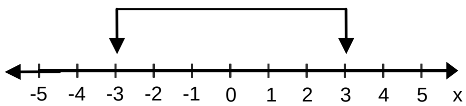
Math 7 Notes – Unit 1: Integers

Integers are defined as the set of whole numbers {0, 1, 2, …} and their opposites. One way to show this is listing its members like this {... ,–3, –2, –1, 0, 1, 2, 3, …} or more simply {…, -1, 0 +1, …} or describing the members in words like this {whole numbers and their opposites}.

Opposites are numbers that are the same distance from 0 on a number line but on the other side of 0.

– 3 and 3 are opposites



NVACS 7.NS.A.1a - Describe situations in which opposite quantities combine to make 0.

Examples: The temperature rises 9 degrees and then falls 9 degrees.

You earn $5 and then you spend $5.

A person loses ten pounds then gains ten pounds.

You enter an elevator on the ground floor and you go up 2 floors and then down 2 floors.

A football team gains 20 yards then loses 20 yards.

A hydrogen atom has 0 charge because its two constituents are oppositely charged. [(It has one proton (+1) and one electron (– 1).]

Other examples may include above and below sea level, credit and debits, deposits and withdrawals, etc.

Examples: Which is a solution of $\mathbf{-15 + x = 0}$

A	–30
B	–15
C	15
D	30

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Examples: Marsha hiked Angel Trails that runs 5 miles north. She then hikes back to where she started from. Which best describes the total change in miles for the entire hike?

A	**10**
B	**5**
C	**0**
D	**-5**

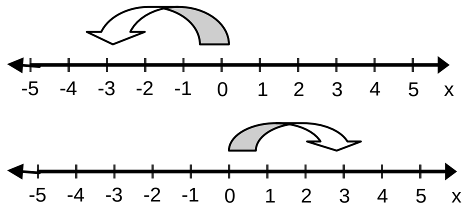
Example (*from OnCore*):

Each hole on a golf course is assigned a number of strokes that a player should need to make to complete the hole. If a hole is par 3, then a player who completes the hole with 5 strokes gets a score of 2 and a player who completes the hole in two strokes gets a score of –1. A player then adds the scores for each hole on the course to find the score for the game.

Half way through a game of golf, a player has a score of three under par. Write an integer to represent this situation. Then find the additive inverse of the integer, and tell what it represents.

A	Integer: –3 Additive Inverse: 3 The additive inverse represents how many extra strokes the player can take and still complete the game in the assigned number of strokes.
B	Integer: 3 Additive Inverse: –3 The additive inverse represents how many extra strokes the player can take and still complete the game in the assigned number of strokes
C	Integer: –3 Additive Inverse: 3 The additive inverse represents how many strokes below par the player must remain and still complete the game in the assigned number of strokes.
D	Integer: 3 Additive Inverse: –3 The additive inverse represents how many strokes below par the player must remain and still complete the game in the assigned number of strokes.

Absolute Value – the distance from 0 on a number line.



Examples: $|-3| = 3$ since –3 is 3 units to the left of 0.

$|3| = 3$ since 3 is 3 units to the right of 0.

So both $|\mathbf{-3}|$ ***and*** $|\mathbf{3}|$ equal 3 because the distance from zero is 3 units. It doesn't matter which direction.

$|+5| = 5$ since 5 is 5 units from 0.

$|\mathbf{-19}| \mathbf{= 19}$ $\quad$ $|0| = 0$

Students need to understand the if $|x| = 5$ then x $= 5$ <u>and</u> x $= -5$.
We can write that as $x = \pm 5$.

CAUTION: Be sure to include absolute values with simple expressions once operations have been taught.

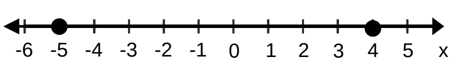
Examples: $|4 + (-2)| = |2| = 2$

$|5(-8)| = |-40| = 40$

$|5 - 7| = |-2| = 2$

$\mathbf{3|-2 - 5| = 3|-7| = 3(7) = 21}$

Example: Find the distance between each set of points on the number line.



$\mathbf{|-5 - 4| = |-9| = 9}$

Adding Integers

When we work with signed numbers, we are often working with two different signs that look exactly alike. They are signs of *value* and signs of *operation*.

- A sign of value indicates whether a number is positive (greater than 0) or negative (less than 0).
- Signs of operation tell you to add, subtract, multiply, or divide.

Example:

Sign of operation

↓

(+3) + (+2)

↓ ↓

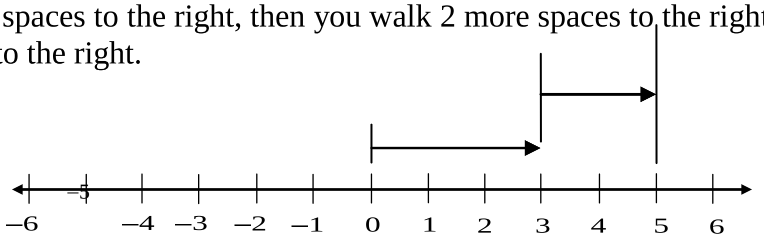
Sign of Value Sign of Value

Notice that the signs of value and the sign of operation are identical.
Note: if a number does NOT have a sign, it is implied to be positive.

Examples: 8 = +8, 19 = +19

NVACS 7.NS.A.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

One way of explaining adding integers is with a number line. Let's say you are standing on zero, and you walk 3 spaces to the right, then you walk 2 more spaces to the right. Where would you be? Five units to the right.



Let's redo that same example, incorporating mathematical notation.
Let's agree that walking to the right is positive, walking to the left is negative.
So, three spaces to the right can be labeled 3R or +3.
Two spaces to the right can be labeled 2R or +2.

Let's **define addition as walking mathematically**.
Translating the problem of walking to the right 3 units, then walking further to the right 2 units, we have:

$$3R + 2R = 5R$$

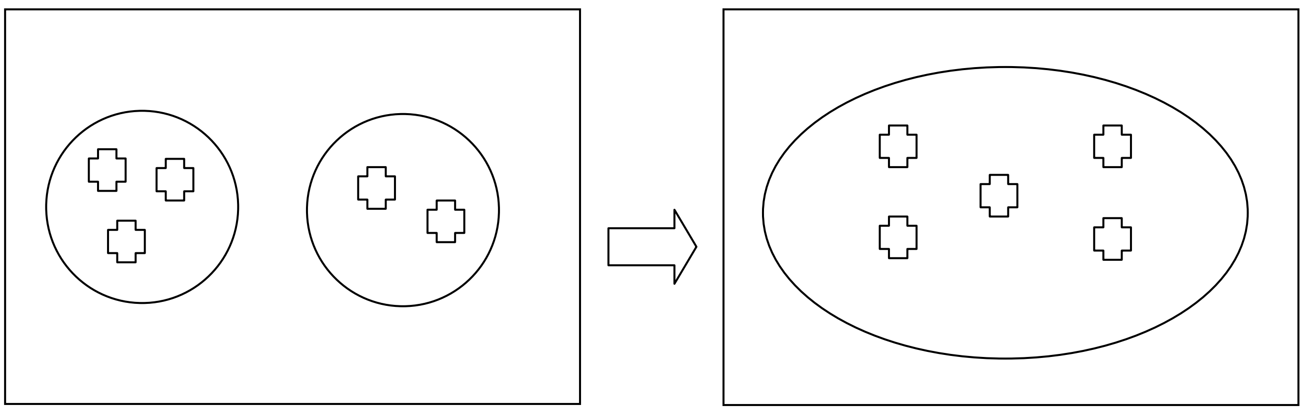
$$(3) + (+2) = (+ 5)$$

The sign of operation tells you to walk; the sign of value tell you which direction.

Another teaching strategy could incorporate the use of positive and negative pieces (tile spacers, purchased at a home improvement store, serve as a great resource for this manipulative) or 2 different colored counters, chips, etc. Establish the meaning of positive and negative pieces with students. Model various ways to show a value of +3. (An example would be six positives and three negatives.) Although there are an infinite number of possibilities, establish that for class purposes, values will be shown in the simplest way, using the least number of manipulative pieces. Therefore, +3 is modeled using three positive pieces/counters. Repeat this process with other values such as –3, 5 and 0. Students must comprehend that zero can be modeled with equal numbers of positive and negative pieces/counters, but again stress the simplest method is one positive and one negative. We call this a "zero pair".

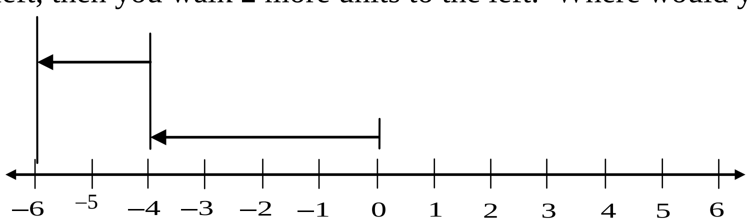
With the expression 3 + 2, have students model the expression. Student should lay out three positives and then, two more positives. Establish the meaning of addition as combining, joining or 'putting together' terms. When the student puts the three positives and two positives together, it is simple to see the result is positive five. Repeat this process with other values.

(+3) + (+2) = +5



Rule 1: **When adding two positive numbers,** ***find the sum of their absolute values, and the answer is positive.***

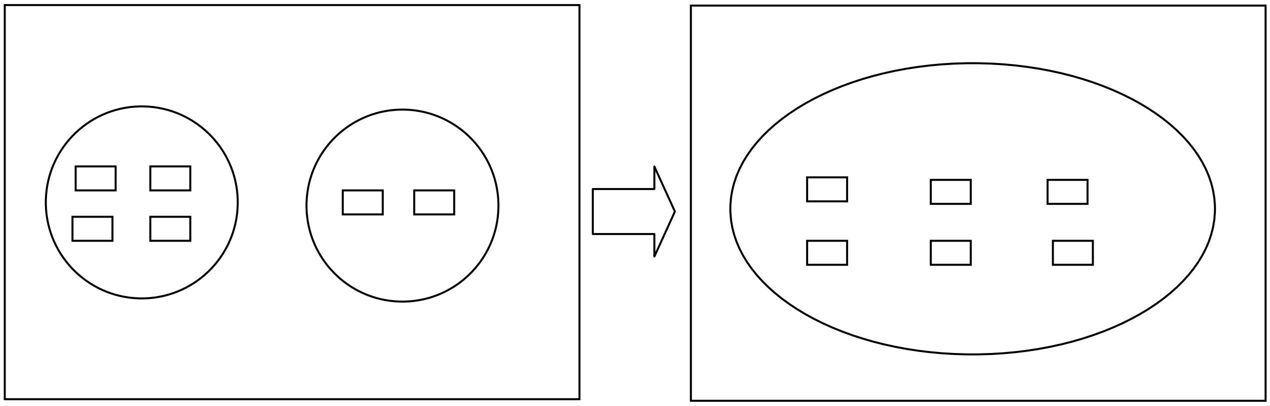
Let's do another problem, this time walking to the left. You are standing on zero, and you walk 4 units to the left, then you walk 2 more units to the left. Where would you be? Six units to the left.



Mathematically, we express it like this: 4L + 2L = 6L
(–4) + (–2) = –6

Using the positive and negative pieces or 2 different colored counters, have students model the expression –4 + –2. Student should lay out four negatives, and then they should lay out two more negatives. When the student puts together four negatives and two negatives it is simple to see the result is negative six. Repeat this process with other values.

(–4) + (–2) = (–6)



Rule 2:	**When adding two negative numbers,**
	find the sum of their absolute values, and the answer is negative.

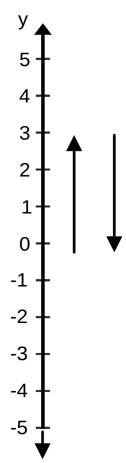
NVACS 7.NS.A.1b – *Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is a positive or negative.* ***Show that a number and its opposite have a sum of 0 (are additive inverses).*** *Interpret sums of rational numbers by describing real-world contexts.*

Example: Show $3 + -3 = 0$

We know … if you only have \$3 and you spend \$3 that you are broke, you have zero dollars.

… that a gain of 3 yards and a loss of three yards is zero.

SHOW… We can show this on a number line …



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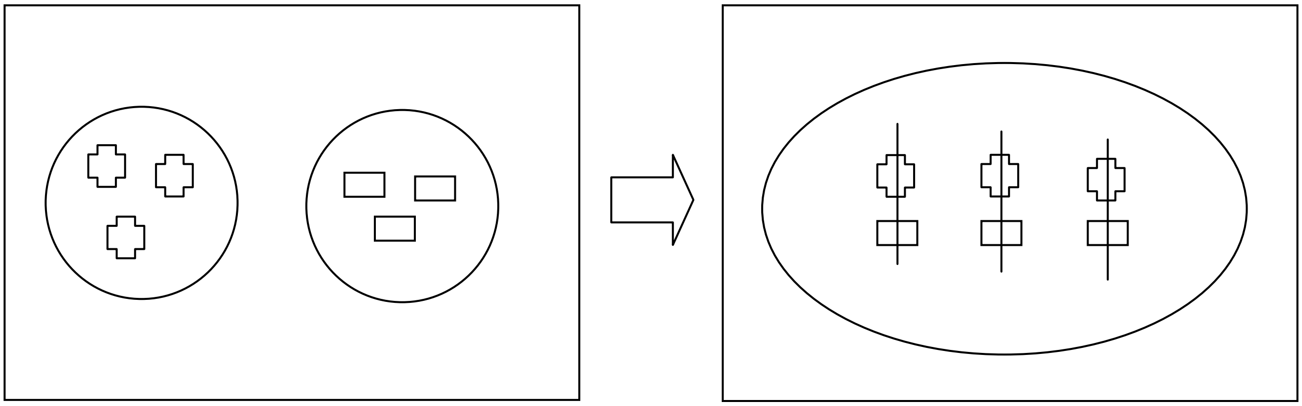
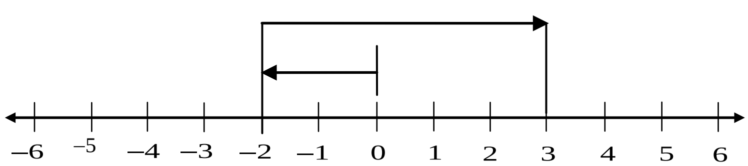
We can also <u>show</u> this using manipulatives and begin to discuss zero pairs - when a pair of one positive and one negative cancel each other out.



$$\mathbf{3 + -3 = 0}$$
$$\mathbf{-9 + 9 = 0}$$
$$\mathbf{5 + -5 = 0}$$
$$\textbf{\textit{any number}} + \textbf{\textit{its opposite}} = \mathbf{0}$$
$$\boldsymbol{n + (-n) = 0}$$

After several examples (using a variety of methods like those above) students should begin to see that adding opposites equals 0. They need to be introduced to the formal property name for this concept at this time…**Additive Inverses.**

Let's do a third addition problem, this time walking in different directions. You are standing on zero, and you walk 2 units to the left, then you walk 5 units to the right. Where would you be? Three units to the right.

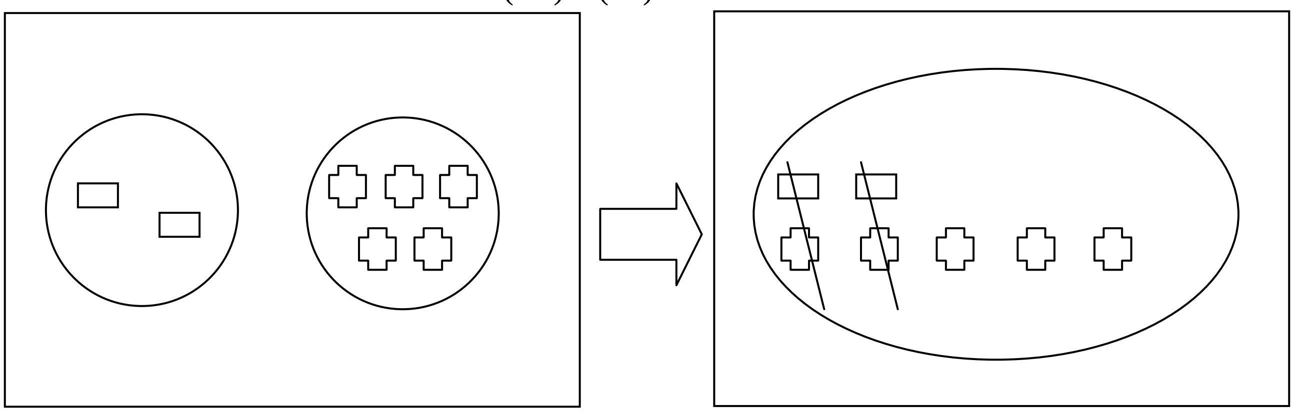


Mathematically, we express it like this: 2L + 5R = 3R
(–2) + (+5) = (+3)

Math 7 Notes – Unit 1: Integers

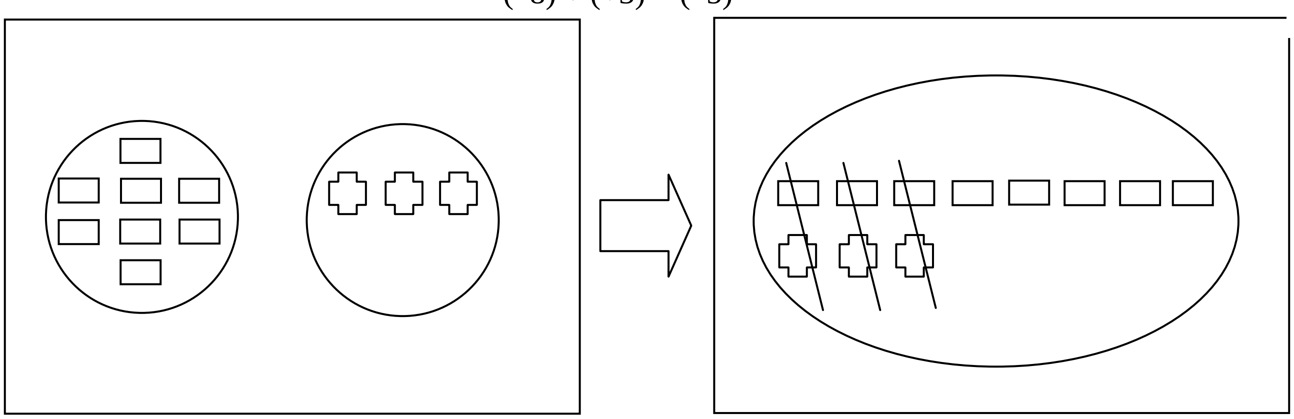
Using the positive and negative pieces or 2 different colored counters, have students model the expression –2 + (+5). Student should lay out two negatives and then five positives. When the student puts together two negatives and five positives, the answer is not simple to see. First they must eliminate the zero pairs; then the result is easy to see. Repeat this process with other values. Make sure you show the mathematical equation along with the model.

(–2) + (+5) = +3

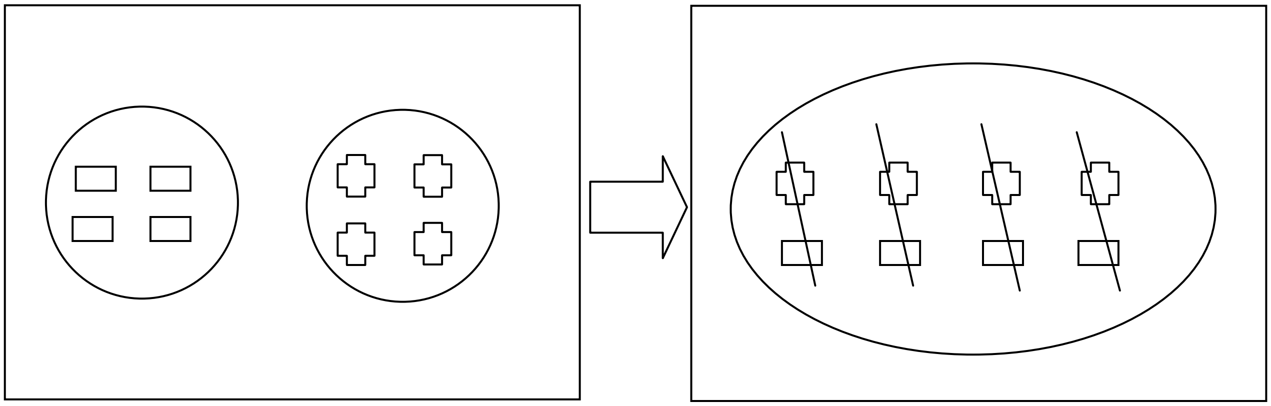


Rule 3: **When adding one positive and one negative number,** ***find the difference of their absolute values, and use the sign of the integer with the greater absolute value.***

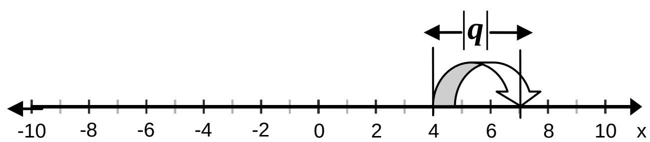
(–8) + (+3) = (–5)



(-4) + (+4) = 0

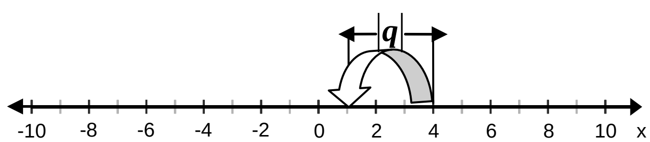


NVACS 7.NS.A.1b - Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. *Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.*



Let p = 4, then looking at the problems such as

$4+3$ and $4+(-3)$,



we start at 4 and move "q" units (in these examples 3 units). If q is positive we move q units right; if q is negative we move q units left.

Subtracting Integers

NVACS 7.NS.A.1c – Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

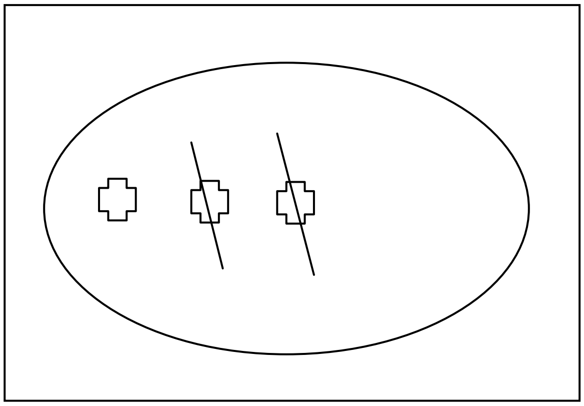
Subtraction – is defined as **adding the opposite.**

Examples: 3 – 2 is the same as 3 plus the opposite of 2 OR 3 + (–2)

4 – (–2) is the same as 4 plus the opposite of –2 OR 4 + 2

Math 7 Notes – Unit 1: Integers

When using the positive and negative pieces or 2 different colored counters, students must be reminded of the meaning of subtraction. Subtraction means "taking away" or "physically removing" a specific amount. With the example 3 – 2, students should begin by modeling +3.

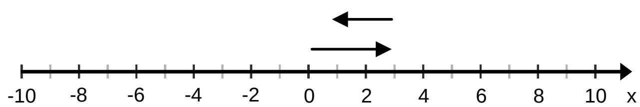


Then the question that needs to be asked is, "Can I physically remove two positives?". If yes, remove those pieces and the result is simple to see.

$$3 - 2 = 1$$

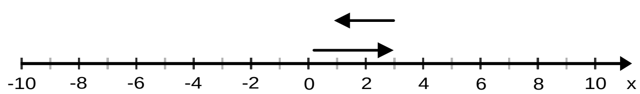
Let's view this on a number line. We show both 3–2 and 3+(–2)

3 – 2 = 1

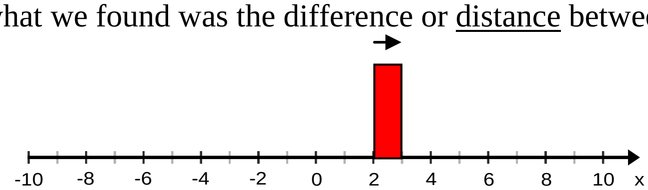


Looking at 3+ (–2) graphically, we see (Start at 0, move 3 units right, then 2 units left)

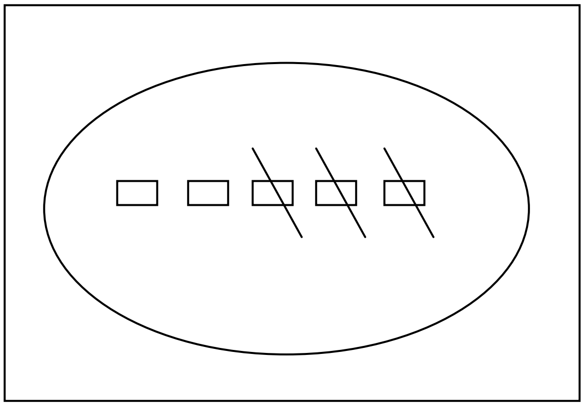
3+(–2) = 1



Keep in mind here what we found was the difference or <u>distance</u> between 2 to 3 graphically.



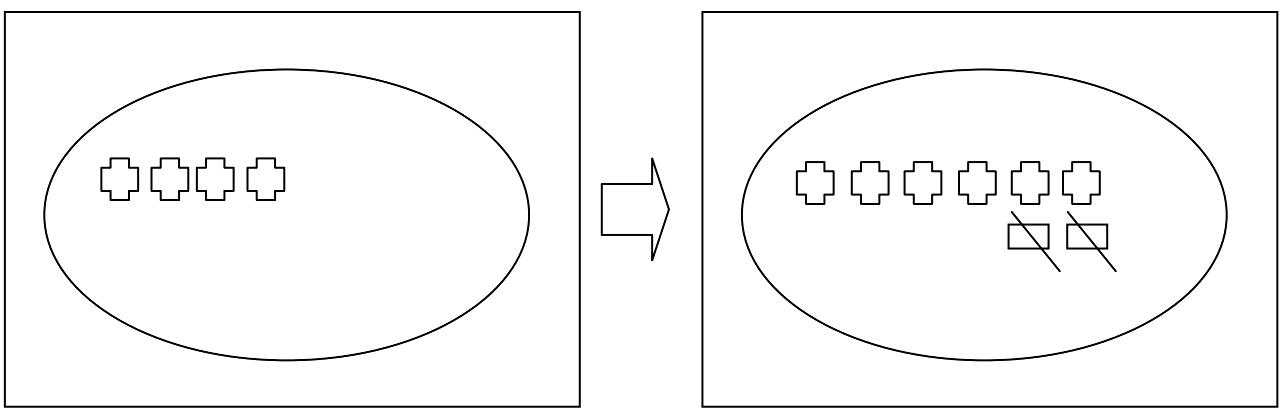
With the example (5) – (–3), we begin by modeling –5. Then the question that needs to be asked is can I “physically remove” three negatives. If yes, remove those pieces and the result is simple to see.



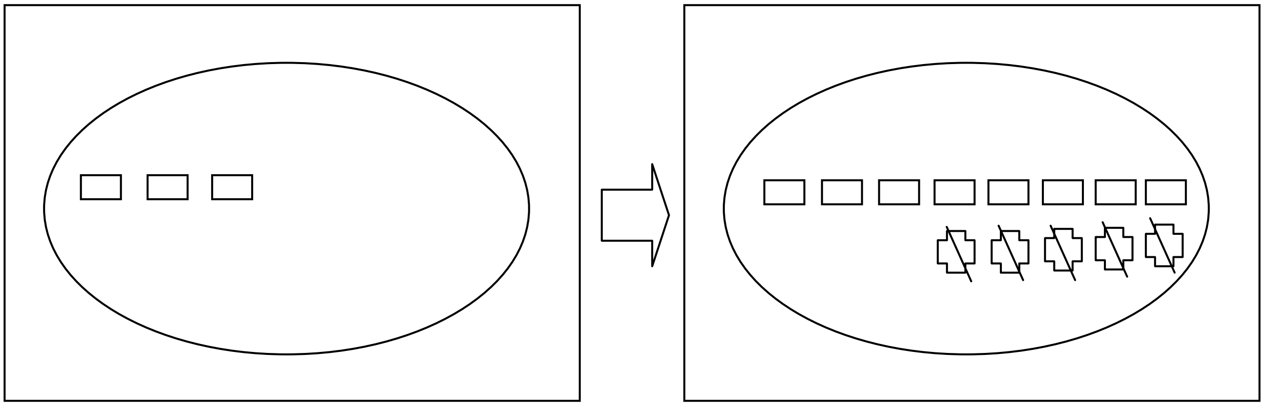
(– 5) – (–3) = – 2

To demonstrate the example 4 – (–2), students should begin by modeling +4. Then the question is, “Can I physically remove two negatives?” In this case you cannot. So what can we do? We can give ourselves a zero pair. Once that is placed we ask, “Now can we physically remove two negatives?” If yes, remove those pieces. If not, continue to give yourself zero pairs until you can physically remove the stated amount. Remind students a “zero pair” consists of one positive and one negative. Repeat this process with other values until it is automatic.

4 – (–2) = 6



(–3) – (+5) = –8



Rule 4: When subtracting signed numbers,

(A) ***change the sign of the subtrahend (second number) and add using rule 1, 2, or 3, whichever applies.***

OR

(B) ***add the opposite.***

***Examples*:** 6 – (+13) ⟶ 6 + (–13)   Change sign and add, according to rule 3.

–7

(–4) – (+2) ⟶ (–4) + (–2)   Change sign and add, according to rule 2.

–6

Math 7 Notes – Unit 1: Integers

Alternative Rules for Adding and Subtracting Integers

1. When the signs are the SAME, find the SUM of their absolute values, and use the common sign.

Examples: 5 + 2 =7 –8 + –3 = –11

2. When the signs are DIFFERENT, find the DIFFERENCE of their absolute values, and use the sign of the number with the greater absolute value.

Examples: 5 + (–12) = –7 (–4) + 10 = +6

Remember to change each subtraction problem to an addition problem, since subtracting is ADDING THE OPPOSITE; then use the rules listed above.

Examples: (–2) – 8 = (–2) + (–8) = –10 3 – (–6) = 3 + 6 = 9

SBAC Example

Identify the number(s) that makes each statement true. You may select more than one number for each statement.

–4 + ☐ = a positive number ◯ **–5** ◯ 5

☐ **– 1** = a negative number ◯ **–3** ◯ **0**

☐ **+5** = zero ◯ **–5** ◯ **5**

–2 – ☐ = a negative number ◯ **–7** ◯ **3**

NVACS 7.NS.A.1b – *Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is a positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses).* ***Interpret sums of rational numbers by describing real-world contexts.***

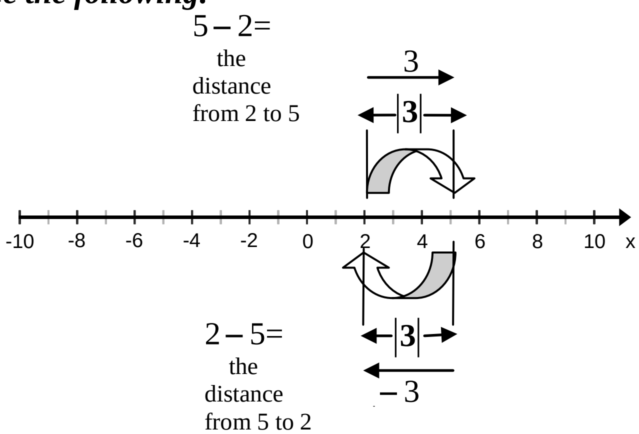
Example: A ship set sail from Hawaii, whose latitude is 20° N. The ship sailed 15° S, then 8° N then 59° S. What was the latitude of the ship at that point?

Example: Stock in LUK Corporation was issued at \$100 per share. In the next three years it went up \$21 per share, down \$15 and down \$17. What was the price of a share of stock at the end of the 3 year period?

Example: An elevator starts at the 12th floor, goes down 7 floors and then up 15 floors. At what floor is the elevator then?

NVACS 7.NS.A.1c – *Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q).* ***Show that the distance between two rational numbers on the number line is the absolute value of their difference,*** *and apply this principle in real-world contexts.*

Notice the following:



In both the examples $5-2$ and $2-5$, we notice the <u>distance</u> between these numbers is $|3|$. If we subtract $5-2$, we get 3. If we subtract $2-5$, we get -3.

NVACS 7.NS.A.1c – *Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference,* and ***apply this principle in real-world contexts.***

Example: From the highest point in California. Mount Whitney, elevation 14,494 ft., one can see the lowest point in the United States, Bad Water, elevation **–282** ft. How much higher is Mount Whitney than Bad Water?

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Example: Jo Chung started a checking account with $400. Later she wrote a check for $150, made a deposit of $230, and wrote another check for $180. How much money was left in Jenny's account?

Example: These announcements were heard at a rocket launch: "Minus 45 seconds and counting" and "We have second-stage ignition at plus 110 seconds". How much time past between the announcements?

Example: A plane flew from Sydney, Australia latitude 34°S, to the North Pole at 90°N. What was the plan's change in latitude?

Example: Paul Garcia's first-of-the-month balance was $268. During the month he withdrew $65 and $258, and made one deposit. How large was the deposit if his end-of-the-month balance was $526?

Multiplying and Dividing Integers

NVACS 7.NS.A.2 – Apply and extend previous understandings of multiplication and division of fractions to multiply and divide rational numbers.

<u>Approach 1 – Travel Examples</u>

For multiplying and dividing we will use the analogy of flying, instead of walking.

Traveling east (right) is positive. Traveling west (left) is negative.

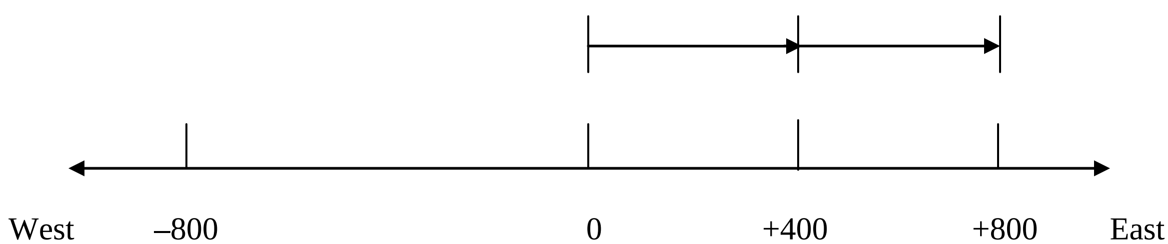
Future time will be defined as a positive number.

Past time will be defined as a negative number.

The starting place will be home, designated as zero.

Example 1: You are at home (zero) and a plane heading east at 400 mph passes directly overhead. Where will it be in 2 hours? Use the formula **distance = rate ● time**.

Translating English to math, going 400 mph East is +400, 2 hours in the future is +2.



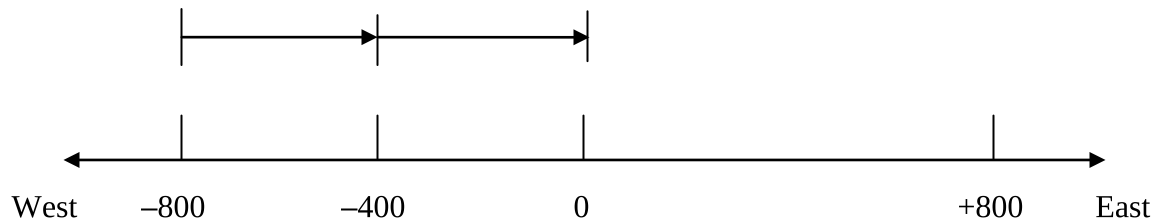
Math 7 Notes – Unit 1: Integers

Standing at 0, the plane is heading east for 2 hours, travelling at 400 mph. It will be 800 miles east in 2 hours. Mathematically, we have: 400 mph east ● 2 hours future = 800 miles east

(+400) ● (+2) = +800

***Example 2**:* The plane is directly over your house, heading east at 400 mph. Where was it 2 hours ago?

Translating English to math, going 400 mph East is +400, 2 hours in the past is –2.

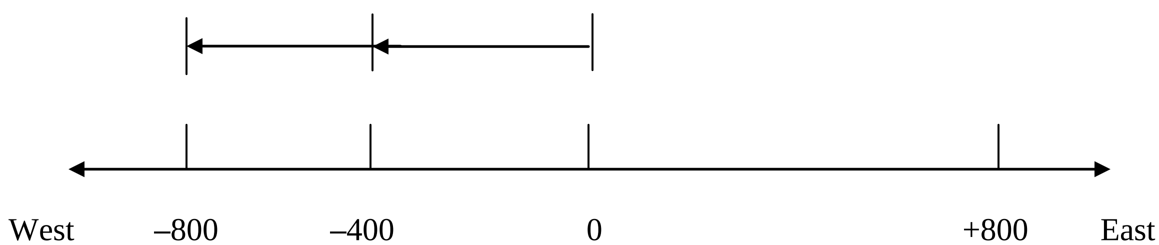


Standing at 0, the plane is heading east for 2 hours, travelling at 400 mph. It would be 800 miles west 2 hours ago. Mathematically, we have: 400 mph east ● 2 hours past = 800 miles west

(+400) ● (–2) = –800

***Example 3**:* The plane is directly over your house, heading west at 400 mph. Where will it be in 2 hours?

Translating English to math, going 400 mph west is –400, 2 hours in the future is +2.

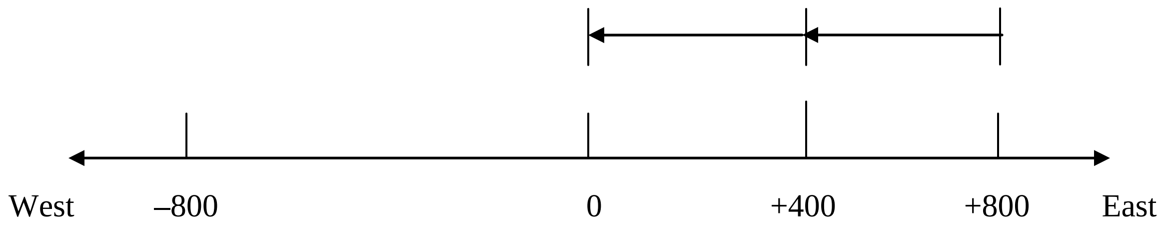


Standing at 0, the plane is heading west for 2 hours, travelling at 400 mph. It will be 800 miles west in 2 hours . Mathematically, we have: 400 mph west ● 2 hours future = 800 miles west

(–400) ● (+2) = –800

Math 7 Notes – Unit 1: Integers

***Example 4**:* The plane is over your house heading west at 400 mph. Where was it 2 hours ago? Translating English to math, going 400 mph west is –400, 2 hours in the past is –2.



Standing at 0, the plane is heading west for 2 hours, travelling at 400 mph. It would be 800 miles east 2 hours ago.   Mathematically, we have: 400 mph west ● 2 hours past = 800 miles east
(–400) ● (–2) = +800

Use the examples to create rules for multiplying and dividing integers.

(+400) ●(+2)	+800
(+400) ●(–2)	–800
(–400)●(+2)	–800
(–400)●(–2)	+800

+•+=+
+•−=−
−•+=−
−•−=+

<u>Approach 2 – Pattern Development</u>

An alternative to using the travel examples is to begin the lesson by modeling something familiar. For example, 3×2. Have the students draw a picture to represent 3×2. Give students time to think, write, and then share their responses. Possible solutions are 3 groups of 2 or repeated addition, 2+2+2. Next ask the students to predict if the model for 2×3 will look the same as the model for 3×2. Allow the students time to create this model to verify their answers.

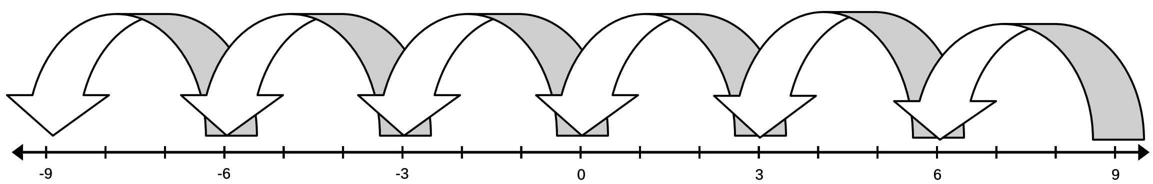
Remind the students that multiplication means “of” so 3×2 means “3 groups of 2”, while 2×3 means “2 groups of 3”. Although the models look different, the product is the same. This can then be related to the Commutative Property of Multiplication.

An additional question would be to ask, “What if we expand this to 3(–2)?” Again give students time to think, write, and share their responses. Possible solutions would be 3 groups of –2 or (–2) + (–2) + (–2).

If we utilize pattern development and place the results on a number line, students can readily visualize these relationships. Look at this pattern. Remind students that multiplication means “groups of”. So as we start this pattern, we begin with 3 groups of 3, then 3 groups of 2, then 3 groups of 1, etc. Notice as we decrease the number of items within the 3 groups the pattern can be shown as seen below.

3 x 3 =
3 x 2 =
3 x 1 =
3 x 0 =
3 x (–1) =
3 x (–2) =
3 x (–3) =

The answers will show this pattern 9, 6, 3, 0, –3, –6, … This pattern can be shown on the number line:



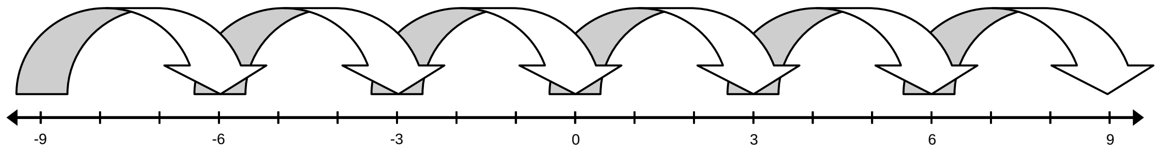
NVACS 7.NS.A.2a – Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. *Interpret products of rational numbers by describing real-world contexts.*

Now let's look at the **opposite** of the pattern above. Above we began with 3 groups of 3, then 3 groups of 2, then 3 groups of 1, etc. For this new pattern we want to examine the pattern the **opposite** of 3 groups of 3, then the **opposite** of 3 groups of 2, then the **opposite** of 3 groups of 1, etc.

Notice our visualization of the pattern is the **opposite** of the one above! The important part of this visual is the continuation of the pattern that shows a negative • negative = positive.

–3 x (3) =
–3 x (2) =
–3 x (1) =
–3 x (0) =
–3 x (–1) =
–3 x (–2) =
–3 x (–3) =

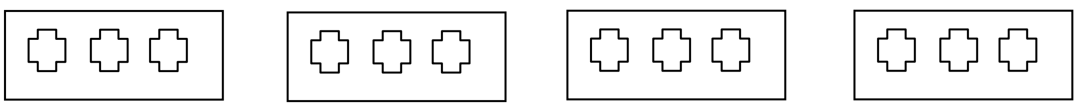
The answers will show this pattern. –9, –6, –3, 0, 3, 6, … This pattern can be shown on the number line:



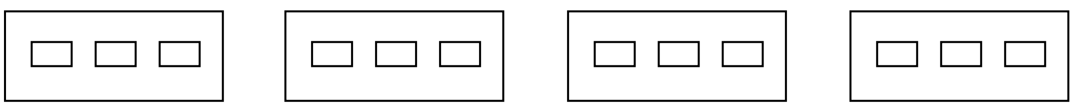
<u>Approach 3 – Positive and Negative Counters</u>

Another approach involves the use of two different colored counters, the use of positive and negative manipulatives, or drawing on the overhead.

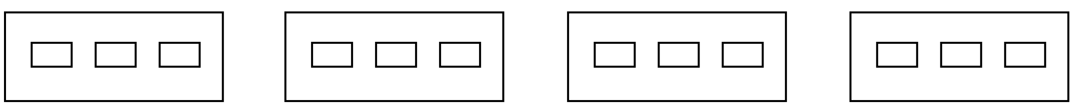
The teacher demonstrates how $4(3)$ can be displayed as *4 groups of 3*.



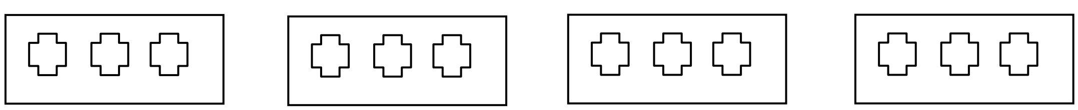
$4(-3)$ would be displayed as *4 groups of negative 3.*



$-4(+3)$ would be displayed as "*the <u>opposite</u> of 4 groups of positive 3*" or "*4 groups of negative 3*".



$-4(-3)$ would be "*the <u>opposite</u> of 4 groups of negative 3*" or "*4 groups of positive 3*".



Continue this technique with several additional examples.

<u>Approach 4 - Concrete Problems to Illustrate</u>

1. If John saves $5 a month, his wealth is increased each month by 5 dollars. In 3 months (+3) his wealth will be increased $15 or +15 dollars.
   Then: $+3(+5) = \$15$.

2. If John spends \$5 a month, his wealth is decreasing each month by 5 dollars. In 3 months (+3), his wealth will be decreased \$15 or –15 dollars.
   Then: $+3(-5) = -\$15$.

3. If John saves \$5 a month, his wealth is increased each month by 5 dollars. Three months ago (–3), his wealth was \$15 less than now or –15 dollars.
   Then: $-3(+5) = -\$15$.

4. If John spends \$5 a month, his wealth is decreased each month by 5 dollars. Three months ago (–3), his wealth was \$15 more than now or +15 dollars.
   Then: –3 x (–5) = 15

Next, the students along with the teacher will formalize the rules for multiplying 2 integers. Teachers should have students record them in their notebooks.

Rules for Multiplication of Two Integers		
positive x positive = positive negative x negative = positive	or	same signs are positive
positive x negative = negative negative x positive = negative	or	different signs are negative

Multiplication and division are inverse operations, so the rules for multiplication and division of integers are the same. For example, since:

$-2 \times (-3) = +6$ then $6 \div (-2) = -3$
$-2 \times (-2) = +4$ then $4 \div (-2) = -2$
$-2 \times (-1) = +2$ then $2 \div (-2) = -1$
$-2 \times 0 = 0$ then $0 \div (-2) = 0$
$-2 \times 1 = -2$ then $-2 \div (-2) = +1$
$-2 \times 2 = -4$ then $-4 \div (-2) = +2$
$-2 \times 3 = -6$ then $-6 \div (-2) = +3$

Continue with additional examples.

$\frac{-8}{4} = -2$ $\quad -15 \div (-3) = 5$ $\quad \frac{56}{-7} = -8$ $\quad \frac{-100}{-25} = 4$ $\quad -91 \div 7 = -13$

Math 7 Notes – Unit 1: Integers

Rules for Division of Two Integers		
positive ÷ positive = positive negative ÷ negative = positive	or	same signs are positive
positive ÷ negative = negative negative ÷ positive = negative	or	different signs are negative

Rule 5: When multiplying or dividing two numbers with the same sign, the answer is positive.

Examples: (+5) •(+4) = +20 (–3) •(–12) = (+36) $12 \div 3 = 4$ $-15 \div -3 = 5$

Rule 6: When multiplying or dividing two numbers with different signs, the answer is negative.

Examples: (+7)•(–6) = (–42) (–8) •(+7) = (–56) $-16 \div 2 = -8$ $30 \div -5 = -6$

Students should also be introduced to the rules of multiplication when there are more than 2 factors. For example, if I need to multiply $5(-4)(-3)(2)(-1)$, students could multiply from left to right.

$$5(-4)(-3)(2)(-1) =$$
$$-20(-3)(2)(-1) =$$
$$60(2)(-1) =$$
$$120(-1) =$$
$$-120$$

When multiplying integers with more than 2 factors it is much more efficient for students to develop the use of the short cut:

1. Count the total number of negative signs in the factors.
2. If the number you counted was even, then the product is positive and you multiply the factors.
   If the number you counted was odd, then the product is negative and you multiply the factors.

So knowing this short cut, to simplify $5(-4)(-3)(2)(-1) =$ I would:

1. count the # of negative signs…. 3 so the answer is negative.
2. multiply the numbers….= 120
   So the answer is -120.

Math 7 Notes – Unit 1: Integers

NVACS 7.NS.A.2b – Understand that integers can be divided, provided that the divisor is not zero, *and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(–q). Interpret quotients of rational numbers by describing real world contexts.*

Students should be able to build on knowledge from work with whole numbers that division by 0 is undefined. Teachers must explicitly review that division by 0 is undefined with integers also. Be sure to show them:

$0\overline{)-15}$ and $\frac{-9}{0}$ are undefined **(**cannot be done) **but also show** $-15\overline{)0}$ *and* $\frac{0}{-9}$ *both* $= 0$

SBAC Example:

Two of these statements are true in **all** cases:

- Statement 1: The greatest common factor of any two distinct prime numbers is 1.
- Statement 2: The greatest common factor of any two distinct composite numbers is 1.
- Statement 3: The product of any two integers is a rational number.
- Statement 4: The quotient of any two integers is a rational number.

Part A: Which two statements are true in *all* cases?

Part B: For both statements that you did <u>not</u> choose in Part A, provide one clear reason and/or example for each statement that proves the statement can be false.

Statement ___ Reason/example

Statement ___ Reason/example

Once again students need to review powers but this time using integers.

Examples:

$(-8)^2 = (-8)(-8) = 64$

$(-5)^3 = (-5)(-5)(-5) = -125$

$-(2)^4 \, or -2^4 = -(2)(2)(2)(2) = -16$

$(-2)^4 = (-2)(-2)(-2)(-2) = 16$

Notice the difference in these two problems. In $-(2)^4$ you are taking the opposite of 2^4, whereas in $(-2)^4$ you are taking -2 to the fourth power.

Math 7 Notes – Unit 1: Integers

Example: $7+(-9\bullet 2)=7+-18=-11$

Example: $-6^2\bullet 2=-36\bullet 2=-72$

Example: $(-6\bullet 2)^2=(-12)^2=144$

Example: $8(-1)^3-7(-1)^2+2(-1)+4=8(-1)-7(1)-2+4=-8-7-2+4=-13$

Example: $\dfrac{3\cdot 10^2-4(-15+6)}{-4^2+3(3+2)+2}=\dfrac{300-4(-9)}{-16+15+2}=\dfrac{300+36}{1}=\dfrac{336}{1}=336$

Examples: Identify the following statements as true or false. If the statement is false give a reason or an example to show it is false.

The square of every negative integer is positive. True - because a $-\bullet-=+$

Every integer and its opposite have equal squares. True

The cube of every negative integer is positive. False

The cube of every negative integer is <u>negative</u>.

$(-2)^3=(-2)(-2)(-2)=-8$

Every integer and its opposite have equal cubes. False

$3^3=27,(-3)^3=-27$

$27\neq -27$

The greater of two integers has the greater square. False

$2>-3$

$2^2=4$

$(-3)^2=9$

$4\ \textit{is NOT} > \textit{than}\ 9$

The greater of two integers has the greater cube. True

Example: Which statement is true?

A	$8\div(-2)=4$
B	$-20\div 4=5$
C	$-14\div(-7)=-2$
D	$15\div(-5)=-3$

NVACS 7.NS.A.2a – *Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers.* ***Interpret products of rational numbers by describing real-world contexts.***

Math 7 Notes – Unit 1: Integers

Example: Your cell phone bill is automatically deducted from your bank account each month. How much will the deductions total for the year?

$$12(-52) = -624$$

The bank will deduct $624 dollars from your account.

Example: Josh made four withdrawals of $150 from his bank account. How much did he withdraw in total?

$$4(-150) = 600$$

Josh withdrew $600.

Example: The price of one share of Arcan Company declined $32 per month for six consecutive months. How much did the price of one share decline in total for that period of six months?

$$6(-32) = -192$$

Each share decreased $192.

NVACS 7.NS.A.2b – *Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number.* ***If p and q are integers, then –(p/q) = (–p)/q = p/(–q).*** *Interpret quotients of rational numbers by describing real world contexts.*

Students should understand that $-\dfrac{15}{3} = \dfrac{-15}{3} = \dfrac{15}{-3}$ each have the same value.

NVACS 7.NS.A.2b – *Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(–q).* ***Interpret quotients of rational numbers by describing real world contexts.***

Examples: It took a submarine 20 seconds to drop to 100 feet below sea level from the surface. What was the rate of the descent?

$-100 \div 20 = -5$ **–5** *feet per second*

NVACS 7.NS.A.3 – Solve real-world and mathematical problems involving the four operations with rational numbers.

NVACS 7.EE.B.3 – Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form *(whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate: and* ***assess the reasonableness of answers using mental***

computation and estimation strategies. ***Of course this should be an on-going skill throughout the year.

Example: In a game one couple made a score of 320, while another couple made a score of –30, what was the difference in scores?

Example: At noon the thermometer stood at $+12°F$, at 5 pm it was $–8°F$. How many degrees had the temperature fallen?

Example: The height of Mt. Everest is 29,000 feet, the greatest known depth of the ocean is 32,000 ft. Find their difference.

Example: On 6 examination questions, Bob received the follow deductions for errors $-4, -2, 0, -5, -0, -8$. What was his grade based on 100 pts?

Example: A team lost 4 yards on the 1st play, gained 12 yards on the 2nd play and lost 5 yards on the 3^{rd} play. What was the net result?

Example: The average temperature of Mars is $–60°F$. The average temperature of Venus is $68°F$. What is the difference in temperature?

Example: How long did a man live who was born in 73 B.C. and died in 25 B.C.?

Example: Roberto traveled from an altitude of 113 ft. below sea level to an altitude of 200 ft below sea level. What was the change in altitude?

NVACS 7.EE.B.3– *Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically.* ***Apply properties of operations to calculate with numbers in any form;*** *convert between forms as appropriate: and assess the reasonableness of answers using mental computation and estimation strategies.*

NVACS 7.NS.A.1d – Apply properties of operations as strategies to add and subtract rational numbers.

NVACS 7.NS.A.2c– Apply properties of operations as strategies to multiply and divide rational numbers.

Example: $\mathbf{3+(2-5)=}$

Using the Associative Property of Addition

$$\mathbf{3+(2-5)=(3+2)-5}$$
$$\mathbf{5-5=}$$
$$\mathbf{0}$$

Example: $3(-5)(-2)(0) =$

Using the Commutative Property of Multiplication $3(-5)(-2)(0) = 0(3)(-5)(-2)$

0

Example: $5(-2+-8) =$

Using the Distributive Property

$5(-2+-8) = 5(-2)+5(-8) =$

$-10 + -40 =$

-50

Example: $|-5|(x+2) =$

Using the Distributive Property

$|-5|(x+2) = 5(x+2) =$

$5(x)+5(2) =$

$5x+10$

MAJOR CAUTION: NVACS 7.EE.B.4a (page 30) clearly states students will fluently solve 2-step equations! The released assessment questions for this grade level from Smarter Balanced Assessment Consortium (SBAC) shows the expectation of many two-step equations and multi-step equations. Therefore you will see our notes, practice test and tests beginning to include two step equations at this time.

Solving Equations Containing Integers

Strategy for Solving Equations: To solve linear equations, put the variable terms on one side of the equal sign, and put the constant (number) terms on the other side. To do this, use opposite (inverse) operations.

Example: x + 5 = –3

$x + 5 = -3$

$\quad -5 \quad -5$ Undo adding 5 by subtracting 5 from both sides.

$x = -8$ Simplify.

Example: $m+(-2)=7$

$m+(-2)=7$

$\quad +2 \quad +2$ Undo adding a negative by adding a positive 2 to both sides.

$m = 9$ Simplify.

Example: $96 = -8n$

$\frac{96}{-8} = \frac{-8n}{-8}$ Undo multiplying by –8 by dividing both sides by –8.

$-12 = n$ Simplify.

OR $n = -12$

Example: $\frac{y}{-5} = -10$

$(-5)\frac{y}{-5} = -10\ (-5)$ Undo dividing by –5, by multiplying both sides by –5.

$y = +50$ Simplify.

Solving 2-Step Equations

The general strategy for solving a multi-step equation in one variable is to rewrite the equation in $ax+b=c$ format (or as CCSS refers to it as $px+q=r$), then solve the equation by isolating the variable using the Order of Operations in reverse and using the opposite operation. (Remember the analogy to unwrapping a gift…)

<u>Order of Operations</u>

1. Parentheses (Grouping)
2. Exponents
3. Multiply/Divide, left to right
4. Add/Subtract, left to right

Evaluating an arithmetic expression using the Order of Operations will suggest how we might go about solving equations in the $ax+b=c$ format (or as CCSS refers to it as $px+q=r$).

To evaluate an arithmetic expression such as $4+2\cdot 5$, we'd use the Order of Operations.

$4+\underline{2\cdot 5}=$ First we multiply, $2\cdot 5$

$4+10=$ Second we add, $4+10$

14

Now, rewriting that expression, we have $2\cdot 5+4=14$, a form that leads to equations written in the form $ax + b = c$ (or as NVACS refers to it as $px+q=r$). If I replace 5 with *n*, I have

$2\cdot n+4=14$ *or*

$2n+4=14$,

an equation in the $ax+b=c$ format (or as NVACS refers to it as px+q=r.

To solve that equation, I am going to "undo" the expression "2*x* + 4". I will isolate the variable by using the Order of Operations in reverse and using the opposite operation.
That translates to getting rid of any addition or subtraction first, then getting rid of any multiplication or division next. Undoing the expression and isolating the variable results in finding the value of *x*.

This is what it looks like:

$$2x+4=14$$
$$2x+4-4=14-4 \quad \text{subtract 4 from each side to "undo" the addition}$$
$$2x=10$$
$$\frac{2x}{2}=\frac{10}{2} \quad \text{divide by 2 to "undo" the mulitplication}$$
$$x=5$$

$$\begin{aligned} 2x+4&=\ 14 \\ -4&=-4 \\ 2x&=10 \\ \frac{2x}{2}&=\frac{10}{2} \\ x&=5 \end{aligned}$$

Check your solution by substituting the answer back into the original equation.

$$2x+4=14 \quad \text{original equation}$$
$$2(5)+4=14 \quad \text{substitute '5' for 'x'}$$
$$10+4=14$$
$$14=14 \quad \text{true statement, so my solution is correct}$$

Example: Solve for *x*. $3x-4=17$.

Using the general strategy, we always want to "undo" whatever has been done in reverse order. We will undo the subtracting first by adding, and then undo the multiplication by dividing.

$$\begin{aligned} 3x-4&=17 \\ 3x-4+4&=17+4 \\ 3x&=21 \\ \frac{3x}{3}&=\frac{21}{3} \\ x&=7 \end{aligned}$$

or

$$\begin{aligned} 3x-4&=17 \\ +4&=+4 \\ 3x&=21 \\ \frac{3x}{3}&=\frac{21}{3} \\ x&=7 \end{aligned}$$

check

$$\begin{aligned} 3x-4&=17 \\ 3(7)-4&=17 \\ 21-4&=17 \\ 17&=17\checkmark \end{aligned}$$

Example: Solve for *x*. $\frac{x}{4}+5=12$

$$\begin{aligned} \frac{x}{4}+5&=12 \\ \frac{x}{4}+5-5&=12-5 \\ \frac{x}{4}&=7 \\ (4)\left(\frac{x}{4}\right)&=(4)(7) \\ x&=28 \end{aligned}$$

or

$$\begin{aligned} \frac{x}{4}+5&=12 \\ \frac{x}{4}+5-5&=12-5 \\ \frac{x}{4}&=7 \\ (4)\left(\frac{x}{4}\right)&=(4)(7) \\ x&=28 \end{aligned}$$

Check:

$$\begin{aligned} \frac{x}{4}+5&=12 \\ \frac{(28)}{4}+5&=12 \\ 7+5&=12 \\ 12&=12\checkmark \end{aligned}$$

Math 7 Notes – Unit 1: Integers

Example: Solve for x. $\frac{x}{3}-7=2$

$$\frac{x}{3}-7=2$$
$$\frac{x}{3}-7+7=2+7$$
$$\frac{x}{3}=9$$
$$\frac{x}{3}\bullet 3=9\bullet 3$$
$$x=27$$

Check:
$$\frac{27}{3}-7=2$$
$$9-7=2$$
$$2=2$$

Example: Solve for *x*. $5(x+2)=-40$

$$5(x+2)=-40$$

$$5(x)+5(2)=-40$$
$$5x+10=-40$$
$$5x+10-10=-40-10$$
$$5x=-50$$
$$\frac{5x}{5}=\frac{-50}{5}$$
$$x=-10$$

or

$$\frac{5(x+2)}{5}=\frac{-40}{5}$$
$$x+2=-8$$
$$x+2-2=-8-2$$
$$x=-10$$

Be sure to have students check their answers.

Example: Solve for *x*. $-4(x-5)=16$

$$-4(x-5)=16$$

$$\frac{-4(x-5)}{-4}=\frac{16}{-4}$$
$$x-5=-4$$
$$x-5+5=-4+5$$
$$x=1$$

or

$$-4(x)-4(-5)=16$$
$$-4x+20=16$$
$$-4x+20-20=16-20$$
$$-4x=-4$$
$$\frac{-4x}{-4}=\frac{-4}{-4}$$
$$x=1$$

Be sure to have students check their answers.

NVACS 7.EE.B.4 – Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations *and inequalities* ***to solve problems by reasoning about the quantities.***

NVACS 7.EE.B.4a – Solve word problems leading to equations of the form px+q=r and p(x+q)=r where p, q and r are specific rational numbers. Solve equations of these forms fluently. *Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.*

Write the equation and then solve.

Example: Two angles of a triangle have a sum of $85°$. The sum of all three angles is $180°$. What is the measure of the third angle?

Let x = the measure of the third angle

$$x + 85 = 180$$
$$-85 \quad -85$$
$$x = 95$$

The third angle is $95°$.

Example: The sum of -14 and a number n is 7.

Let n = the number

$$n + 14 = 7$$
$$-14 \quad -14$$
$$n = -7$$

The number is -7.

Example: The product of 6 and a number t is 84.

Let t = the number

$$6t = 84$$
$$\frac{6t}{6} = \frac{84}{6}$$
$$t = 14$$

The number is 14.

If given as a word problem, many students would be able to solve the problems below mentally or mathematically. Since we require them to <u>write and solve an</u> <u>equation</u>, students may require a model to help them organize the information. As a way to scaffold this standard for students a model is used below as an example to help them move towards the 'write an equation and solve' requirement.

Example: Mary earns \$34 less a week than her brother Tim. Their combined salaries are \$94. How much does each of them earn per week?

Let t = *the money Tim earns*

Let $\boldsymbol{t-34}$ = *the money Mary earns* ***NOTE: ANOTHER 2 step***

Tim's salary	Maria's salary
94	

t	$t-34$
94	

t	$t-34$	+34
94		+34

$2t$
128

t	t
64	*64*

$$\boldsymbol{t+t-34=94}$$
$$\boldsymbol{2t-34=94}$$
$$\boldsymbol{+34\ \ +34}$$
$$\boldsymbol{2t = 128}$$
$$\boldsymbol{\frac{2t}{2} = \frac{128}{2}}$$
$$\boldsymbol{t = 64}$$
$$\boldsymbol{t-34=64-34=30}$$

Check:

$$t+t-34=94$$
$$64+30=94$$
$$94=94$$

Tim earns \$64 a week. Mary earns \$30 a week.

Example: The youth group is going on a trip to the state fair. The trip costs \$51. Included in that price is \$11 for a concert ticket and the cost of 2 passes, one for the rides and one for the game booths. Each of the passes cost the same price. Write an equation representing the cost of the trip and determine the price of one pass.

NOTE: ANOTHER 2 step

Solution

x	x	11
51		

Let x = the cost of 1 pass

$$\boldsymbol{2x+11=51}$$

x	x	11
51		
40		11
20	20	11

$$2x+11=51$$
$$-11 \quad -11$$

$$2x=40$$

$$x=20$$

$20 per pass

Example: **Mr. Green bought 50 shares of Trie stock for $1,650. What was the selling price of the stock per share?**

A.	$1,600
B.	$330
C.	$50
D.	$33

Let x = price per share of stock

$$50x=1,650$$
$$\frac{50x}{50}=\frac{1,650}{50}$$
$$x=33$$

The selling price of each share of stock is $33. The answer is D.

Example: **Abol has $60 to spend on clothes. He buys a pair of jeans for $28. The rest will buy t-shirts costing $8 each. Write an equation and solve for the number of t-shirts he can purchase. Let t = # of t-shirts**

NOTE: ANOTHER 2 step

Cost of t-shirts	Cost of jeans
60	

8*t* ($8 per shirt)	28
60	

8t	28
32	28

t	*t*	*t*	*t*	*t*	*t*	*t*	*t*	28
32								28

t	*t*	*t*	*t*	*t*	*t*	*t*	*t*	28
4	*4*	*4*	*4*	*4*	*4*	*4*	*4*	28

$$\begin{aligned} 8t + 28 &= 60 \\ -28\ &\ -28 \\ 8t\ &= 32 \\ \frac{8t}{8} &= \frac{32}{8} \\ t &= 4 \end{aligned}$$

Abol can buy 4 t-shirts.

SBAC Example: In the following equation, *a* and *b* are both integers.

$$a(3x-8) = b-18x$$

What is the value of ***a?*** [ ]

What is the value of ***b?*** [ ]

At first glance we may look at this problem and wonder "where did this come from"? Although some may consider this problem out of context in this unit, upon closer inspection we begin to notice the *3x* and the $-18x$. What would we have to multiply the *3x* by to equal −*18x*? Well, times −6. That will give me $-18x$ on both sides of the equation; which will cancel out. Then we would be left with $6(-8) = b$.

Solution $\boldsymbol{a = -6}$ ***and*** $\boldsymbol{b = 48}$