

GEOMETRY

Special Right Triangles



OBJECTIVE #: G.SRT.C.8

OBJECTIVE

Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
*(Modeling Standard)

BIG IDEA (Why is this included in the curriculum?)

- To derive special right triangles and use the side length relationships to solve problems.
- To find the missing side lengths in special right triangles. If students are given or are able to find one side in any special right triangle (45° - 45° - 90° ; 30° - 60° - 90°) they can find the lengths of the other two sides by applying the pattern of special right triangles. Students will use their knowledge of special right triangles to solve problems and to help prove other theorems.

PREVIOUS KNOWLEDGE (What skills do they need to have to succeed?)

- The student must know the Triangle Sum Theorem.
- The student must know how to apply the Pythagorean Theorem to find the missing side of any right triangle.
- The student must know how to simplify radical expressions including rationalizing the denominator.

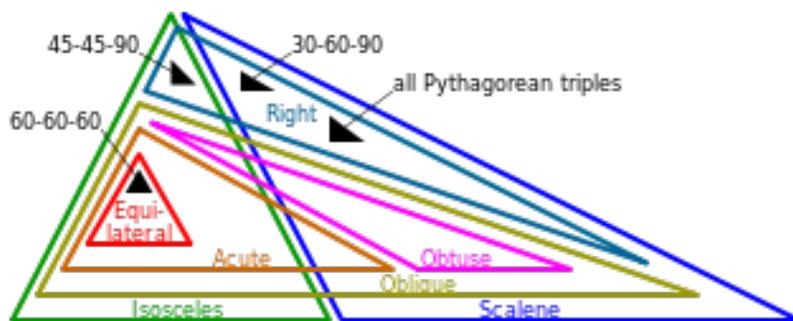
VOCABULARY USED IN THIS OBJECTIVE (What terms will be essential to understand?)

PREVIOUS VOCABULARY (Terms used but defined earlier)

- Congruent Triangles: Students should be able to prove if two triangles are congruent by SSS, SAS, ASA, AAS, or HL.
- Equilateral Triangle: Triangle that has three congruent sides.
- Isosceles Triangle: Any triangle with at least two congruent sides.
- Isosceles Triangle Theorem: If two or more sides in a triangle are congruent then the angles opposite those sides are congruent.
- Perpendicular: Two lines/segments/rays that intersect to form right angles.
- Pythagorean Theorem: In any right triangle, the sum of the square of the legs is equal to the square of the hypotenuse.
- Right Triangle: Any triangle with exactly one right angle.
- Triangle Sum Theorem: The sum of the angles of any triangle is 180° .

NEW VOCABULARY (New terms and definitions introduced in this objective)

- Special Right Triangles: A special right triangle is a right triangle with some regular feature that makes calculations on the triangle easier, or for which simple formulas exist. For example, a right triangle may have angles that form simple relationships, such as 45° - 45° - 90° . This is called an "angle-based" right triangle. A "side-based" right triangle is one in which the lengths of the sides form ratios of whole numbers, such as 3:4:5, or of other special numbers such as the golden ratio. Knowing the relationships of the angles or ratios of sides of these special right triangles allows one to quickly calculate various lengths in geometric problems without resorting to more advanced methods.



SKILLS (What will they be able to do after this objective?)

- The student will be able to find the missing sides in any special right triangle given just one side in the triangle.
- The student will be able to derive the basic formulas for a 45°-45°-90° and a 30°-60°-90° triangle.

SHORT NOTES (A short summary of notes so that a teacher can get the basics of what is expected.)

- In an Isosceles Right Triangle (45°-45°-90°) when calculating the hypotenuse from one of the legs you multiply by $\sqrt{2}$ and when going from the hypotenuse to one of the legs you divide by $\sqrt{2}$.
 - Remind students that there should never be a radical in the denominator for the problem to be completely simplified (they must rationalize the denominator).
- In a 30°-60°-90° triangle when calculating the longer leg from the shorter leg you multiply by $\sqrt{3}$ and when calculating the shorter leg from the longer leg you divide by $\sqrt{3}$.
 - Remind students that there should never be a radical in the denominator for the problem to be completely simplified (they must rationalize the denominator).
- Also in a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shortest leg and conversely the shortest leg is one-half the length of the hypotenuse.
- Discuss the value of exact answers. Show the difference between an answer found with a calculator and an answer that follows the special right triangle patterns.

MISCONCEPTIONS (What are the typical errors or difficult areas? Also suggest ways to teach them.)

- Remind students that the hypotenuse of the right triangle is **ALWAYS** the longest side of any given right triangle.
- When given an isosceles right triangle (45°-45°-90°) some students tend to multiply by 2 instead of $\sqrt{2}$ when moving from a leg to the hypotenuse. Also students tend to multiply by $\sqrt{2}$ when moving from the hypotenuse to a leg. Remind them that they are going backwards, and the opposite of multiplication is division, so they need to divide by $\sqrt{2}$.
- Also when given a 30°-60°-90° triangle with the leg across from the 60° angle students will tend to treat this leg as the side across from 30° and solve it by multiplying by $\sqrt{3}$ instead of dividing by $\sqrt{3}$. When not given the side across from the 30° angle some students are unsure what to do first. It may be easier to remind them to find the shorter leg (across from the 30° angle) first and then find the third side.
- Caution: Once students learn how to manipulate special right triangles and their properties they tend to try to make all triangles fit the pattern. Remind students that these patterns only work with these specific right triangles.

FUTURE CONNECTIONS (What will they use these skills for later?)

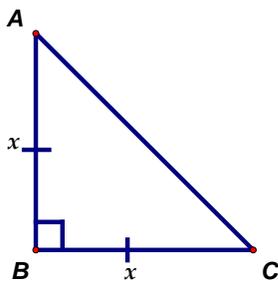
- Students will continue to use special right triangles to solve right triangles, in Geometric proofs and also throughout the trigonometry portion of Algebra 2.

ADDITIONAL EXTENSIONS OR EXPLANATIONS (What needs greater explanation?)

- Additional examples are helpful in showing students all the different possible ways they will use special right triangles.
- Students should see examples when given either leg of an isosceles right triangle and also given the hypotenuse. They should also see examples for when the leg is a radical expression, or the hypotenuse is a whole number.
- Students should see examples when given the longer leg, or hypotenuse, of a 30°-60°-90° triangle. This includes examples when these sides are both whole numbers and radical expressions. Students should also see examples when given the shorter leg as a radical expression.

DERIVING THE PROPERTIES of Special Right Triangles:

45°-45°-90°



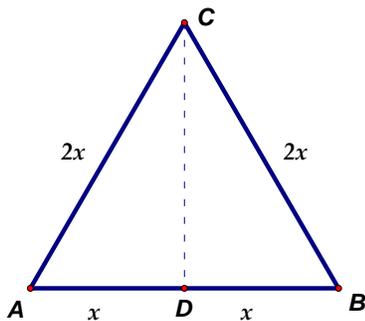
By applying the Pythagorean Theorem we can solve for \overline{AC} .

$$AC^2 = x^2 + x^2$$

$$AC^2 = 2x^2$$

$$AC = \sqrt{2} \cdot \sqrt{x^2} = \sqrt{2}x$$

30°-60°-90°



By bisecting an angle of an equilateral triangle, we create two 30°-60°-90° triangles.

The altitude of the equilateral triangle becomes the longer leg of the 30°-60°-90° triangle and the bisected side becomes the shorter leg.

By applying the Pythagorean Theorem we can solve for \overline{CD} , the longer leg.

$$(2x)^2 = x^2 + CD^2$$

$$4x^2 = x^2 + CD^2$$

$$3x^2 = CD^2$$

$$\sqrt{3} \cdot \sqrt{x^2} = CD$$

$$\sqrt{3}x = CD$$

ASSESSMENTS (Questions that get to the heart of the objective – multiple choice, short answer, multi-step)

1. If the area of a square is 121 ft^2 , what is the length of the diagonal?

$11\sqrt{2} \text{ ft}$

2. Find the area of a square with a diagonal of $20\sqrt{2}$ inches.

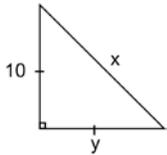
400 in^2

3. What is the ratio between the hypotenuse and the longer leg in a 30°-60°-90° triangle?

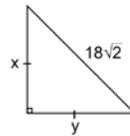
$2:\sqrt{3}$

4. What is the ratios between the hypotenuse and a leg in a $45^\circ-45^\circ-90^\circ$
 $\sqrt{2}:1$
5. What would you multiply the hypotenuse by to find the longer leg of a $30^\circ-60^\circ-90^\circ$ triangle?
 $\frac{\sqrt{3}}{2}$
6. What would you multiply the longer leg of a $30^\circ-60^\circ-90^\circ$ triangle by to find the hypotenuse?
 $\frac{2}{\sqrt{3}}$ or $\frac{2\sqrt{3}}{3}$
7. A construction sign is an equilateral triangle with side lengths of 22 inches. What is the height of the sign?
 $11\sqrt{3}$ inches
8. A rope is tied to the bottom of a post sticking in the ground. The rope forms a 90° angle with the ground and reaches the top of a building that is 45 feet away. How tall is the building?
 $15\sqrt{3}$ feet

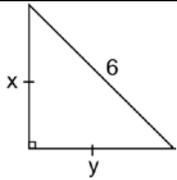
9. Find the values x and y in each of the following:



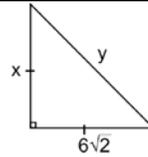
$x: 10\sqrt{2}$
 $y: 10$



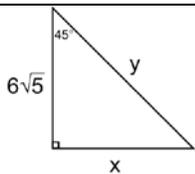
$x: 18$
 $y: 18$



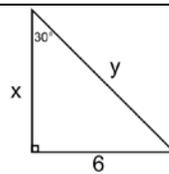
$x: 3\sqrt{2}$
 $y: 3\sqrt{2}$



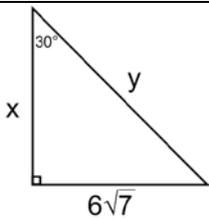
$x: 6\sqrt{2}$
 $y: 12$



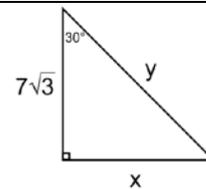
$x: 6\sqrt{5}$
 $y: 6\sqrt{10}$



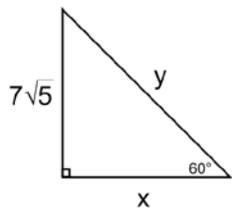
$x: 6\sqrt{3}$
 $y: 12$



$x: 6\sqrt{21}$
 $y: 12\sqrt{7}$

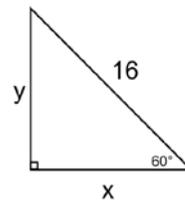


$x: 7$
 $y: 14$



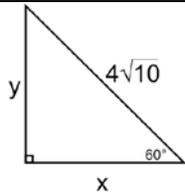
$$x: \frac{7\sqrt{15}}{3}$$

$$y: \frac{14\sqrt{15}}{3}$$



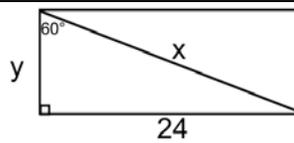
$$x: 8$$

$$y: 8\sqrt{3}$$



$$x: 2\sqrt{10}$$

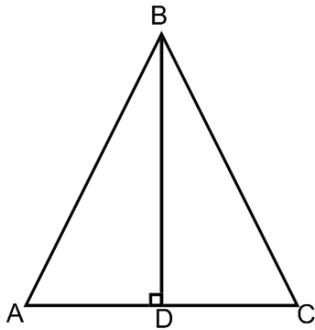
$$y: 2\sqrt{30}$$



$$x: 16\sqrt{3}$$

$$y: 8\sqrt{3}$$

10. In the figure below, $\triangle ABC$ is an equilateral triangle. How can we prove that $\triangle ADB \cong \triangle CDB$?



There are multiple ways to prove this. $\overline{DB} \cong \overline{DB}$, $\angle ADB \cong \angle CDB$; $\angle A \cong \angle C$; $\overline{AB} \cong \overline{BC}$ so you can use AAS, SAS (proving that $\angle ABD$ must be congruent to $\angle DBC$), or ASA (also proving that $\angle ABD$ must be congruent to $\angle DBC$)

This proof is helpful in deriving the formulas for a 30° - 60° - 90° triangle. (see above)