

GEOMETRY

Cofunctions of Trigonometric Ratios



OBJECTIVE #: G.SRT.C.7

OBJECTIVE

Explain and use the relationship between the sine and cosine of complementary angles.

BIG IDEA (Why is this included in the curriculum?)

- To define sine and cosine as cofunctions.
- To see the relationship between the sine and cosine of complementary angles in right triangles.

PREVIOUS KNOWLEDGE (What skills do they need to have to succeed?)

- The student must know how to find trig ratios in right triangles.
- The student must know the difference in the parts of right triangles (legs and hypotenuse; opposite, adjacent, and hypotenuse).
- The student must know that non-right angles in right triangles are acute and complementary.
- The student must know how to find the sine/cosine/tangent of given angle measures.

VOCABULARY USED IN THIS OBJECTIVE (What terms will be essential to understand?)

PREVIOUS VOCABULARY (Terms used but defined earlier)

- Adjacent: Sharing a vertex and common side.
- Complementary Angles: Two angles whose sum is 90° .
- Cosine: The trigonometric function that is equal to the ratio of the non-hypotenuse side (leg) adjacent a given angle (in a right triangle) to the hypotenuse.
- Hypotenuse: Longest side in a right triangle. The side directly across from the right angle.
- Right Triangle: Any triangle with exactly one right angle.
- Sine: The trigonometric function that is equal to the ratio of the side (leg) opposite a given angle (in a right triangle) to the hypotenuse.
- SOH CAH TOA: Mnemonic for remembering the ratios of the three trig functions (Sine Opposite Hypotenuse; Cosine Adjacent Hypotenuse; Tangent Opposite Adjacent)
- Tangent: The trigonometric function that is equal to the ratio of the side (leg) opposite a given angle (in a right triangle) to the non-hypotenuse side (leg) adjacent to the given angle.
- Triangle Sum Theorem: The three angles in any triangle have a sum of 180° .

NEW VOCABULARY (New terms and definitions introduced in this objective)

- Cofunctions: The congruent trigonometric function of the complement of an angle. The value of a trig function of an angle equals the value of the cofunctions of the complement of the angle.

SKILLS (What will they be able to do after this objective?)

- The student will be able to identify congruent trig values without calculating ratios or typing trig values into the calculator.
- The student will be able to name, define, and identify cofunctions.

SHORT NOTES (A short summary of notes so that a teacher can get the basics of what is expected.)

- Given different right triangles, have students find the sine and cosine of the different acute angles. Discuss the similarities and difference between them. Students should start to see that the sine of on acute angle is always equal to the cosine of the other acute angle in any right triangle. Start to discuss

with them what the measure of the two angles will add up to and identify them as always being complementary.

- Discuss the special right triangles and how the $\sin 45^\circ = \cos 45^\circ$, $\sin 30^\circ = \cos 60^\circ$, $\sin 60^\circ = \cos 30^\circ$. Explain how these are exact examples of cofunctions. Define the trig cofunctions to students (two acute angles, A and B, in any right triangle are always complementary, and $\sin(A) = \cos(B)$).
- Have students identify different pairs of complementary angles and use their calculators to practice identifying cofunctions and see how they are equal.

MISCONCEPTIONS (What are the typical errors or difficult areas? Also suggest ways to teach them.)

- Remind students that they need to find the complement of the angle, and that the only time (under 90°) that the sine and cosine are equal is at 45° . For example, $\sin 33^\circ \neq \cos 33^\circ$.
- Students tend to forget to change the trig function when finding the complement (for instance, students tend to say $\sin 33^\circ = \sin 57^\circ$).

FUTURE CONNECTIONS (What will they use these skills for later?)

- Students will continue to use basic trigonometry ratios in Geometric proofs and throughout the trigonometry portion of Algebra 2. When finding the sides of right triangles student will need to choose between their different trig ratios and need to understand that it doesn't matter which trig ratio they use as long as they use the appropriate one in the appropriate place.

ADDITIONAL EXTENSIONS OR EXPLANATIONS (What needs greater explanation?)

- Using a calculator let students explore the trig values and type in different cofunctions bettering their understanding of what cofunctions are and how they are congruent.
- Ask students if this also works for supplementary angles. Why? Is it possible to have a pair of supplementary angles in any triangle?

ASSESSMENTS (Questions that get to the heart of the objective – multiple choice, short answer, multi-step)

1. What does it mean to be cofunctions?

The complementary angle to a given angle. If A and B are complementary, then $\sin(A) = \cos(B)$ and the $\cos(A) = \sin(B)$

2. What is unique about the Sin 58° and Cos 32° ?

They are the equal because sin and cosine are cofunctions.

3. Find the given trig values. Round to the nearest thousandth:

$$\begin{array}{l} \sin 34^\circ \\ 0.559 \end{array}$$

$$\begin{array}{l} \cos 40^\circ \\ 0.766 \end{array}$$

$$\begin{array}{l} \sin 87^\circ \\ 0.997 \end{array}$$

$$\begin{array}{l} \cos 56^\circ \\ 0.559 \end{array}$$

$$\begin{array}{l} \cos 3^\circ \\ 0.997 \end{array}$$

$$\begin{array}{l} \sin 51^\circ \\ 0.777 \end{array}$$

$$\begin{array}{l} \cos 49^\circ \\ 0.656 \end{array}$$

$$\begin{array}{l} \sin 40^\circ \\ 0.643 \end{array}$$

4. Which of the trig values above are cofunctions?

$\sin 87^\circ$ and $\cos 3^\circ$ and also $\sin 34^\circ$ and $\cos 56^\circ$

5. $\sin 14^\circ = \cos$ _____ $^\circ$ 76°

6. \sin _____ $^\circ = \cos 65^\circ$ 15°

7. \sin _____ $^\circ = \cos 38.3^\circ$ 51.7°

8. $\sin 9.4^\circ = \cos$ _____ $^\circ$ 80.6°

9. Are the following cofunctions?

no $\cos 45^\circ, \sin 55^\circ$	yes $\sin 70^\circ, \cos 20^\circ$	no $\cos 33^\circ, \cos 57^\circ$
yes $\cos 61^\circ, \sin 29^\circ$	yes $\cos 70^\circ, \sin 20^\circ$	no $\sin 8^\circ, \sin 82^\circ$

10. If the $\sin(X) = 0.2$, what is the cosine of $90^\circ - X$? **0.2**

11. Write $\sin(30^\circ)$ in terms of its co-function. **$\cos(60^\circ)$**