GEOMETRY   Applying Trigonometric Functions

OBJECTIVE #: G.SRT.C.8

OBJECTIVE

Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
*(Modeling Standard)

BIG IDEA (Why is this included in the curriculum?)
- To use the trigonometric functions to find unknown sides of a right triangle.
- To use the trigonometric inverse functions to find unknown angles of a right triangle.
- To solve applications of right triangles using trigonometric ratios and the Pythagorean Theorem.

PREVIOUS KNOWLEDGE (What skills do they need to have to succeed?)
- The student must know how to find trig ratios in right triangles. (SOH CAH TOA)
- The student must be able to identify the parts of right triangles (legs and hypotenuse; opposite, adjacent, and hypotenuse). They must also be able to locate these parts of the right triangle with respect to a given angle of the triangle.
- The student must know how to find the sine/cosine/tangent of given angle measures.

VOCABULARY USED IN THIS OBJECTIVE (What terms will be essential to understand?)

PREVIOUS VOCABULARY (Terms used but defined earlier)
- Adjacent: Sharing a vertex and common side.
- Complementary Angles: Two angles whose sum is 90°.
- Cosine: The trigonometric function that is equal to the ratio of the non-hypotenuse side (leg) adjacent a given angle (in a right triangle) to the hypotenuse.
- Hypotenuse: Longest side in a right triangle. The side directly across from the right angle.
- Right Triangle: Any triangle with exactly one right angle.
- Sine: The trigonometric function that is equal to the ratio of the side (leg) opposite a given angle (in a right triangle) to the hypotenuse.
- SOH CAH TOA: Mnemonic for remembering the ratios of the three trig functions (Sine Opposite Hypotenuse: Cosine Adjacent Hypotenuse; Tangent Opposite Adjacent)
- Tangent: The trigonometric function that is equal to the ratio of the side (leg) opposite a given angle (in a right triangle) to the non-hypotenuse side (leg) adjacent to the given angle.
- Triangle Sum Theorem: The three angles in any triangle have a sum of 180°.

NEW VOCABULARY (New terms and definitions introduced in this objective)
- Angle of Depression: The "downwards" angle from the horizontal line of sight from the observer to some point of interest located below the observer.
- Angle of Elevation: the angle between the horizontal and the line from the object to the observer's eye (the line of sight) when the point of interest is located above the observer.
- Angle of Depression = Angle of Elevation
- Inverse Trigonometric Functions: Also known as arcsine, arccosine, arctangent. Uses the ratio of the appropriate sides (sine is opposite over hypotenuse, etc.) to find the missing angle measures (using $sin^{-1}, cos^{-1}, tan^{-1}$)

SKILLS (What will they be able to do after this objective?)
- The student will be able to find missing sides in right triangles using Pythagorean Theorem and trig ratios.
- The student will be able to find missing angles in right triangles using trig ratios.

- The student will be able to apply trig ratios in real-world scenarios to find missing angles and sides in right triangles.

SHORT NOTES (A short summary of notes so that a teacher can get the basics of what is expected.)
- Give students different types of right triangles where they are given different measures of sides and angles. Help students identify which trig ratio to use and write an equation with the sides and angles that are given and the side that is needed and then solve for the needed side(s). Students should be able to leave their answers in simplest expression notation and find the answer to a certain decimal place.
- Give students examples of right triangles where they only know two sides and need to find the third side and the two acute angles. Remind them that they can use the Pythagorean Theorem to find the third side. Define the inverse trig functions and show students how they use their trig ratios (SOH CAH TOA) but with the inverse functions to find the given angle measures.
- Using real-world scenarios have students draw figures identifying the right triangles in the problems and the parts of the triangles that they are given in the problem. Discuss which parts of the triangle they have and which parts they are trying to find. Have students apply their trig ratios and inverse trig ratios to find the needed side(s) and angle(s).

MISCONCEPTIONS (What are the typical errors or difficult areas? Also suggest ways to teach them.)
- If students are having a hard time getting the correct answer have them verify that their calculators are in degree mode.
- Remind students that they need to use the inverse trig functions with the ratios of the correct trig function (SOH CAH TOA) when finding the angles measures, not the regular trig functions.
- Students tend to struggle with what trig function to use and when. Remind them to identify what they have and what they are trying to find (either opposite, adjacent, or hypotenuse) and use the correct trig function.

FUTURE CONNECTIONS (What will they use these skills for later?)
- Students will continue to use basic trigonometry ratios in Geometric proofs and throughout the trigonometry portion of Algebra 2. Students will find missing parts of triangles in parts of circles and when proving the Law of Sines, Law of Cosines, and finding the Trig Formula for the Area of a Triangle.

ADDITIONAL EXTENSIONS OR EXPLANATIONS (What needs greater explanation?)
- Give students different examples of finding the sides of triangles when given an angle and one other side (switching between giving students the opposite, adjacent, and hypotenuse sides).
- Give students different examples of finding the measures of angles when given different pairs of sides in a triangle (opposite and adjacent; opposite and hypotenuse; adjacent and tangent).
- When giving students real-world scenarios use terminology including angle of elevation, angle of depression, and switch between having students find angles and sides.

ASSESSMENTS (Questions that get to the heart of the objective – multiple choice, short answer, multi-step)

1. Most airplanes take off at an angle of elevation between 10° and 15°. What is the possible range of miles, in feet, across the ground would the airplane travel before reaching a height of 10,000 feet?
   37,320.5 feet – 56,712.8 feet

2. A photographer is standing on a cliff looking out at an angle of depression of 25° to another (shorter) cliff that is 550 yards away. How tall is the cliff that the photographer is standing on?
   1,179.5 yards

3. You are building a skateboard ramp. The ramp itself will be made out of a piece of wood that is 4.5 meters long. The angle that the ramp will form with the ground is going to be 22°. What are the lengths of the ramp's base and height?
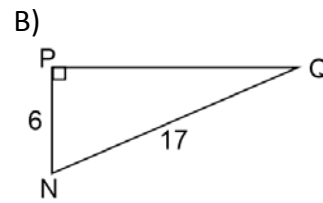**Height: 1.686 meters, Base: 4.172 meters**

4. Solve the right triangles by finding the missing angle measures and the missing side measures in each of the following triangles:

A)
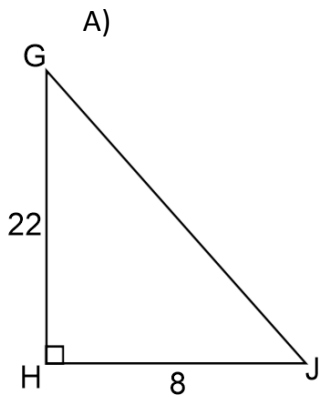
a. $m\angle J$
   70.017°

b. $m\angle G$
   19.983°

c. $\overline{GJ}$
   $\sqrt{548} \approx 23.409$

B)

a. $m\angle N$
   69.333°

b. $m\angle Q$
   20.667°

c. $\overline{PQ}$
   $\sqrt{253} \approx 15.906$

5. Find each angle measure:

a. $sin^{-1}(0.56)$
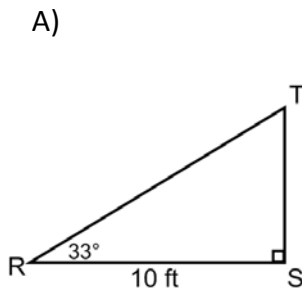   **34.056°**

b. $cos^{-1}(0.83)$
   33.901°

c. $tan^{-1}(2.4)$
   67.380°

d. $sin^{-1}(\frac{3}{7})$
   25.977°

e. $cos^{-1}(\frac{8}{17})$
   61.928°

f. $tan^{-1}(\frac{3}{11})$
   15.255°

6. Find the missing angles and sides in each of the following triangles. Write each side as a simplified expression and solve to the nearest thousandth. For each angle measure (when given two sides) write an inverse trig ratio and find the angle measure:

A)

a. $m\angle T$
   57°

b. $\overline{ST}$
   ST
   $= 10\tan(33)$
   ≈ 6.494

c. $\overline{RT}$
   $RT = \dfrac{10}{\cos(33)}$
   ≈ 15.397

B)

a. $m\angle T$
   10°

b. $\overline{TU}$
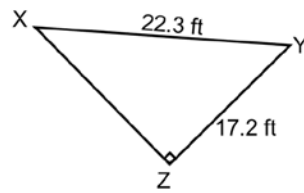 $ST = 10\tan(80)$
   ≈ 56.713

c. $\overline{TV}$
   $RT = \dfrac{10}{\cos(80)}$
   ≈ 57.588

C)

a. $m\angle Y$
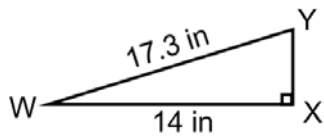
   $m\angle Y = sin^{-1}(\frac{14}{17.3})$

   54.023°

b. $m\angle W$

   $m\angle W = cos^{-1}(\frac{14}{17.3})$

   35.977°

c. $\overline{XY}$

   $\sqrt{103.29} \approx 10.163$

D)

a. $m\angle Y$

   $m\angle Y = cos^{-1}(\frac{17.2}{22.3})$

   39.529°

b. $m\angle X$

   $m\angle X = sin^{-1}(\frac{17.2}{22.3})$

   50.471°

c. $\overline{XZ}$

   $\sqrt{201.45} \approx 14.193$