

**OBJECTIVE #:** G.SRT.D.9/10 (**HONORS**)**OBJECTIVE**

Derive the formula  $A = 1/2 ab \sin(C)$  for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.

Prove the Laws of Sines and Cosines and use them to solve problems.

**BIG IDEA** (Why is this included in the curriculum?)

- To prove the Law of Sines and use to solve problems.
- To prove the Law of Cosines and use to solve problems.
- To derive and then use the formula for the area of any triangle,  $A = 1/2 ab \sin C$ .

**PREVIOUS KNOWLEDGE** (What skills do they need to have to succeed?)

- The student must know how to find trig ratios in right triangles.
- The student must know how to find the sine/cosine/tangent of given angle measures.
- The student must know how to solve equations for given variables.
- The student must know the formula for the area of a triangle and how to apply it.

**VOCABULARY USED IN THIS OBJECTIVE** (What terms will be essential to understand?)**PREVIOUS VOCABULARY** (Terms used but defined earlier)

- Adjacent: Sharing a vertex and common side.
- Altitude: Height of a triangle. Line segment connecting a vertex to the opposite side of a triangle forming a right angle.
- Cosine: The trigonometric function that is equal to the ratio of the non-hypotenuse side adjacent a given angle (in a right triangle) to the hypotenuse.
- Hypotenuse: Longest side in a right triangle. The side directly across from the right angle.
- Right Triangle: Any triangle with exactly one right angle.
- Sine: The trigonometric function that is equal to the ratio of the side opposite a given angle (in a right triangle) to the hypotenuse.
- SOH CAH TOA: Synonym for remembering the ratios of the three trig functions (Sine Opposite Hypotenuse; Cosine Adjacent Hypotenuse; Tangent Opposite Adjacent)
- Tangent: The trigonometric function that is equal to the ratio of the side opposite a given angle (in a right triangle) to the non-hypotenuse side adjacent to the given angle.

**NEW VOCABULARY** (New terms and definitions introduced in this objective)

- Law of Sines: the relationship between the sides and angles of non-right (oblique) triangles . Simply, it states that the ratio of the length of a side of a triangle to the **sine** of the angle opposite that side is the same for all sides and angles in a given triangle.
- Law of Cosines: for calculating one side of a triangle when the angle opposite and the other two sides are known. Can be used in conjunction with the **law** of sines to find all sides and angles.
- Trig Formula for the Area of a Triangle:

**SKILLS** (What will they be able to do after this objective?)

- The student will be able to prove and apply the Law of Sines.
- The student will be able to prove and apply the Law of Cosines.
- The student will be able to prove and apply the trig formula for the area of a triangle.

**SHORT NOTES** (A short summary of notes so that a teacher can get the basics of what is expected.)

- Give students a triangle that is not a right triangle with either two sides and the included angle, or two angles and a side (any side) and ask them how they can find the rest of the angles and sides. Give students the Law of Sines and show students to find the missing sides and angles when given triangles in either ASA, or AAS format, we use the Law of Sines to find the missing values. Work students through the proof of the Law of Sines and give them a few examples of applying.
- Give students a triangle with all three sides given to them and ask them to find all three angles. Show the students that they need to use the Law of Cosines to find the angles in a problem like this and how to use it. Work students through the proof of the Law of Cosines and remind them they need to use it when they are given only three sides of any triangle.
- Give students a triangle without the altitude given to them and discuss how to find the area of the triangle and what would be needed. Give students the trig formula for the area of a triangle and work them through the proof.

**MISCONCEPTIONS** (What are the typical errors or difficult areas? Also suggest ways to teach them.)

- Students tend to forget to use the Law of Sines and the Law of Cosines. Remind them that the Law of Sines needs to be used when given any one of the given formats of a triangle: ASA, and AAS. They use the Law of Cosines when given the formats SSS and SAS.
- When finding the area of a triangle students tend to misplace what values go where. Remind students they want to use two sides with the included angle measure to substitute into the formula.
- When proving remind students that the base depends on what altitude you are using in the triangle.

**FUTURE CONNECTIONS** (What will they use these skills for later?)

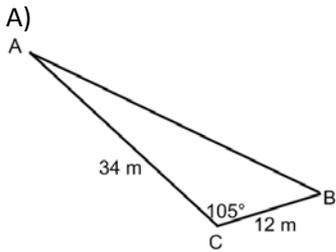
- Students will continue to use basic trigonometry ratios in Geometric proofs and throughout the trigonometry portion of Algebra 2. Students will find missing parts of triangles in parts of circles.
- Students will extend their knowledge on area and volume later in Geometry.

**ADDITIONAL EXTENSIONS OR EXPLANATIONS** (What needs greater explanation?)

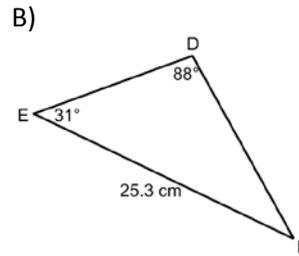
- Give students different examples of when to use the Law of Sines to find the sides and angles of triangles when given all different types of triangles (ASA, and AAS)
- Give students different examples of finding the measures of angles when given different types of triangles (acute, right, obtuse) to use the Law of Cosines (given SAS and SSS).
- When giving students different problems where they may need to find sides and/or angles first to get the needed information to substitute into the area formula.

**ASSESSMENTS** (Questions that get to the heart of the objective – multiple choice, short answer, multi-step)

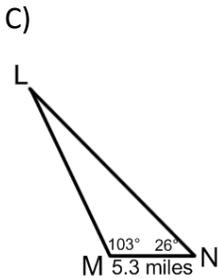
1. Find the missing sides, angles, and area of each of the following triangles:



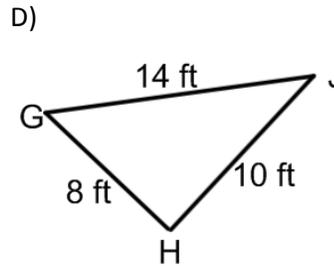
- a.  $\overline{AB}$   
 $AB^2 = 34^2 + 12^2 - 2(34)(12)\cos 105^\circ$   
 $AB \approx 38.874$
- b.  $m\angle B$   
 $34^2 = 12^2 + 38.874^2 - 2(38.874)(12)\cos B$   
 $m\angle B \approx 57.653^\circ$
- c.  $m\angle A$   
 $12^2 = 34^2 + 38.874^2 - 2(38.874)(34)\cos B$   
 $m\angle A \approx 17.348^\circ$
- d. Area of  $\triangle ABC$ :  
 $197.049 m^2$



- a.  $m\angle F$   
 $m\angle F = 61^\circ$
- b.  $\overline{DF}$   
 $DF = \frac{25.3\sin(31)}{\sin(88)}$   
 $13.038 cm$
- c.  $\overline{DE}$   
 $DE = \frac{25.3\sin(61)}{\sin(88)}$   
 $22.141 cm$
- d. Area of  $\triangle DEF$ :  
 $144.254 cm^2$



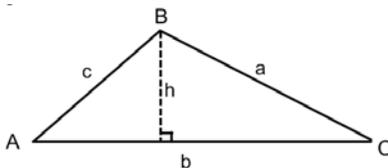
- a.  $m\angle L$   
 $51^\circ$
- b.  $\overline{LN}$   
 $LN = \frac{5.3\sin(103)}{\sin(51)}$   
 $6.645 miles$
- c.  $\overline{LM}$   
 $LM = \frac{5.3\sin(26)}{\sin(51)}$   
 $2.990 miles$
- d. Area of  $\triangle LMN$ :  
 $7.719 miles$



- a.  $m\angle G$   
 $10^2 = 14^2 + 8^2 - 2(14)(8)\cos G$   
 $m\angle G \approx 44.415^\circ$
- b.  $m\angle H$   
 $14^2 = 10^2 + 8^2 - 2(10)(8)\cos H$   
 $m\angle H \approx 101.537^\circ$
- c.  $m\angle J$   
 $8^2 = 14^2 + 10^2 - 2(14)(10)\cos J$   
 $m\angle J \approx 34.048^\circ$
- d. Area of  $\triangle GHJ$ :  
 $39.192 ft^2$

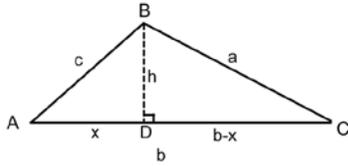
## HONORS

### 2. Prove the Law of Sines.



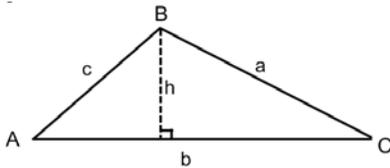
Given  $\triangle ABC$ , draw an altitude from  $B$  to side  $AC$  (side  $h$ ). Using the sine function,  $\sin A = \frac{h}{c}$  and  $\sin C = \frac{h}{a}$ . Solving each equation for  $h$  we get:  $h = c \sin A$  and  $h = a \sin C$ . Using substitution and dividing each side by  $ac$  we get  $\frac{\sin A}{a} = \frac{\sin C}{c}$ . Similarly using the altitude from  $A$  to side  $BC$  proves  $\frac{\sin B}{b} = \frac{\sin C}{c}$ . By substitution we get  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ .

3. Prove the Law of Cosines.



Given  $\triangle ABC$ , draw an altitude from  $B$  to side  $AC$  (side  $h$ ). If  $x$  is the length of  $AD$  and  $b$  is the length of  $AC$  then  $b - x$  is the length of  $DC$ . Using the cosine function in  $\triangle ABD$ ,  $\cos A = \frac{x}{c}$ . Solving for  $x$  we get  $x = c \cos A$ . Using the Pythagorean Theorem we get  $x^2 + h^2 = c^2$ . Using the Pythagorean Theorem for  $\triangle DBC$  we get  $(b - x)^2 + h^2 = a^2$ . Simplifying we get  $b^2 - 2bx + x^2 + h^2 = a^2$ . Substituting  $c^2$  in for  $x^2 + h^2$  we get  $b^2 - 2bx + c^2 = a^2$ , or  $b^2 + c^2 - 2bx = a^2$ . Since  $x = c \cos A$ , then using substitution we get:  
 $a^2 = b^2 + c^2 - 2bc \cos A$ . By using the other two altitudes and similar reasoning we can show that  $b^2 = a^2 + c^2 - 2ac \cos B$  and  $c^2 = b^2 + a^2 - 2ba \cos C$ .

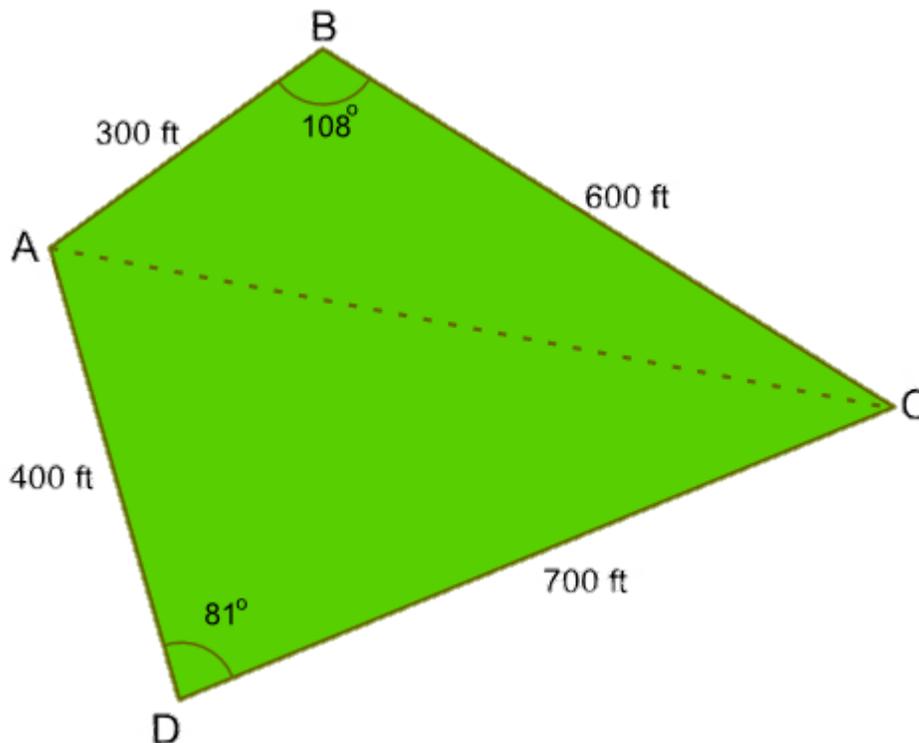
4. Prove the trig formula for the area of a triangle.



The formula for the area of any triangle is given by  $A = \frac{1}{2}bh$ . Given  $\triangle ABC$ , draw an altitude from  $B$  to side  $AC$  (side  $h$ ). Using the sin function, we get  $\sin A = \frac{h}{c}$ . Solving for  $h$  we get  $h = c \sin A$ . By substituting the value of  $h$  in for  $h$  in the area formula we get:  
 $A = \frac{1}{2}b(c \sin A) = \frac{1}{2}bc \sin A$ .

5. Farmer Jones owns a triangular piece of land. The length of the fence  $AB$  is 150 m. The length of the fence  $BC$  is 231 m. The angle between fence  $AB$  and fence  $BC$  is 123. How much land does Farmer Jones own?  
**Farmer Jones has 14,530 m<sup>2</sup> of land.**

6. Find the area of the field to the nearest square foot.



**Approximately 223,871 ft<sup>2</sup>**