Mathematics II Resources for EOC Remediation

S-CP Probability Cluster:

S-CP.A.2

S-CP.A.3

S-CP.A.4

S-CP.B.7

The information in this document is intended to demonstrate the depth and rigor of the Nevada Academic Content Standards. The items are **not** to be interpreted as indicative of items on the EOC exam. These are a collection of standard-based items for students and **only** include those standards selected for the Math II EOC examination.

Initial Draft: Dec. 2016

S-CP.A.2 Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

1. A fair 6-sided number cube is rolled twice.

Part A:
What is the probability the second roll is odd, given that the first roll was a six.
Part B:
What is the probability that the second roll results in a number greater than two, given that the first roll was even?
Part C:
What is the probability of rolling a 2 on the first roll, given that the first roll was a prime number?

Answer: Part A: $\frac{1}{2}$, Part B: $\frac{2}{3}$, Part C: $\frac{1}{3}$

2. Which of the following represent independent events?

A) $P(A) = 0.2,\ P(B) = 0.4,\ P(A \text{ and } B) = 0.8$

B) $P(A) = 0.3,\ P(B) = 0.3,\ P(A \text{ and } B) = 0.6$

C) $P(A) = 0.5,\ P(B) = 0.2,\ P(A \text{ and } B) = 0.1$

D) $P(A) = 0.5,\ P(B) = 0.2,\ P(A \text{ and } B) = 0.3$

Answer: C

3. One card is selected at random from the following set of 6 cards, each of which has a number and a black or white symbol: {2△, 4□, 8■, 8●, 5□, 5■}

Part A: Let *B* be the event that the selected card has a black symbol and *F* be the event that the selected card has a 5. Are the events *B* and *F* independent? Justify your answer with appropriate calculations.

Part B: Let *B* be the event that the selected card has a black symbol and *E* be the event that the selected card has an 8. Are the events *B* and *E* independent? Justify your answer with appropriate calculations.

$$P(B)=\frac{3}{6} \qquad P(F)=\frac{2}{6} \qquad P(B\,\&\,F)=\frac{1}{6}$$

Answer: Part A: $P(B)\bullet P(F)=\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)=\frac{1}{6} \quad P(B)\bullet P(F)=P(B\,\&\,F)$

Independent

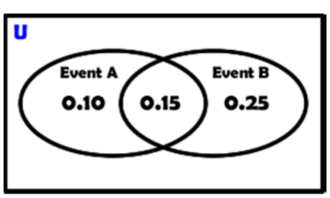
$$P(B)=\frac{3}{6} \qquad P(E)=\frac{2}{6} \qquad P(B\,\&\,E)=\frac{2}{6}$$

Part B: $P(B)\bullet P(E)=\left(\frac{3}{6}\right)\left(\frac{2}{6}\right)=\frac{1}{6} \quad P(B)\bullet P(F)\neq P(B\,\&\,F)$

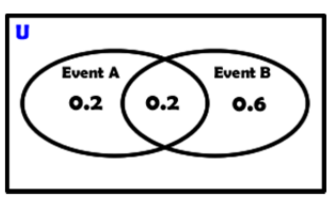
Not Independent

4. Which of the following Venn diagrams represent independence between event *A* and event *B*?

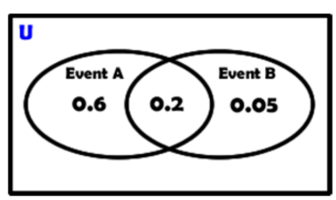
A)



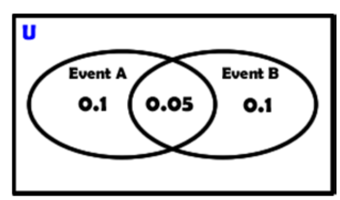
B)



C)



D)



Answer: C

5. In contract negotiations between a local government agency and its workers, it is estimated that there is a 50% chance that an agreement will be reached on the salaries of the workers. It is estimated that there is a 70% chance that there will be an agreement on the insurance benefits. There is a 20% chance that no agreement will be reached on either issue. Find the probability that an agreement will be reached on ***both*** issues. Based on this answer, determine whether the agreement on salaries and the agreement on insurance are independent events. Justify your answer.

Answer: Using a Venn Diagram, the $P(S \text{ and } I) = 0.4$. However, $(0.5)(0.7) = 0.35 \neq 0.4$, therefore salary and insurance are not independent.

S-CP.A.3 Understand the conditional probability of *A* given *B* as $\dfrac{P(A \text{ and } B)}{P(B)}$, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.

1. Of the light bulbs available at a store, 42% are fluorescent, 23% are labeled as long life, and 12% are fluorescent and long life.

Part A:
A light bulb is selected at random from the light bulbs at this store. Rounding your answer to the nearest thousandth where necessary, find the probability that:

1. The selected light bulb is fluorescent.
2. The selected bulb is fluorescent, given that it is labeled as long life.

Part B:
Are the events "fluorescent" and "long life" independent?

Answer: Part A1: 0.42, Part A2: $P(F \mid LL) = \dfrac{0.12}{0.23} = 0.522$

Part B: No, if the two events were independent, then the answers to A1 and A2 would be the same.

2. Using the formulas, show that the two formulas are equivalent when *P*(*A*) and *P*(*B*) are independent.

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$
$$P(B \mid A) = P(B)$$

Answer: $P(B \mid A) = \dfrac{P(A) \cdot P(B)}{P(A)} \quad \Rightarrow \quad P(B \mid A) = \dfrac{\cancel{P(A)} \cdot P(B)}{\cancel{P(A)}}$

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3. If events *A* and *B* are independent, then which statement is not always true?

A) $P(A) \bullet P(B) = P(A \text{ and } B)$

B) $P(B \mid A) = \dfrac{P(A \, and \, B)}{P(B)}$

C) $P(A \mid B) = P(A)$

D) $P(B \mid A) = P(B)$

Answer: B

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4. Sean's team has a baseball game tomorrow. He pitches 50% of the games. There is a 40% chance of rain during the game tomorrow. If the probability that it rains given that Sean pitches is 40%, it can be concluded that these two events are

A. dependent

B. independent

C. mutually exclusive

D. complements

Answer: B

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5. A bag contains five green gumdrops and six red gumdrops. If Kim pulls a green gumdrop out of the bag and eats it, what is the probability that the next two gumdrops she pulls out will be red?

A. $\frac{3}{10}$ B. $\frac{9}{25}$ C. $\frac{1}{3}$ D. $\frac{2}{9}$

Answer: C

S-CP.A.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*

1. The table represents the results of a survey given to 153 high school juniors about whether they like Burgers or Pizza. What is the *P*(Pizza | Girl)?

	Boys	Girls	Total
Burgers	35	54	89
Pizza	41	23	64
Total	76	77	153

A) $\frac{23}{77}$ B) $\frac{64}{153}$ C) $\frac{23}{64}$ D) $\frac{23}{153}$

Answer: A

2. The table shows the results of a survey of engineering students and what the engineers said their favorite subject was in middle school.

	Math	**Science**	**Total**
Electrical	85	90	175
Chemical	80	91	171
Mechanical	89	81	170
Total	254	262	516

Part A:
A student is randomly selected from the group that is surveyed. What is the probability that the student's favorite subject was math, given that the student is a mechanical engineer?

Part B:
A student is randomly selected from the group that is surveyed. What is the probability that the student is a chemical engineer, given that the student's favorite subject in middle school was science?

Answer: Part A: $\frac{89}{170}$, Part B: $\frac{91}{262}$

3. The frequency table below shows the number of freshmen and sophomores in a Geometry class that passed or failed the final exam.

	Pass	**Fail**
Freshman	137	8
Sophomore	111	13

If a student is chosen at random, which of the following probabilities are true? Select **ALL** that apply. Probabilities are rounded to the nearest hundredth.

A. The probability that the student failed given that they are a freshman is 0.62.
B. The probability that the student is a sophomore given that they pass is 0.90.
C. The probability that the student passed given that they are a sophomore is 0.90.
D. The probability that the student is a freshman given that they failed is 0.38.
E. The probability that the student is a sophomore given that they failed is 0.10.
F. The probability that the student passed given that they are a freshman is 0.94.

Answer: C, D and F

4. Determine the information from the two way frequency table.

Young adults (ages 18 – 25) were surveyed on the East and West Coasts of the USA about their music preference. The two way table shows the results of the survey.

A. What is the P(Rap)?
B. What is the P(East Coast)?
C. What is the P(East Coast and Punk)?
D. What is the P(East Coast | Punk)?
E. What is the P(Rap | West Coast)?

	Rap	Punk	Total
East Coast	56	14	70
West Coast	42	27	69
Total	98	41	139

F. Is being from the East Coast independent of choosing Rap?
Yes or No Use calculations to explain why.

Answer: A. $\frac{98}{139}$, B. $\frac{70}{139}$, C. $\frac{14}{139}$, D. $\frac{14}{41}$, E. $\frac{42}{69}$

F. $P(\text{East}) = \frac{70}{139}$; $P(\text{Rap}) = \frac{98}{139}$; $\left(\frac{70}{139}\right)\left(\frac{98}{139}\right) \neq \frac{56}{139} \Rightarrow \text{Not independent}$

5. The set of data in the table below shows the results of a survey on the number of messages that people of different ages text on their cell phones each month.

Age Group	Text Messages per Month		
	0–10	11–50	Over 50
15–18	4	37	68
19–22	6	25	87
23–60	25	47	157

If a person from this survey is selected at random, what is the probability that the person texts over 50 messages per month given that the person is between the ages of 23 and 60?

A. $\frac{157}{229}$ B. $\frac{157}{312}$ C. $\frac{157}{384}$ D. $\frac{157}{456}$

Answer: A

S-CP.B.7 Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.

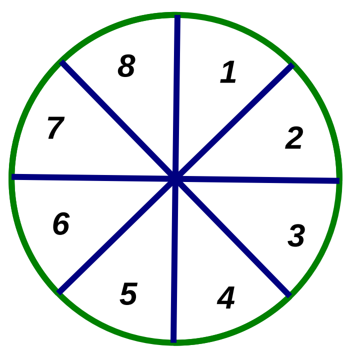
1. If events *A* and *B* are independent and the $P(A) = 0.7$ and $P(A \text{ or } B) = 0.9$, what is the *P(B)?*

Answer: 0.67

2. Donald has ordered a computer and a desk from two different stores. Both items are to be delivered on Tuesday. The probability that the computer will be delivered before noon is 0.6 and the probability that the desk will be delivered before noon is 0.8. If the probability that either the computer or the desk will be delivered before noon is 0.9, what is the probability that both will be delivered before noon?

Answer: 0.50

3. It is equally probable that the pointer will land on any one of the 8 regions, numbered 1 thru 8. Find the probability of getting an even number or a number greater than 5 on one spin.



Answer: $\frac{5}{8}$

4. A suburban high school has a population of 1376 students. The number of students who participate in sports is 649. The number of students who participate in music is 433. If the probability that a student participates in either sports or music is $\frac{974}{1376}$, what is the probability that a student participates in both sports and music?

Answer: $\frac{108}{1376}$

5. The Bar Harbor Boating Company in Maine offers 3 boating tours: Just Whale Sightings, Just Puffin Sightings, and Whale and Puffin Sightings. The manager would like to guarantee the sighting of whales and/or puffins on all of the tours or he would give his customers a full refund, however, he is worried he might lose too much money. If the proportion of sighting only a whale is 0.75, the proportion of sighting only a puffin is 0.95, and the proportion of sighting a whale and puffin is 0.72, set up a two-way table. Then, use what you know about calculating probability to make a recommendation to the manager whether or not he should guarantee sightings for each tour. Include a two-way table to support your recommendation.

Answer:

	Puffins	No Puffins	Total:
Whale	0.72	0.03	0.75
No Whales	0.23	0.02	0.25
Total:	0.95	0.05	1.00

$P(\text{Puffin or Whale}) = 0.98$; $P(\text{Puffin and Whale}) = 0.72$

I would recommend that the manager guarantee the sighting of puffins on the Just a Puffin tour, since the probability of seeing a puffin on a random day is 0.95. I would not recommend the manager offer the guarantee for the sighting of whales on Just the Whale tour, since the probability of seeing a whale on a random day is only 0.75. For the Whale and Puffin tour, I would recommend that the manager offer a guarantee for the sighting of either puffins or whales, since the probability of sighting both a puffin and whale on a random day is only 0.72, but the probability of sighting a puffin or a whale is 0.98.