall: You have multiplied a monomial by a polynomial using the distributive property.

Ex 7: Find the product. $4x^2(x^2 - 3x + 1)$

Multiply each term of the trinomial by the monomial.

$$4x^2 \cdot x^2 + 4x^2 \cdot (-3x) + 4x^2 \cdot 1$$

$$= 4x^4 - 12x^3 + 4x^2$$



Method 1: To multiply by a binomial, use the distributive property twice, then combine like terms.

Ex 8: Find the product. $(3x - 5)(2x^2 + 8x + 3)$

$$\begin{aligned} \text{Distributive property: } & 3x \cdot 2x^2 + 3x \cdot 8x + 3x \cdot 3 + (-5) \cdot 2x^2 + (-5) \cdot 8x + (-5) \cdot 3 \\ & = 6x^3 + 24x^2 + 9x - 10x^2 - 40x - 15 \end{aligned}$$

$$\text{Combine like terms: } = \boxed{6x^3 + 14x^2 - 31x - 15}$$

Method 2: Multiply polynomials vertically.

$$\begin{array}{r} 32 \\ \times 21 \\ \hline 32 \\ 64 \\ \hline 672 \end{array}$$

Recall: When multiplying whole numbers, we use a vertical method.

Ex 9: Find the product. $(3x + 2)(2x + 1)$

$$\begin{array}{r} 3x + 2 \\ \times 2x + 1 \\ \hline 3x + 2 \quad \text{multiply by 1} \\ 6x^2 + 4x \quad \text{multiply by 2x} \\ \hline \boxed{6x^2 + 7x + 2} \quad \text{add like terms} \end{array}$$

Method 3: FOIL – This method can be used only when multiplying *two binomials*. This acronym comes from the distributive property in the order of First Outer Inner Last.

Ex 10: Find the product. $(3x + 8)(x - 1)$

$$\begin{array}{ccccccc} \text{First} & (3x)(x) & \text{Outer} & (3x)(-1) & \text{Inner} & (8)(x) & \text{Last} & (8)(-1) \end{array}$$

$$3x^2 - 3x + 8x - 8 = \boxed{3x^2 + 5x - 8}$$

Teacher Note: If you draw arcs over each product (arcs above for First and Last, arcs below for Outer and Inner) it will make the “smiling man”. Try it – the kids will love it!



Ex 11: Multiply the polynomials vertically. $(4y^2 + y - 7)(2y - 8)$

$$\begin{array}{r}
 4y^2 + y - 7 \\
 \times \quad 2y - 8 \\
 \hline
 -32y^2 - 8y + 56 \quad \text{multiply by } -8 \\
 8y^3 + 2y^2 - 14y \quad \text{multiply by } 2y \\
 \hline
 8y^3 - 30y^2 - 22y + 56 \quad \text{add like terms}
 \end{array}$$

Polynomials in Real Life

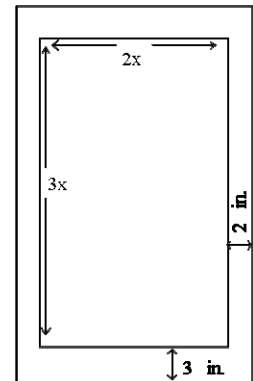
Ex 12: A piece of paper has margins that are 2 inches on the sides and 3 inches at the top and bottom. The height-to-width ratio of the usable part of the paper is 3:2 as shown in the diagram. Write a polynomial expression that represents the total area of the paper, including the margins. Find the area when $x = 10$.

$$\text{Height of Paper} = 3x + 6 \quad \text{Width of Paper} = 2x + 4$$

$$\text{Area of Paper} = \text{Height} \times \text{Width} =$$

$$(3x + 6)(2x + 4) = 6x^2 + 12x + 12x + 24 = \boxed{6x^2 + 24x + 24}$$

$$\text{When } x = 10: 6(10)^2 + 24(10) + 24 = 600 + 240 + 24 = \boxed{864 \text{ sq. in.}}$$



You Try: Write a polynomial expression for the area of a trapezoid with bases $(2x + 1)$ and $(3x + 4)$, and a height of $(4x + 8)$.

QOD: List some advantages and disadvantages of each method of multiplying polynomials: horizontal, vertical, and FOIL.

Sample Exam Questions

1. Multiply the binomials $(2x + 3)(3x - 1)$.

A. $5x^2 + 7x - 3$

B. $5x^2 + 5x - 3$

C. $6x^2 + 7x - 3$

D. $6x^2 + 5x - 3$

Ans: C



2. Multiply the polynomials:

$$(x + 5)(2x^2 - 3x + 4)$$

- A. $2x^3 + 7x^2 - 11x + 20$
- B. $2x^3 + 7x^2 - 19x + 20$
- C. $2x^3 + 13x^2 - 11x + 20$
- D. $2x^3 + 13x^2 - 19x + 20$

Ans: A

3. Under what operations is the system of polynomials NOT closed?

- A. addition
- B. subtraction
- C. multiplication
- D. division

Ans: D

4. Let $x^2 + y^2 = 23$ and $xy = 6$. What is the value of $(x + y)^2$?

- A. 9
- B. 23
- C. 29
- D. 35

Ans: D

5. Which is equivalent to $3x(x^2y + 2xy^2)$?

- A. $3x^2y + 6xy^3$
- B. $3x^3y + 2xy^2$
- C. $3x^3y + 6x^2y^2$
- D. $9x^4y^3$

Ans: C



6. Expand the expression $(3x - 7)^2$.

- A. $9x^2 - 42x - 49$
- B. $9x^2 - 42x + 49$
- C. $9x^2 - 49$
- D. $9x^2 + 49$

Ans: B

7. Given $ax^2 + bx + c = 2(1.2x + 0.3)(x - 0.5) + (0.5x^2 + 2.5x - 1.3)$.

What are the values of a , b , and c ?

Ans: $a = 2.9$, $b = 1.9$, and $c = -2.6$

8. Given $f(x) = 2x - 3$, $g(x) = \frac{x}{3} + 2$, and $h(x) = 3x^2 - x - 4$, find:

- A. $f(x) \cdot g(x)$
- B. $f(x) + h(x)$
- C. $f(x) - g(x)$

Ans:

$$A. f(x) \cdot g(x) = (2x - 3)\left(\frac{x}{3} + 2\right) = \frac{2}{3}x^2 + 3x - 6$$

$$B. f(x) + h(x) = (2x - 3) + (3x^2 - x - 4) = 3x^2 + x - 7$$

$$C. f(x) - g(x) = (2x - 3) - \left(\frac{x}{3} + 2\right) = \frac{5x}{3} - 5$$

9. Expand the expression $(3x - 5)^2$.

- A. $9x^2 + 25$
- B. $9x^2 - 25$
- C. $9x^2 + 30x + 25$
- D. $9x^2 - 30x + 25$

Ans: D



Skill: multiply polynomials – recognizing special products.

Exploration: Find the products using FOIL.

$$(x+2)(x-2) = x^2 - 2x + 2x - 4 = x^2 - 4 = (x)^2 - (2)^2$$

$$(2x+1)(2x-1) = 4x^2 - 2x + 2x - 1 = 4x^2 - 1 = (4x)^2 - (1)^2$$

$$(5t-3)(5t+3) = 25t^2 + 15t - 15t - 9 = 25t^2 - 9 = (5t)^2 - (3)^2$$

$$(a+b)(a-b) = a^2 - ab + ab - b^2 = a^2 - b^2$$

Special Pattern – Sum and Difference Pattern $(a+b)(a-b) = a^2 - b^2$

Ex 13: Find the product without using FOIL. $(3t+4)(3t-4)$

Use the sum and difference pattern. $(3t+4)(3t-4) = (3t)^2 - (4)^2 = 9t^2 - 16$

Exploration: Find the products using FOIL.

$$(x+3)^2 = (x+3)(x+3) = x^2 + 3x + 3x + 9 = x^2 + 6x + 9$$

$$(x-5)^2 = (x-5)(x-5) = x^2 - 5x - 5x + 25 = x^2 - 10x + 25$$

$$(2x+7)^2 = (2x+7)(2x+7) = 4x^2 + 14x + 14x + 49 = 4x^2 + 28x + 49$$

$$(a+b)^2 = (a+b)(a+b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

Special Pattern – Square of a Binomial Pattern

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

Ex 14: Find the product without using FOIL. $(8y-3)^2$

Use the square of a binomial pattern. $(8y-3)^2 = (8y)^2 - 2(8y)(3) + (3)^2 = 64y^2 - 48y + 9$

Ex 15: Find the product without using FOIL. $(4y+5)^2$

Use the square of a binomial pattern. $(4x+5)^2 = (4x)^2 + 2(4x)(5) + (5)^2 = 16x^2 + 40x + 25$



Using Special Products to Multiply Numbers

Ex 16: Multiply $32 \cdot 28$ using mental math.

Rewrite the two factors.

$$32 = 30 + 2 \quad \text{and} \quad 28 = 30 - 2$$

Multiply using the sum and difference pattern. $(30 + 2)(30 - 2) = (30)^2 - (2)^2 = 900 - 4 = 896$

Ex 17: Multiply 25^2 using the square of a binomial pattern.

Rewrite 25 as a sum (or difference).

$$25 = 20 + 5$$

Multiply using the square of a binomial pattern. $(20 + 5)^2 = (20)^2 + 2(20)(5) + (5)^2 = 400 + 200 + 25 = 625$

$$(3b - 7)(3b + 7)$$

You Try: Find the product of the following using special patterns. $\left(5n + \frac{1}{2}\right)^2$

QOD: Is the following statement true or false? If it is false, give a counterexample. $(a + b)^2 = a^2 + b^2$

NOTE:

The following projects can be found on the website. They may be used to give students quick practice adding and subtracting numbers. This skill will be very useful for quadratics (add to get the middle term coefficient; multiply to get the last term coefficient, or constant).

- *Sum and Product Practice – X Factor*



Unit 9.4 To factor polynomials

Review: Greatest Common Factor (GCF)

Ex 18: Find the GCF of 45 and 60.

Recall: The GCF is the largest factor that the 2 numbers have in common.

We can write the prime factorization of each: $45 = 3 \cdot 3 \cdot 5$ $60 = 2 \cdot 2 \cdot 3 \cdot 5$

Now write all of the factors they have in common: $3 \cdot 5 = 15$

The GCF is 15.

Ex 19: Find the GCF of $6x^2y$ and $16x^3$.

Write out each term as a product of factors: $6x^2y = 2 \cdot 3 \cdot x \cdot x \cdot y$ $16x^3 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot x \cdot x \cdot x$

Write all of the factors they have in common: $2 \cdot x \cdot x = \boxed{2x^2}$

Factoring Using the Distributive Property

Ex 20: Factor the polynomial $2x^2 + 8x$.

Step One: Find the GCF of the terms. $2x^2 = 2 \cdot x \cdot x$ $8x = 2 \cdot 2 \cdot 2 \cdot x$ GCF = $2x$

Step Two: Use the distributive property to factor the GCF out of the polynomial.

$$2x(x + 2 \cdot 2)$$

$$= \boxed{2x(x + 4)}$$

Ex 21: Factor the polynomial $14x^3y - 21x^2y^2 + 7x^2y$.

Step One: Find the GCF of the terms.

$$14x^3y = 2 \cdot 7 \cdot x \cdot x \cdot x \cdot y$$

$$-21x^2y^2 = -3 \cdot 7 \cdot x \cdot x \cdot y \cdot y$$

$$7x^2y = 7 \cdot x \cdot x \cdot y$$

GCF = $7x^2y$

Step Two: Use the distributive property to factor the GCF out of the polynomial.

$$7x^2y(2 \cdot x - 3 \cdot y + 1)$$

$$= \boxed{7x^2y(2x - 3y + 1)}$$

***Note: You can check your answers by multiplying using the distributive property.



Factoring by Grouping

Ex 22: Factor the polynomial $x^3 + 2x^2 + 3x + 6$.

Step One: Group the first two terms and last two terms. $(x^3 + 2x^2) + (3x + 6)$

Step Two: Factor the GCF from both sets of terms. $x^2(x + 2) + 3(x + 2)$

Step Three: Factor the common factor using the distributive property. $(x + 2)(x^2 + 3)$

Ex 23: Factor the polynomial $n^3 + 6n^2 - 3n - 18$.

Step One: Group the first two terms and last two terms. $(n^3 + 6n^2) + (-3n - 18)$

Step Two: Factor the GCF from both sets of terms. $n^2(n + 6) - 3(n + 6)$

Step Three: Factor the common factor using the distributive property. $(n + 6)(n^2 - 3)$

You Try: Find the GCF and factor it out of the polynomial. $18x^6 + 6x^4y - 24x^2y^2$

QOD: How can you find the GCF of a variable expression without writing out all of the variables as factors?

Factoring a Quadratic Polynomial

Multiply $(x + p)(x + q)$ using FOIL. $x^2 + qx + px + pq = x^2 + (q + p)x + pq$

To factor $x^2 + bx + c$ into two binomials, $(x + p)(x + q)$, we must find values for p and q such that $b = q + p$, and $c = pq$.

Ex 24: Factor $x^2 + 5x + 4$.

Step One: Find values for p and q such that $pq = 4$ and $p + q = 5$.

We will list all of the factors of 4: $4 = 1 \cdot 4 = 2 \cdot 2 = -1 \cdot -4 = -2 \cdot -2$

The two factors that have a sum of 5 are 1 and 4, so $p = 1$ and $q = 4$.

Step Two: Write each factor $(x + p)(x + q)$ $(x + 1)(x + 4)$

***Note: Because multiplication is commutative, we could also write our answer as $(x + 4)(x + 1)$



Ex 25: Factor $x^2 - 10x + 24$.

Step One: Find values for p and q such that $pq = 24$ and $p + q = -10$.

$$\begin{aligned} \text{We will list all of the factors of 24:} \quad & 24 = 1 \cdot 24 = 2 \cdot 12 = 3 \cdot 8 = 4 \cdot 6 \\ & = -1 \cdot -24 = -2 \cdot -12 = -3 \cdot -8 = -4 \cdot -6 \end{aligned}$$

The two factors that have a sum of -10 are -4 and -6 , so $p = -4$ and $q = -6$.

Step Two: Write each factor $(x + p)(x + q)$ $(x - 4)(x - 6)$

Check your answer using FOIL!

Ex 26: Factor $x^2 - 8x - 9$.

Step One: Find values for p and q such that $pq = -9$ and $p + q = -8$.

$$\text{We will list all of the factors of } -9: \quad -9 = -1 \cdot 9 = -9 \cdot 1 = 3 \cdot -3$$

The two factors that have a sum of -8 are -9 and 1 , so $p = -9$ and $q = 1$.

Step Two: Write each factor $(x + p)(x + q)$ $(x - 9)(x + 1)$

Check your answer using FOIL!

Factoring Quadratic Trinomials in the Form $ax^2 + bx + c$, $a \neq 1$

Ex 27: Factor the trinomial $2x^2 + 11x + 5$.

Step One: Multiply ac . Find values for p and q such that $pq = ac$, and $p + q = b$

$$ac = 10 = 1 \cdot 10 = 2 \cdot 5 = -1 \cdot -10 = -2 \cdot -5$$

The two factors that have a sum of 11 are 1 and 10 .

Step Two: Split the middle term (bx) into two terms $px + qx$.

$$2x^2 + 1x + 10x + 5$$

Step Three: Factor by grouping.

$$\begin{aligned} & (2x^2 + 1x) + (10x + 5) \\ & = x(2x + 1) + 5(2x + 1) \\ & = \boxed{(2x + 1)(x + 5)} \end{aligned}$$

Check your answer using FOIL!

Ex 28: Factor $6n^2 - 11n - 2$.

Step One: Multiply ac . Find values for p and q such that $pq = ac$, and $p + q = b$

$$ac = -12 = 1 \cdot -12 = -1 \cdot 12 = -2 \cdot 6 = 2 \cdot -6 = -3 \cdot 4 = 3 \cdot -4$$

The two factors that have a sum of -11 are 1 and -12 .



Step Two: Split the middle term (bx) into two terms $px + qx$.

$$6n^2 + 1n - 12n - 2$$

Step Three: Factor by grouping.

$$\begin{aligned} & (6n^2 + 1n) + (-12n - 2) \\ &= n(6n + 1) - 2(6n + 1) \\ &= \boxed{(6n + 1)(n - 2)} \end{aligned}$$

Check your answer using FOIL!

Ex 29: Factor $4t^2 - 9t + 5$.

Step One: Multiply ac . Find values for p and q such that $pq = ac$, and $p + q = b$

$$ac = 20 = 1 \cdot 20 = -1 \cdot -20 = 2 \cdot 10 = -2 \cdot -10 = 4 \cdot 5 = -4 \cdot -5$$

The two factors that have a sum of -9 are -4 and -5 .

Step Two: Split the middle term (bx) into two terms $px + qx$.

$$4t^2 - 4t - 5t + 5$$

Step Three: Factor by grouping.

$$\begin{aligned} & (4t^2 - 4t) + (-5t + 5) \\ &= 4t(t - 1) - 5(t - 1) \\ &= \boxed{(t - 1)(4t - 5)} \end{aligned}$$

Check your answer using FOIL!

Ex 30: Factor the trinomial $6x^2 + 19x + 10$.

Step One: Multiply ac . Find values for p and q such that $pq = ac$, and $p + q = b$

$$ac = 60 = 1 \cdot 60 = 2 \cdot 30 = 3 \cdot 20 = 4 \cdot 15$$

The two factors that have a sum of 19 are 4 and 15 .

Note: Because 60 has so many factors, we did not list all of them. b is positive, so we only need to list the positive factors and stop when we find the two that add up to 19 .

Step Two: Split the middle term (bx) into two terms $px + qx$.

$$6x^2 + 4x + 15x + 10$$

Step Three: Factor by grouping.

$$\begin{aligned} & (6x^2 + 4x) + (15x + 10) \\ &= 2x(3x + 2) + 5(3x + 2) \\ &= \boxed{(3x + 2)(2x + 5)} \end{aligned}$$

You Try: Factor the following trinomials.

1. $m^2 - 6m + 5$

2. $6y^2 - 2y - 8$

3. $21x^2 + 8x - 4$

QOD: Are all quadratic trinomials factorable? If not, write a trinomial that cannot be factored.



Skill: factor polynomials – recognizing special products.

Recall: Sum and Difference Pattern $(a + b)(a - b) = a^2 - b^2$

Square of a Binomial Pattern:

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - 2ab + b^2$$

Previously we learned to multiply using the special products patterns. Now we will factor from these special products.

Special Factoring Patterns

Difference of Two Perfect Squares: $a^2 - b^2 = (a + b)(a - b)$

Perfect Square Trinomial:

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Ex 31: Factor $x^2 - 9$.

Recognize this as the difference of two perfect squares. $x^2 - 9 = (x)^2 - (3)^2$

Factor using the special factoring pattern. $= (x + 3)(x - 3)$

Check your answer using FOIL!

Ex 32: Factor $16 - 25y^2$.

Recognize this as the difference of two perfect squares. $16 - 25y^2 = (4)^2 - (5y)^2$

Factor using the special factoring pattern. $= (4 + 5y)(4 - 5y)$

Check your answer using FOIL!

Ex 33: Factor $y^2 + 6y + 9$.

Recognize this as a perfect square trinomial. $y^2 + 6y + 9 = (y)^2 + 2(y)(3) + (3)^2$

Factor using the special factoring pattern. $= (y + 3)^2$



Ex 34: Factor $9x^2 - 12x + 4$.

Recognize this as a perfect square trinomial.

$$9x^2 - 12x + 4 = (3x)^2 - 2(3x)(2) + (2)^2$$

Factor using the special factoring pattern.

$$= \boxed{(3x - 2)^2}$$

Ex 35: Factor $-\frac{9}{16} + f^2$.

Recognize this as the difference of two perfect squares.

$$-\frac{9}{16} + f^2 = f^2 - \frac{9}{16} = (f)^2 - \left(\frac{3}{4}\right)^2$$

Factor using the special factoring pattern.

$$= \boxed{\left(f + \frac{3}{4}\right)\left(f - \frac{3}{4}\right)}$$

Ex 36: Factor $4y^2 + 20xy + 25x^2$.

Recognize this as a perfect square trinomial.

$$4y^2 + 20xy + 25x^2 = (2y)^2 + 2(2y)(5x) + (5x)^2$$

Factor using the special factoring pattern.

$$= \boxed{(2y + 5x)^2}$$

You Try: Factor the following.

1. $49n^2 - 225m^2$

2. $49 - 56x + 16x^2$

QOD: Describe how to check your answers when factoring.

Factoring Completely: putting it all together

Step One: Factor the GCF using the distributive property. ****IMPORTANT!****

Step Two: Factor the polynomial that remains using the *ac* method or special products.

Ex 37: Factor completely. $4x^2 - 36$

Step One: Factor the GCF, which is 4. $= 4(x^2 - 9)$

Step Two: The remaining polynomial is a difference of two perfect squares. Use the special factoring pattern.

$$= \boxed{4(x + 3)(x - 3)}$$



Ex 38: Factor completely. $2n^3 - 10n^2 - 12n$

Step One: Factor the GCF, which is $2n$. $= 2n(n^2 - 5n - 6)$

Step Two: Factor the remaining trinomial. $= \boxed{2n(n-6)(n+1)}$

Ex 39: Factor $-30y^4 + 58y^3 - 24y^2$.

Step One: Factor the GCF, which is $-2y^2$. $= -2y^2(15y^2 - 29y + 12)$

Step Two: Factor the remaining trinomial. $= \boxed{-2y^2(5y-3)(3y-4)}$

Note: Because of the large numbers for a and c in the trinomial, a trial and error approach may work best for factoring.

Ex 40: Factor $3x^3 - 15x^2 - 12x + 60$ completely.

Step One: Factor the GCF, which is 3. $= 3(x^3 - 5x^2 - 4x + 20)$

$$= 3[(x^3 - 5x^2) + (-4x + 20)]$$

Step Two: Factor the remaining polynomial by grouping. $= 3[x^2(x-5) - 4(x-5)]$

$$= 3[(x-5)\underline{(x^2-4)}]$$

Step Three: The underlined factor is a difference of two perfect squares. Use the special factoring pattern to factor completely.

$$= \boxed{3(x-5)(x+2)(x-2)}$$

Ex 41: Factor $2x^5 - 32x^4 + 128x^3$.

Step One: Factor the GCF, which is $2x^3$. $= 2x^3(x^2 - 16x + 64)$

Step Two: The remaining polynomial is a perfect square trinomial. Use the special factoring pattern.

$$= \boxed{2x^3(x-8)^2}$$



Quadratic Trinomials: Factorable or Not Factorable?

Discriminant: In the quadratic trinomial $ax^2 + bx + c$, the *discriminant* is equal to $b^2 - 4ac$

If the trinomial can be factored, the *discriminant* must be a **perfect square**.

Ex 42: Is the trinomial factorable? If yes, factor it. $5x^2 + 11x + 2$

Find the discriminant. $a = 5, b = 11, c = 2$ $b^2 - 4ac = (11)^2 - 4(5)(2) = 121 - 40 = 81$
81 is a perfect square, so the trinomial **is** factorable.

Use the ac method. $ac = 10 = 1 \cdot 10$

$$\begin{aligned} & (5x^2 + 1x) + (10x + 2) \\ &= x(5x + 1) + 2(5x + 1) \\ &= \boxed{(5x + 1)(x + 2)} \end{aligned}$$

Ex 43: Is the trinomial factorable? If yes, factor it. $3r^2 - 5r - 1$

Find the discriminant. $a = 3, b = -5, c = -1$ $b^2 - 4ac = (-5)^2 - 4(3)(-1) = 25 + 12 = 37$
37 is not a perfect square, so the **trinomial is not factorable**.

Application Problem:

Ex 44: The volume of a box can be modeled by the polynomial $x^3 + 3x^2 - 4x - 12$. Write three linear binomials that could represent the length, width, and height of the box.

The volume of the box was found by multiplying the length, width, and height. To find the binomials, we will factor the expression that represents the volume.

Factor by grouping.

$$\begin{aligned} & (x^3 + 3x^2) + (-4x - 12) \\ &= x^2(x + 3) - 4(x + 3) \\ &= (x + 3)\underline{(x^2 - 4)} \end{aligned}$$

The underlined factor is a difference of two perfect squares. Use the special pattern to factor.

$$= (x + 3)(x + 2)(x - 2)$$

The length, width, and height of the box are $x + 3, x + 2,$ and $x - 2$

You Try: List all of the factors of $24b^3 + 6b^2 - 9b$.

QOD: Why is it important to factor the GCF as the first step in factoring a polynomial?

**Sample Exam Questions**

1. The area of a rectangular tabletop is represented by $x^2 + 5x - 24$. Which pair of expressions could represent the dimensions of the tabletop?

- A. $(x + 6), (x - 4)$
- B. $(x + 8), (x - 3)$
- C. $(x + 12), (x - 2)$
- D. $(x + 24), (x - 1)$

Ans: B

2. Which of the following is a factor of $5x^2 + 23x + 12$?

- A. $(5x + 2)$
- B. $(5x + 3)$
- C. $(5x + 4)$
- D. $(5x + 6)$

Ans: B

3. Which expression is equivalent to $xc + xb + yc + yb$?

- A. $(x + b)(y + c)$
- B. $(x + c)(y + b)$
- C. $(x + y)(b + c)$

Ans: C

4. Which is equivalent to $(4x^2 - 9y^4)$

- A. $(2x - 3y^2)^2$
- B. $(2x - 3y^2)(2x + 3y^2)$
- C. $(2x + 3y^2)(2x - 3y)(2x + 3y)$

Ans: B



For questions 5-7, use the expression $x^4 - y^4$.

5. $(x^2 - y^2)(x^2 + y^2)$ is equivalent to the given expression.

- A. True
- B. False

Ans: A

6. $(x - y)(x + y)(x^2 + y^2)$ is equivalent to the given expression.

- A. True
- B. False

Ans: A

7. $(x - y)(x + y)^3$ is equivalent to the given expression.

- A. True
- B. False

Ans: B

8. Which of these is NOT a factor of $12x^2 + 6x - 90$?

- A. 6
- B. $2x$
- C. $x + 3$
- D. $2x - 5$

Ans: B

9. If $(x - 7)$ is a factor of $2x^2 - 11x + k$, what is the value of k ?

- A. -21
- B. -7
- C. 7
- D. 28

Ans: A



10. Factor $25x^2 + 4$.

- A. $(5x + 2)(5x - 2)$
- B. $(5x + 2)^2$
- C. The expression is not factorable with real coefficients.

Ans: C

11. Factor $9x^2 - 16$.

- A. $(3x + 4)(3x - 4)$
- B. $(3x - 4)^2$
- C. The expression is not factorable with real coefficients.

Ans: A

12. Which expression is equivalent to $35x^2 + 26x - 16$?

- A. $(7x - 2)(5x + 8)$
- B. $(7x + 2)(5x - 8)$
- C. $(7x - 8)(5x + 2)$
- D. $(7x + 8)(5x - 2)$

Ans: D

13. What value of c makes the expression $y^2 - 9y + c$ a perfect trinomial square?

- A. -9
- B. $-\frac{9}{2}$
- C. 81
- D. $\frac{81}{4}$

Ans: D