Math 7: Expressions, Equations and Inequalities
Notes

***Prep** for 7.EE.B.4*

A **numerical expression** is simply a name for a number. For example, 4 + 6 is a numerical expression for 10, and $400 \bullet 4$ is a numerical expression for 1,600, and $5 - 3 + 2$ is a numerical expression for 4. Numerical expressions include numbers and operations and do not include an equal sign and an answer. In English, we use expressions such as "Hey", "Awesome", "Cool", "Yo". Notice they are not complete sentences.

In this unit we will begin working with variables. **Variables** are letters and symbols we use to represent one or more numbers. An expression, such as $100 \times n$, that contains a variable is called a **variable expression**.

Students have been working with variables in problems like $2 \times \boxed{} = 8$ and $5 + \underline{} = 9$, where a box, or a circle, or a line represents the missing value. Now we will begin to replace those symbols with letters, so we will see $2m = 8$ and $5 + y = 9$. To avoid some initial confusion it is important to stress that the variables represent different values that make each statement true. Some students want to make an incorrect connection such as $a = 1$, $b = 2$, $c = 3$, …

When working with variables, as opposed to numerical expressions, we omit the multiplication sign so $100 \times n$ is written $100n$. The expression $x \times y$ is written xy. Hopefully you can see the confusion that could be caused using variables with the "$\times$" multiplication sign. Stress to students the need to omit the use of the "$\times$" sign with variables, but allow them time to adapt. Remind them in a numerical expression for a product such as 100×4, we **must** use a multiplication sign to avoid confusion. This may also be a great opportunity to begin to have students use the **raised dot** as a multiplication sign, so 100×4 may be written $100 \bullet 4$.

We **simplify** or **evaluate** numeric expressions when we replace it with its simplest name. For example, when we simplify the expression 4 + 6 we replace it with its simplest name, 10.

***Prep* for 7.EE.B.4** Order of Operations

The Order of Operations is just an agreement to compute problems the same way so everyone gets the same result, like wearing a wedding ring on the left ring finger or driving (in the US) on the right side of the road.

Order of Operations (**PEMDAS** or **P**lease **e**xcuse **m**y **d**ear **A**unt **S**ally's **l**oud **r**adio)*

1. Do all work inside the grouping symbols and/or **P**arentheses.

   Grouping symbols include $[\ ]$, $(\ )$, and $\frac{x}{y}$.

2. Evaluate **E**xponents.

3. **M**ultiply/**D**ivide from left to right.*

4. **A**dd/**S**ubtract from left to right.*

*Emphasize that it is NOT always multiply then divide, but rather which ever operation occurs first (going from left to right). Likewise, it is NOT always add-then subtract, but which of the two operations occurs first when looking from left to right.

Example: Simplify the following expression.

$$4+5\times 6-7$$

$$4+\underline{5\times 6}-7$$
$$\underline{4+30}-7$$
$$\underline{34-7}$$
$$27$$

Underline the first step.

Simplify underlined step, and then underline next step.

Repeat; simplify and underline next step until finished.

Note: each line is simpler than the line above it.

Example: Simplify the following expressions.

(a) $3+5\times 2$

Work:

$3+\underline{5\times 2}=$

$3+10=$

13

(b) $4+24\div 6\times 2+1$

Work:

$4+\underline{24\div 6}\times 2+1=$

$4+\underline{4\times 2}+1=$

$\underline{4+8}+1=$

$12+1=$

13

(c) $8\div(1+3)\times 5^2-8$

Work:

$8\div\underline{(1+3)}\times 5^2-8=$

$8\div 4\times\underline{5^2}-8=$

$\underline{8\div 4}\times 25-8=$

$\underline{2\cdot 25}-8=$

$50-8=$

42

Example: **Evaluate** $\dfrac{3\bullet 6-2}{4}$

Work:

Be sure to point out to students that the numerator must be simplified before they can divide by 4.

$$\frac{3 \bullet 6 - 2}{4} = \frac{18-2}{4} = \frac{16}{4} = 4$$

Example: $(5+3)\times(9-2)+9$

Work:

$$(5+3)\times(9-2)+9 =$$
$$8 \times 7 + 9 =$$
$$56 + 9 =$$
$$65$$

Example: Evaluate $\mathbf{9^2 \bullet 2^5 \bullet 0^8 =}$

$\mathbf{81 \bullet 32 \bullet 0 =}$

$\mathbf{0}$

It would be more efficient to use the Commutative Property and place the zero factor first to minimize the need for work, since 0 times any number(s) is zero.

Example: Evaluate $\mathbf{5^2 \bullet 10^3 =}$

$\mathbf{25 \bullet 1{,}000 =}$

$\mathbf{25{,}000}$

Example: Evaluate $\mathbf{7 + 9 \bullet 2^2 =}$

$\mathbf{7 + 9 \bullet 4 =}$

$\mathbf{7 + 36 =}$

$\mathbf{43}$

Example: Evaluate $\mathbf{5 + 5 \bullet 8 \div 2^2 =}$

$\mathbf{5 + 5 \bullet 8 \div 4 =}$

$\mathbf{5 + 40 \div 4 =}$

$\mathbf{5 + 10 =}$

$\mathbf{15}$

Example: Evaluate $7\left[5(6-1)+3(2+3)\right]=$

$$7\left[5(6-1)+3(2+3)\right]=$$
$$7\left[5(5) + 3(5)\right]=$$
$$7\left[25 + 15\right]=$$
$$7\left[40\right]=$$
$$280$$

To **evaluate an algebraic expression**, substitute a given value for the variable, then follow the Order of Operations to evaluate the arithmetic expression.

Example: If $w = 3$, evaluate $w + 5$.

$$w+5$$
$$=3+5$$
$$=8$$

Example: If $h = 6$, evaluate the algebraic expression $h \div 2$.

$$h \div 2$$
$$=6 \div 2$$
$$=3$$

Example: Find the value of $2b + 4$, if $b = 3$.

$$2b+4$$
$$=2(3)+4$$
$$=6+4$$
$$=10$$

Example: If $x = 3$, evaluate $x^2 - 3$.

$$x^2-3$$
$$=3^2-3$$
$$=9-3$$
$$=6$$

Example: Evaluate $2a + 3b - 4c$ when $a = 5$, $b = 10$ and $c = 7$.

$$2a+3b-4c$$
$$=2(5)+3(10)-4(7)$$
$$=10+30-28$$
$$=40-28$$
$$=12$$

Example: Evaluate $4s$, when $s = 8$.

$$4s$$
$$=4 \bullet 8$$
$$=32$$

Example: Evaluate $2l + 2w$, when $l = 10$ and $w = 5$.

$$2l+2w$$
$$=2 \bullet 10+2 \bullet 5$$
$$=20+10$$
$$=30$$

Example: Which is the greatest?

A.	2^5
B.	3^4
C.	4^3
D.	5^2

A. $2^5=32$

B. $3^4=3 \bullet 3 \bullet 3 \bullet 3=81$

C. $4^3=4 \bullet 4 \bullet 4=64$

D. $5^2=5 \bullet 5=25$

So the answer is B

A ***variable*** is defined as a letter or symbol that represents a number that can change.

Examples: $a, b, c, \_, \triangle, \square, \ldots$

An ***algebraic*** *(variable)* ***expression*** is an expression that consists of numbers, variables, and operations.

Examples: $a+b,\ 4x,\ 5+z$

A ***constant*** is a quantity that does not change, like the number of cents in one dollar.

Examples: 5, 12

Terms of an expression are a part or parts that can stand alone or are separated by the + (or –) symbol. (In algebra we talk about monomials, binomials, trinomials, and polynomials. Each term in a polynomial is a monomial.)

Example: The expression ***9+a*** *has 2 terms 9 and a.*

Example: The expression ***3ab*** *has 1 term.*

Example: The expression $7\boldsymbol{a}^2\boldsymbol{b}-2\boldsymbol{a}$ *has 2 terms,* $7a^2b$ *and 2a.*

Example: The expression $\mathbf{2}\boldsymbol{a} + \mathbf{3}\boldsymbol{b} - \mathbf{4}\boldsymbol{c}$ *has 3 terms, 2a, 3b and 4c.*

A ***coefficient*** is a number that multiplies a variable.

Example: In the expression ***3ab,*** the coefficient is *3*

Example: In the expression $\mathbf{-2}\boldsymbol{ab}\mathbf{+1}$**,** the coefficient is -2**.** (Note: 1 is a constant.)

Example: In the expression $\frac{\boldsymbol{x}}{\mathbf{2}}$**,** the coefficient is $\frac{1}{2}$.

In review, in the algebraic expression $\boldsymbol{x}^{\mathbf{2}}+\mathbf{6}\boldsymbol{x}+\mathbf{2}\boldsymbol{y}+\mathbf{8}$

- the variables are *x* and *y*.
- there are 4 terms, x^2, $6x$, $2y$ and 8.
- the coefficients are 1, 6, and 2 respectively.
- There is one constant term, 8.
- This expression shows a sum of 4 terms.

Prep for 7.EE.B

Word Translations

Words/Phrases that generally mean:

ADD : total, **sum**, add, in all, altogether, more than, increased by
SUBTRACT: **difference**, less, less than, minus, take away, decreased by, words ending in "er"
MULTIPLY: times, **product,** multiplied by,
DIVIDE: **quotient**, divided by, per, each, goes into

Examples:

Operation	Verbal Expression	Algebraic Expression
Addition +	a number plus 7	*n* + 7
Addition +	8 added to a number	*n* + 8
Addition +	a number increased by 4	*n* + 4

Addition +	5 more than a number	$n + 5$
Addition +	the sum of a number and 6	$n + 6$
Addition +	Tom's age 3 years from now	$n + 3$
Addition +	two consecutive integers	$n, n+1$
Addition +	two consecutive odd integers	Let $x = $1st odd, $x + 2 = $2nd odd
Addition +	2 consecutive even integers	Let $x = $1st even, $x + 2 = $2nd even
Subtraction –	a number minus 7	$x - 7$
Subtraction –	8 subtracted from a number*	$x - 8$
Subtraction –	a number decreased by 4	$x - 4$
Subtraction –	4 decreased by a number	$4 - x$
Subtraction –	5 less than a number*	$x - 5$
Subtraction –	the difference of a number and 6	$x - 6$
Subtraction –	Tom's age 3 years ago	$x - 3$
Subtraction –	separate 15 into two parts*	$x, 15 - x$
Multiplication • ( )	12 multiplied by a number	$12n$
Multiplication • ( )	9 times a number	$9n$
Multiplication • ( )	the product of a number and 5	$5n$
Multiplication • ( )	Distance traveled in x hours at 50 mph	$50x$
Multiplication • ( )	twice a number	$2n$
Multiplication • ( )	half of a number	$\frac{n}{2}$ or $\frac{1}{2}n$
Multiplication • ( )	number of cents in x quarters	$25x$
Division ÷	a number divided by 12	$\frac{x}{12}$
Division ÷	the quotient of a number and 5	$\frac{x}{5}$
Division ÷	8 divided into a number	$\frac{x}{8}$

*Be aware that students have difficulty with some of these expressions. For example, "five less than a number" is often incorrectly written as $5 - n$, and should be written $n - 5$.

Example: Write a word translation for the expression $\mathbf{5n + 2}$.

5 times a number increased by 2

Example: Write the expression for ***seven more than the product of fourteen and a number x*** $7 + 14x$

Example: Write the expression for ***3 times the sum of a number and 9.***

$$\mathbf{3(n+9)}$$

Example: Write the expression for ***5 less than the product of 2 and a number x*** and evaluate it when ***x = 7.***

$$\begin{aligned} &2x-5 \\ &=2\bullet 7-5 \\ &=14-5 \\ &=9 \end{aligned}$$

Example: Write an expression for ***3 times the sum of a number and 9,*** and simplify the expression.

$$3(n+9) \quad = \quad \begin{array}{r} n+9 \\ n+9 \\ \underline{+\ n+9} \end{array} \quad = \quad 3\bullet n+3\bullet 9 \quad = \quad 3n+27$$

Direct translation

Repeated addition – 3 groups of $n+9$

Using the Distributive Property

Example: Write possible equivalent expressions for $24x+18y$.

Solution:

$$24x+18y=2(12x+9y)$$
$$24x+18y=3(8x+6y)$$
$$24x+18y=6(4x+3y)$$

NVACS 7.EE.B.4 *Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.*

Prep for 7.EE.B — Solving One-Step Equations

An **equation** is a mathematical statement using the equal sign between two mathematical expressions naming the same number. For example, 3 + 5 = 8, $x+3=12$ or $3x - 1 = 17$ are equations.

Solving Equations – finding the value(s) of *x* which make the equation a true statement.

Strategy for Solving Equations: To solve linear equations, put the variable terms on one side of the equal sign, and put the constant (number) term on the other side. To do this, use OPPOSITE (or INVERSE) OPERATIONS.

Let's look at a gift wrapping analogy to better understand this strategy. When a present is wrapped, it is placed in a box, the cover is put on, the box is wrapped in paper, and finally a ribbon is added to complete the project. To get the present out of the box, everything would be done in reverse order, performing the OPPOSITE (INVERSE) OPERATION. First we take off the ribbon, then take off the paper, next take the cover off, and finally take the present out of the box.

To **solve equations in the form of *x* + *b* = *c***, we will "undo" this algebraic expression to isolate the variable. To accomplish this, we will use the opposite operation to isolate the variable.

Example: Solve for *x* in the equation $x-5=8$.

$$\begin{array}{r} x-5= \;\; 8 \\ \underline{+5=+5} \\ x=13 \end{array}$$

To isolate the *x* term, undo "subtracting 5" by "adding 5" to both sides.

$13-5=8$ ✓

Check to see that the answer is a solution.

It is also common practice to show the work this way:

$$\begin{aligned} x-5&=8 \\ x-5+5&=8+5 \\ x&=13 \end{aligned}$$

Example: Solve: $x+7=16$.

$$\begin{array}{r} x+7= \;\; 16 \\ \underline{-7=-7} \\ x= \;\; 9 \end{array}$$

To isolate the *x* term, undo "adding 7" by "subtracting 7" from both sides.

$9+7=16$ ✓

Check to see that the answer is a solution.

It is also common practice to show the work this way:

$$\begin{aligned} x+7&=16 \\ x+7-7&=16-7 \\ x&=9 \end{aligned}$$

Example: Solve: $3x=27$.

$$\begin{aligned} 3x&=27 \\ \frac{3x}{3}&=\frac{27}{3} \\ x&= \;\; 9 \end{aligned}$$

To isolate the *x* term, undo "multiplying by 3" by "dividing both sides by 3".

$3(9)=27$ ✓

Check to see that the answer is a solution

Example: Solve: $\frac{x}{4}=12$.

$$\begin{aligned} \frac{x}{4}&=12 \\ (4)\frac{x}{4}&=12(4) \\ x&= \;\; 48 \end{aligned}$$

To isolate the *x* term, undo "dividing by 4" by "multiplying both sides by 4".

$\frac{48}{4}=12$ ✓

Check to see that the answer is a solution

Example: Solve: $x+3=-7$

$$x+3=-7$$
$$x+3+{\color{red}-3}=-7+{\color{red}-3}$$
$$x=-10$$

Undo add 3 by adding -3 to both sides. Simplify.

Check: $-10+3=-7$
$-7=-7$

Example: Solve: $x-(-4)=-14$

$$x-(-4)=-14$$
$$x-(-4){\color{red}+(-4)}=-14{\color{red}+(-4)}$$
$$x=-18$$

Undo subtract -4 by adding -4 to both sides. Simplify.

Check: $-18--4=-14$
$-18+4=-14$

****Example:*** Solve: $7-x=4$

$$7-x=4$$
$$7-x{\color{red}+x}=4{\color{red}+x}$$
$$7=4+x$$

OR

$$7{\color{red}-4}=4+x{\color{red}-4}$$
$$3=x$$

$$7-x=4$$
$$7-x{\color{red}-7}=4{\color{red}-7}$$
$$-x=-3$$
$$x=3$$

Check: $7-3=4$
$4=4$

Example: Solve: $-5x=15$

$$-5x=15$$
$$\frac{-5x}{\color{red}-5}=\frac{15}{\color{red}-5}$$
$$x=-3$$

Undo multiply by -5 by dividing by -5 to both sides. Simplify.

Check: $-3(-5)=15$
$15=15$

Example: Solve: $-2x=-22$

$$-2x=-22$$
$$\frac{-2x}{\color{red}-2}=\frac{-22}{\color{red}-2}$$
$$x=11$$

Undo multiply by -2 to dividing by -2 to both sides. Simplify.

Check: $-2(11)=-22$
$-22=-22$

Example: Solve: $\frac{x}{-4}=-6$

$$\frac{x}{-4}=-6$$
$${\color{red}(-4)}\frac{x}{-4}=-6{\color{red}(-4)}$$
$$x=24$$

Undo divide by -4 to multiply by -4 to both sides. Simplify.

Check: $\frac{24}{-4}=-6$
$-6=-6$

***Example*:** Solve: $x-\frac{1}{3}=\frac{2}{5}$.

$$x-\frac{1}{3}=\frac{2}{5}$$
$$x-\cancel{\frac{1}{3}+\frac{1}{3}}=\frac{2}{5}+\frac{1}{3}$$
$$x=\frac{6}{15}+\frac{5}{15}$$
$$x=\frac{11}{15}$$

Undo subtracting one-third by adding one-third to both sides of the equation; make equivalent fractions with a common denominator of 15; add.

Check:
$$\frac{11}{15}-\frac{1}{3}=\frac{2}{5}$$
$$\frac{11}{15}-\frac{5}{15}=\frac{6}{15}$$
$$\frac{6}{15}=\frac{6}{15}=\frac{2}{5}\ \checkmark$$

***Example*:** Solve: $4\frac{2}{5}=x+\frac{9}{10}$

$$4\frac{2}{5}=x+\frac{9}{10}$$
$$4\frac{2}{5}-\frac{9}{10}=x+\cancel{\frac{9}{10}-\frac{9}{10}}$$
$$4\frac{4}{10}-\frac{9}{10}=x$$
$$3\frac{14}{10}-\frac{9}{10}=x$$
$$3\frac{5}{10}=x$$
$$3\frac{1}{2}=x$$

Undo adding $\frac{9}{10}$ by subtracting $\frac{9}{10}$ from both sides.
Make equivalent fractions with a common denominator of 10, regroup and subtract.
Simplify.

Check:
$$4\frac{2}{5}=3\frac{1}{2}+\frac{9}{10}$$
$$4\frac{4}{10}=3\frac{5}{10}+\frac{9}{10}$$
$$4\frac{4}{10}=3\frac{14}{10}=4\frac{2}{5}\ \checkmark$$

***Example*:** Solve: $x+0.6=1.5$

$$x+0.6=1.5$$
$$x+\cancel{0.6-0.6}=1.5-0.6$$
$$x=0.9$$

Undo adding 0.6 by subtracting 0.6 from both sides.
Simplify.

Check:
$$0.9+0.6=1.5\ \checkmark$$
$$1.5=1.5$$

***Example*:** Solve: $x-1.2=2.3$

$$x-1.2=2.3$$
$$x-\cancel{1.2+1.2}=2.3+1.2$$
$$x=3.5$$

Undo subtracting 1.2 by adding 1.2 from both sides.
Simplify.

Check:
$$3.5-1.2=2.3$$
$$2.3=2.3\ \checkmark$$

***Example*:** Solve: $\frac{x}{5}=\frac{2}{3}$

$$\frac{x}{5}=\frac{2}{3}$$

$$\left(\frac{\cancel{5}}{1}\right)\left(\frac{x}{\cancel{5}}\right)=\frac{2}{3}\left(\frac{5}{1}\right)$$

$$x=\frac{10}{3}\ or\ 3\frac{1}{3}$$

To undo division by 5, multiply both sides of the equation by 5. Put the answer in simplest form.

$$\frac{3\frac{1}{3}}{5}=\frac{2}{3}$$

$$\text{Check: } 3 \bullet 3\frac{1}{3}=5 \bullet 2$$

$$10=10 \ \checkmark$$

***Example*:** Solve: $\frac{2}{5}x=3$

$$\frac{2}{5}x=3$$

$$\frac{2}{5}x\div\frac{2}{5}=3\div\frac{2}{5}$$

$$\frac{2}{5}x\cdot\frac{5}{2}=3\cdot\frac{5}{2}$$

$$\frac{\cancel{2}}{\cancel{5}}x\cdot\frac{\cancel{5}}{\cancel{2}}=\frac{3}{1}\cdot\frac{5}{2}$$

$$x=\frac{15}{2}\ or\ 7\frac{1}{2}$$

To undo multiplication by $\frac{2}{5}$, divide both sides of the equation by $\frac{2}{5}$.

Dividing by $\frac{2}{5}$ is the same as multiplying by the reciprocal $\frac{5}{2}$.

Simplify the answer as needed.

Check: $\frac{\cancel{2}^{1}}{\cancel{5}_{1}} \bullet \frac{\cancel{15}^{3}}{\cancel{2}_{1}}=3$

$$3=3 \ \checkmark$$

***Example*:** Solve: $4x=\frac{6}{7}$

$$4x=\frac{6}{7}$$

$$\cancel{4}x\cdot\frac{1}{\cancel{4}}=\frac{{}^{3}\cancel{6}}{7}\cdot\frac{1}{\cancel{4}_{2}}$$

$$x=\frac{3}{14}$$

To undo multiplying by 4, we divide by 4. Dividing by 4 is the same as multiplying by the reciprocal $\frac{1}{4}$.

Cancel and simplify.

Check: $\frac{\cancel{4}^{2}}{1} \bullet \frac{3}{\cancel{14}_{7}}=\frac{6}{7}$

$$\frac{6}{7}=\frac{6}{7} \ \checkmark$$

***Example*:** Solve: $0.4x=1.6$

$$0.4x=1.6$$

$$\frac{0.4x}{0.4}=\frac{1.6}{0.4}$$

$$x=4$$

To undo multiplying by 0.4, we divide both sides by 0.4. Simplify.

Check: $0.4 \bullet 4=1.6$

$$1.6=1.6 \ \checkmark$$

Example: Solve: $\dfrac{x}{2.5} = 6$

$$\frac{x}{2.5} = 6$$

$$\frac{x}{2.5} \cdot 2.5 = 6 \cdot 2.5$$

$$x = 15.0 = 15$$

To undo dividing by 2.5, we multiply both sides by 2.5. Simplify.

Check: $\dfrac{15}{2.5} = 6$

$6 = 6$ ✓

OnCore examples:

Example: Which is the solution to the equation $x - 5 = 12$?

A.	$x = 7$
B.	$x = 8$
C	$x = 17$
D.	$x = 18$

Example: Solve $p + 2\frac{4}{5} = 4\frac{1}{2}$

A.	$p = 1\frac{3}{10}$
B.	$p = 1\frac{7}{10}$
C.	$p = 2\frac{3}{10}$
D.	$p = 2\frac{7}{10}$

Example: Solve $-7.2h = 57.6$

A.	$h = -0.8$
B.	$h = -8$
C.	$h = -50.4$
D.	$h = -80$

Now we use our skills in translating from words to math expressions to form equations to help us solve word problems. Look for the key word "is" "the result is", "equals", "will be", 'was", etc. to help place the "=" symbol.

Example: When 15 is subtracted from a number, the result is 56. Write an equation that can be used to find the original number. Then find the original number.

Let x = the original number.

$x-15=56$ is translated from "when 15 is subtracted from a number"

$$\begin{aligned} x-15&=56\\ \underline{+15}&\underline{=+15}\\ x&=71 \end{aligned}$$

The original number is 71.

NVACS 7.EE.B.4a *Solve word problems leasing to equations of the form* $px+q=r$ *and* $p(x+q)=r$ *where* p, q and r *are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.*

Solving 2-Step Equations

The general strategy for solving a multi-step equation in one variable is to rewrite the equation in $ax+b=c$ format, then solve the equation by isolating the variable using the Order of Operations **in reverse** and **using the opposite operation.**

Once again, let's look at a gift wrapping analogy to better understand this strategy. When a present is wrapped, it is placed in a box, the cover is put on, the box is wrapped in paper, and finally a ribbon is added to complete the project. To get the present out of the box, everything would be done in reverse order, performing the OPPOSITE (INVERSE) OPERATION. First we take off the ribbon, then take off the paper, next take the cover off, and finally take the present out of the box.

Order of Operations

1. Parentheses (Grouping)
2. Exponents
3. Multiply/Divide, left to right
4. Add/Subtract, left to right

Evaluating an arithmetic expression using the Order of Operations will suggest how we might go about solving equations in the $ax+b=c$ format.

To evaluate an arithmetic expression such as $4+2\cdot 5$, we'd use the Order of Operations.

$4+\underline{2\cdot 5}=$	First we multiply, $2\cdot 5$
$4+10=$	Second we add, $4+10$
14	

Now, rewriting that expression, we have $2 \cdot 5 + 4 = 14$, a form that leads to equations written in the form $ax + b = c$. If I replace the number 5 with the variable n, we have:

$$2 \cdot n + 4 = 14 \textit{ or}$$
$$2n + 4 = 14,$$
an equation in the $ax + b = c$ format.

To solve that equation, I am going to “undo” the expression “$2n + 4$”. I will isolate the variable by using the Order of Operations in reverse and using the opposite operation.

That translates to getting rid of any addition or subtraction first, then getting rid of any multiplication or division next. Undoing the expression and isolating the variable results in finding the value of n.

This is what it looks like:

$2n + 4 = 14$		$2n + 4 = 14$
$2n + 4 - 4 = 14 - 4$	subtract 4 from each side to "undo" the addition	$\underline{\quad -4 = -4}$
$2n = 10$		$2n = 10$
$\dfrac{2n}{2} = \dfrac{10}{2}$	divide by 2 to "undo" the mulitplication	$\dfrac{2n}{2} = \dfrac{10}{2}$
$n = 5$		$n = 5$

Check your solution by substituting the answer back into the original equation.

$2n + 4 = 14$	original equation
$2(5) + 4 = 14$	substitute '5' for 'n'
$10 + 4 = 14$	
$14 = 14$	true statement, so my solution is correct

Example: Solve for x, $3x - 4 = 17$.

Using the general strategy, we always want to “undo” whatever has been done in reverse order. We will undo the subtracting first by adding, and then undo the multiplication by dividing.

$$
\begin{aligned}
3x-4&=17\\
3x-4+4&=17+4\\
3x&=21\\
\frac{3x}{3}&=\frac{21}{3}\\
x&=7
\end{aligned}
\qquad or \qquad
\begin{aligned}
3x-4&=17\\
\underline{+4}&\underline{=+4}\\
3x&=21\\
\frac{3x}{3}&=\frac{21}{3}\\
x&=7
\end{aligned}
$$

Check:
$$
\begin{aligned}
3x-4&=17\\
3(7)-4&=17\\
21-4&=17\\
17&=17\checkmark
\end{aligned}
$$

Example: Solve for *x*, $\frac{x}{4}+5=12$

$$
\begin{aligned}
\frac{x}{4}+5&=12\\
\frac{x}{4}+5-5&=12-5\\
\frac{x}{4}&=7\\
(4)\left(\frac{x}{4}\right)&=(4)(7)\\
x&=28
\end{aligned}
\qquad or \qquad
\begin{aligned}
\frac{x}{4}+5&=12\\
\underline{-5}&\underline{=-5}\\
\frac{x}{4}&=7\\
(4)\left(\frac{x}{4}\right)&=(4)(7)\\
x&=28
\end{aligned}
$$

Check:
$$
\begin{aligned}
\frac{x}{4}+5&=12\\
\frac{28}{4}+5&=12\\
7+5&=12\\
12&=12\checkmark
\end{aligned}
$$

*****NOTE:** Knowing how to solve equations in the $ax+b=c$ format (or as our standards refer to it as $px+q=r$), is extremely important for success in algebra. All other equations will be solved by converting equations to $ax+b=c$. To solve systems of equations, we rewrite the equations into one equation in the $ax+b=c$ form and solve. (In the student's algebraic future, to solve quadratic equations we will rewrite the equation into factors using $ax+b=c$, then solve the resulting equation letting $c=0$. It is important that students are comfortable solving equations in the $ax+b=c$ format.)

Example: Solve $-10+6g=110$

$$-10+6g=110$$
$$-10+10+6g=110+10$$
$$6g=120$$
$$\frac{6g}{6}=\frac{120}{6}$$
$$g=20$$

Check:
$$-10+6(20)=110$$
$$-10+120=110$$
$$110=110$$

Example: Solve $4(x-7)=12$

Using the Distributive Property.

$$4(x-7)=12$$
$$4(x)-4(7)=12$$
$$4x-28=12$$
$$4x-28+28=12+28$$
$$4x=40$$
$$\frac{4x}{4}=\frac{40}{4}$$
$$x=10$$

OR

Using the inverse of order of operations

$$4(x-7)=12$$
$$\frac{4(x-7)}{4}=\frac{12}{4}$$
$$x-7=3$$
$$x-7+7=3+7$$
$$x=10$$

Check:
$$4(10-7)=12$$
$$4(3)=12$$
$$12=12$$

Example: Solve $c+18+3c=74$

$$c+18+3c=74$$
$$4c+18=74$$
$$4c+18-18=74-18$$
$$4c=56$$
$$\frac{4c}{4}=\frac{56}{4}$$
$$c=14$$

Check:
$$14+18+3(14)=74$$
$$14+18+42=74$$
$$74=74$$

Example: Solve $2x-5+3x=10$

$$2x-5+3x=10$$
$$5x-5=10$$
$$5x-5+5=10+5$$
$$5x=15$$
$$\frac{5x}{5}=\frac{15}{5}$$
$$x=3$$

Check:
$$2(3)-5+3(3)=10$$
$$6-5+9=10$$
$$1+9=10$$
$$10=10$$

Example: Solve

$$5(p-2)+36=-4$$
$$5p-5(2)+36=-4$$
$$5p-10+36=-4$$
$$5p+26=-4$$
$$5p+26-26=-4-26$$
$$5p=-30$$
$$\frac{5p}{5}=\frac{-30}{5}$$
$$p=-6$$

Check:

$$5(-6-2)+36=-4$$
$$5(-8)+36=-4$$
$$-40+36=-4$$
$$-4=-4$$

Example: Solve $15=-7+\frac{x}{3}$

$$15=-7+\frac{x}{3}$$
$$15+7=-7+\frac{x}{3}+7$$
$$22=\frac{x}{3}$$
$$22\cdot 3=\frac{x}{3}\cdot 3$$
$$66=x$$

OR

Clear the fractions by multiplying by the LCD on both sides.

$$15=-7+\frac{x}{3}$$
$$3(15)=3\left(-7+\frac{x}{3}\right)$$
$$45=3(-7)+3\left(\frac{x}{3}\right)$$
$$45=-21+x$$
$$45+21=-21+x+21$$
$$66=x$$

Check:

$$15=-7+\frac{66}{3}$$
$$15=-7+22$$
$$15=15$$

Example: Solve $\frac{r}{7}+\frac{1}{2}=\frac{5}{14}$

$$\frac{r}{7}+\frac{1}{2}=\frac{5}{14}$$
$$\frac{r}{7}+\frac{1}{2}-\frac{1}{2}=\frac{5}{14}-\frac{1}{2}$$
$$\frac{r}{7}=\frac{5}{14}-\frac{7}{14}$$
$$\frac{r}{7}=\frac{-2}{14}$$
$$\frac{7}{1}\left(\frac{r}{7}\right)=\left(\frac{-2}{14}\right)\frac{7}{1}$$
$$r=\frac{-2}{2}$$
$$r=-1$$

OR

Clear the fractions by multiplying by the LCD on both sides.

$$\frac{r}{7}+\frac{1}{2}=\frac{5}{14}$$
$$14\left(\frac{r}{7}\right)+14\left(\frac{1}{2}\right)=14\left(\frac{5}{14}\right)$$
$$14\left(\frac{r}{7}+\frac{1}{2}\right)=\left(\frac{5}{14}\right)14$$
$$2r+7=5$$
$$2r+7-7=5-7$$
$$2r=-2$$
$$\frac{2r}{2}=\frac{-2}{2}$$
$$r=-1$$

Check:

$$\frac{-1}{7}+\frac{1}{2}=\frac{5}{14}$$
$$\frac{-2}{14}+\frac{7}{14}=\frac{5}{14}$$
$$\frac{5}{14}=\frac{5}{14}$$

Example: Write an equivalent equation that does not contain fractions for $\frac{1}{2}x - 5 = \frac{1}{3}$.

Example: Translate the statement into an equation. Then solve the equation.

The sum of 8 and three times a number is 23.

A.	$3x - 8 = 23$; 5
B.	$8+3x = 23$; 5
C.	$3x = 8+23$; 10
D.	$(8+3)x = 23$; 2

Application Problems

Example: Between the hours of 6 a.m. and 9 p.m., 8 buses that were filled to capacity left the terminal. Since the capacity of each bus is the same and 392 tickets were sold, how many passengers were on each bus?

Solution: *Let p = the number of passengers on each bus*
We know the total is 392 tickets.
There were 8 buses.

$$8p = 392$$
$$\frac{8p}{8} = \frac{392}{8}$$
$$p = 49$$

49 *passengers per bus*

Example: The area of a rectangle is 42 square meters. Its width is 7 meters. What is its length?

Let *l* represent the length.

$A = l \times w$
$42 = l \times 7$

is translated from "the area of the rectangle is 42" and "width is 7"

$$42 = l \times 7$$
$$\frac{42}{7} = \frac{l \times 7}{7}$$
$$6 = l$$

The length of the rectangle is 6 meters.

Emphasize to students that they need to include the label "meters".

Example: In a fish tank, $\frac{8}{11}$ of the fish have a red stripe on them. If 16 of the fish have red stripes, how many total fish are in the tank?

A.	20 fish
B.	21 fish
C.	22 fish
D.	26 fish

Example: You are working as an assistant to a chef. The chef made $\frac{3}{4}$ of a recipe and used 20 cups of milk. He wrote the following equation to show how many cups of milk he would use if he made the whole recipe. $\frac{3}{4} = 20x$ Did he write the equation correctly? If not, write the equation as it should be.

Example: Black rhinos usually live to be 50 years old. A black rhino at the zoo is 23 years old. Write an equation and solve it to show how many years longer the black rhino will most likely live.

Example: Two stacks of books together are $7\frac{1}{2}$ inches tall. If one stack is $4\frac{3}{4}$ inches tall, how tall is the other? Write an equation and solve it.

Example: Sami is inviting her friends to the roller-skating rink for her birthday party.

Part A: If the skating rink charges \$6 per person and the birthday party will cost \$72, how many skaters will be at Sami's party? Use an equation to solve and check your answer.

Part B: What will be the new cost of the birthday party if each skater receives \$2 spending money? Check your answer.

Example: A car rental costs \$30.00 per day plus an additional \$0.25 per mile. What is the cost of renting a car for one day and driving 50 miles?

Example: Margie earns 1.5 times her normal hourly rate for each hour over 40 hours in a week. She worked 60 hours this week and earned \$840. What is her normal hourly rate?

Example: Jason went to the amusement park and sent \$28.00. The cost to ride each ride is \$1.75. He spent \$7.00 on food and all the rest on rides. How many rides did he go on?

Example: A rectangle has a length of $x+4$ centimeters and width of x centimeters. The perimeter of the rectangle is 28 centimeters. What is the value of x?

Example: Hector is an electrician. He charges an initial fee of \$24, plus \$33 per hour. If Hector earned \$189 on a job, how long did the job take?

Example: Stacey just bought a new television for \$629.00. She made a down payment of \$57.00 and will pay monthly payments of \$26.00 until it is paid off. How many months will Stacey be paying? (Assume that that no interest is charged.)

Example: Members at a dance school pay \$7 per class plus a one-time \$120 membership fee. If Mia paid a total of \$302 dollars for the year, how many classes did she attend?

Example: Keith is given the following math problem to solve.

Enya has \$160 in her bank account. During March, she earns \$7.50 per hour baby-sitting. After depositing her earnings in the account, she now has \$340. How many hours did Enya bay-sit in March?

Keith used the numeric method below to solve the problem. Which algebraic solution matches his numeric method?

1) Subtract 160 from 340 to find the amount Enya earned during March.

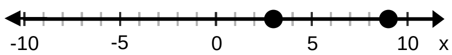
2) Divide the result by 7.5 to find the number of hours Enya spent baby-sitting.

A.	$160+7.5h = 340$ $-160 \quad = -160$ $7.5h = 180$ $h = \dfrac{180}{7.5}$	C.	$h+(7.5)160 = 340$ $-160 = -160$ $h+7.5 = 180$ $h = \dfrac{180}{7.5}$
B.	$7.5h-160 = 340$ $+160 = +160$ $7.5h = 500$ $7.5h = \dfrac{500}{7.5}$	D.	$160h+7.5 = 340$ $-160 \quad = -160$ $h+7.5 = 180$ $h = \dfrac{180}{7.5}$

Prep for 7.EE.B.4b

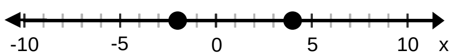
Inequalities

Example: Write an inequality for the two positive numbers on the number line below:



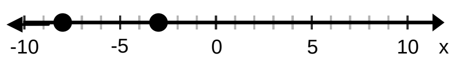
In addition to being able to write $3 < 9$ and $9 > 3$, students should be able to state that 3 is less than 9, 3 is located to the left of 9, and/or 9 is greater than 3 and 9 is located to the right of 3.

Example: Write an inequality for the numbers on the number line below.



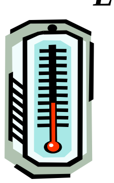
In addition to being able to write $-2 < 4$ and $4 > -2$, students should be able to state that -2 is less than 4, -2 is located to the left of 4, and/or 4 is greater than -2 and 4 is located to the right of -2.

Example: Write an inequality for the numbers on the number line below.



In addition to being able to write $-8 < -3$ and $-3 > -8$, students should be able to state that -8 is less than -3, -8 is located to the left of -3, and/or -3 is greater than -8 and -3 is located to the right of -8.

Example:



Given 2 thermometer readings (or vertical number lines) students should be able to interpret which temperature is colder/lower, write statements that compare the temperatures, and write inequalities to represent the situation.

For example, given two thermometers, one reading $-5°$ Fahrenheit and the second reading $3°$ Fahrenheit, students should be able to state $-5 < 3$, $3 > -5$, $3°$ is warmer than $-5°$ and $-5°$ is colder than $3°$

Example: What statement is true?

A.	$-13>11$
B.	$-8<-2$
C.	$-1<-19$
D.	$-5>16$

Solution: B

Example: The level of the top of the water in the ocean is considered to be an altitude of zero (0) feet. The ocean floor at a particular dive site is -25 feet. A diver at the site is located at -8 feet.

Write an inequality that represents the relationship between the location of the diver to the dive site. Solution: $-8>-25$ *or* $-25<-8$

Interpret/Explain in words the location of the diver to the dive site.

Solution: Although answers may vary, several possible solutions are:

The diver is closer to the top of the water than the dive site is to the top of the water. **OR**
The dive site is below the diver. **OR**
The diver is above the dive site. **OR**
The diver is descending to the dive site.

Prep for 7.EE.B.4b

Solving One-Step Inequalities

We use inequalities in real life all the time. Let's say you are going to purchase a \$2 candy bar and you do not have to use exact change. How would you list all the amounts of money that are enough to buy the item? You might start a list: \$3, \$4, \$5, \$10; quickly you would discover that you could not list all possibilities. However, you could make a statement like "any amount of money \$2 or more" and that would describe all the values.
In algebra, we use inequality symbols to compare quantities when they are not equal, or compare quantities that may or may not be equal.

This symbol	means	and can be disguised in word problems as
$<$	is less than	below, fewer than, less than
$>$	is greater than	above, must exceed, more than
$\leq$	is less than or equal to	at most, cannot exceed, no more than
$\geq$	is greater than or equal to	at least, no less than

An ***inequality*** is a mathematical sentence that shows the relationship between quantities that are *not* equal. For example, m>5, $2x<8$ and 4x $-7\geq 35$ are inequalities.

Our strategy to solve inequalities will be to isolate the variable on one side of the inequality and numbers on the other side by using the opposite operation (same as equations).

Example: Solve the inequality for x: $3x \le 27$.
Isolate the variable by dividing both sides by 3 of the inequality

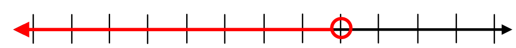
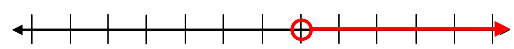
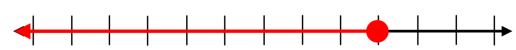
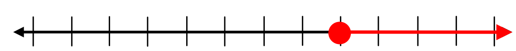
$$3x \le 27$$
$$\frac{3x}{3} \le \frac{27}{3}$$
$$x \le 9$$

Example: Solve the inequality for y: $y + 6 > 10$.
Isolate the variable by subtracting 6 from both sides of the inequality.

$$y + 6 > 10$$
$$y + 6 - 6 > 10 - 6$$
$$y > 4$$

Graphing Solutions of Equations and Inequalities in One Variable

The *solution* of an inequality with a variable is the set of all numbers that make the statement true. You can show this solution by graphing on a number line.

inequality	in words	graph
$x < 2$	all numbers less than two	
$x > 1$	all numbers greater than one	
$x \le 3$	all numbers less than or equal to three	
$x \ge 2$	all numbers greater than or equal to two	

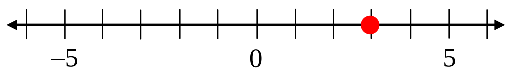
Note that an open circle ○ *is used in the "is less than" or "is greater than" graphs, indicating that the number* <u>*is not*</u> *included in the solution. A closed circle* ● *is used in the "is greater than or equal to" or "is less than or equal to" graphs to indicate that the number* <u>*is*</u> *included in the solution.*

We can solve linear inequalities the same way we solve linear equations. We use the Order of Operations in reverse, using the opposite operation. Linear inequalities look like linear equations with the exception they have an inequality symbol ($<$, $>$, $\le$, or $\ge$) rather than an equal sign.

Note: We will be graphing our solutions using the set of real numbers.

Example: Linear Equation:

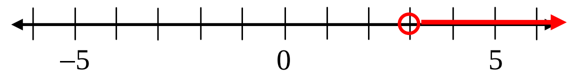
$$x-2=\ 1$$
$$x-2{\color{red}+2}=1{\color{red}+2}$$
$$x=\ 3$$



Notice the graph on the left only has the one point representing 3 plotted; one solution. That translates to $x=3$.

Linear Inequality:

$$x-2>1$$
$$x-2\ {\color{red}+2}>1{\color{red}+2}$$
$$x>3$$

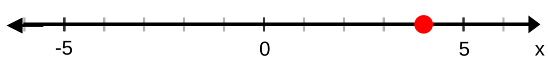


Notice the graph on the right has a dot on 3 which is not shaded because 3 is not part of the solution. Also notice that there is a solid line to the right of the open dot representing all values greater than 3 are part of the solution.

Example:

Linear Equation:

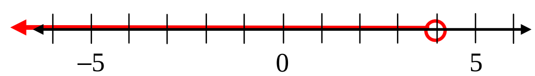
$$x+3=\ 7$$
$$x+3{\color{red}-3}=7{\color{red}-3}$$
$$x=\ 4$$



The graph on the left has a point plotted at 4 indicating there is only one solution. That translates to *x* = 4.

Linear Inequality:

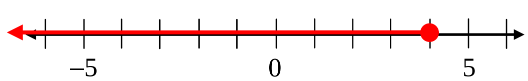
$$x+3<7$$
$$x+3{\color{red}-3}<7{\color{red}-3}$$
$$x<4$$



The graph on the right has an open circle on 4, which is *not* shaded because 4 is not included as part of the solution. Also notice that there is a solid line to the left of the open dot, representing all the values less than 4 are part of the solution set.

If the last example contained the symbol "$\le$", $x+3\le 7$, then everything would be done the same in terms of solving the inequality, but the solution and graph would look a little different. It would include 4 as part of the solution set (closed/shaded circle) and all the values less than 4 are part of the solution set.

The solution $x \le 4$.



Example: <u>Solve</u> the inequality for t: $t-6<9$ <u>and graph</u> the solution.

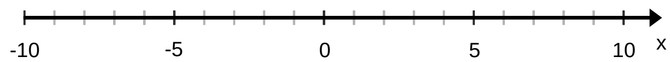
Solution: Isolate the variable by adding 6 to both sides of the inequality.

$$t-6<\ 9$$
$$t-6{\color{red}+6}=9{\color{red}+6}$$
$$t<15$$

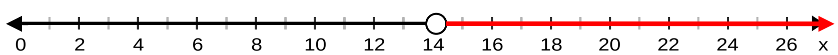
Example: <u>Solve</u> the inequality for z: $z+8\le 9$ <u>and graph</u> the solution.

Solution: Isolate the variable by subtracting 8 to both sides of the inequality.

$$z+8\le 9$$
$$z+8-8=9-8$$
$$z\le 1$$

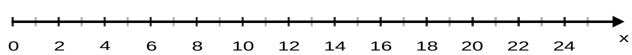
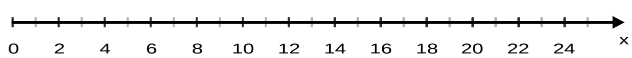
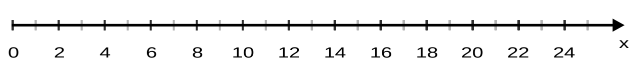
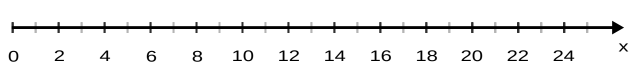


Example: Write an inequality that represents the graph below if the endpoint is moved 5 units to the left.



A.	$x>14$
B.	$x<14$
C.	$x>9$
D.	$x>17$

Example: Dominica made more than \$8 babysitting yesterday. Her sister Tamara made \$13 more than Dominica did. Write an inequality to represent the amount of money Tamara could have made. Graph the inequality on the number line. Could \$15 be the amount of money Tamara made?

A.	$T>8$  Yes; 15 is greater than 8.
B.	$T>13$  Yes; 15 is greater than 13.
C.	$T>21$  No; 15 is not greater than 21.
D.	$T\ge 21$  No; 15 is not greater than or equal to 21.

Example: What is the greatest possible integer solution of the inequality $8.904b < 18.037$?

A.	2.03
B.	1
C.	2
D.	3

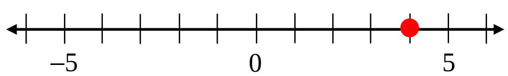
NVACS 7.EE.B.4b *Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$ where p, q and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.*

We can solve multi-step linear inequalities the same way we solve multi-step linear equations. We use the Order of Operations in reverse, using the opposite operation. Linear inequalities look like linear equations with the exception they have an inequality symbol ($<$, $>$, $\le$, or $\ge$) rather than an equal sign.

Example:

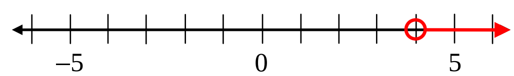
Linear Equation:

$$3x - 2 = 10$$
$$+2 = +2$$
$$3x = 12$$
$$\frac{3x}{3} = \frac{12}{3}$$
$$x = 4$$



Linear Inequality:

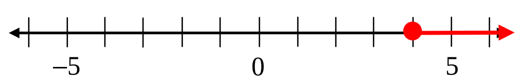
$$3x - 2 > 10$$
$$+2 = +2$$
$$3x > 12$$
$$\frac{3x}{3} > \frac{12}{3}$$
$$x > 4$$



Notice the graph on the left only has the point representing 4 plotted. That translates to $x = 4$. The graph on the right has a dot on 4, which is not shaded because 4 is not included as part of the solution. Also notice that all there is a solid line to the right of the open dot, representing all the numbers greater than 4 that are part of the solution set.

If the inequality contained the symbol "$\ge$", $3x - 2 \ge 10$, then everything would be done the same in terms of solving the inequality, except the answer and graph would look a little different. It would include 4 as part of the solution set. To show 4 was included, we would shade it.

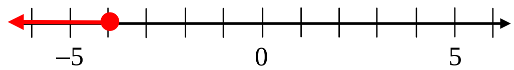
The solution x ≥ 4.



What would happen if we had an inequality such as 6 > 5 and multiplied both sides of the inequality by (–2)? In other words is $(-2)(6) > (-2)(5)$?

Multiplying, would the result –12 > –10 be true? The answer is no. Notice that –12 would be to the left of –10 on the number line. Numbers to the left are smaller. When we multiply or divide by a negative number, to make the statement true, the inequality sign must be reversed, i.e., –12 < –10.

Example: Find the solution set for $-3x-2>10$.



$$\begin{aligned} -3x-2&>10 \\ +2&=+2 \\ -3x&>12 \\ \frac{-3x}{-3}&<\frac{12}{-3} \\ x&<-4 \end{aligned}$$

Notice that when we divided by a negative number, we reversed the inequality symbol.

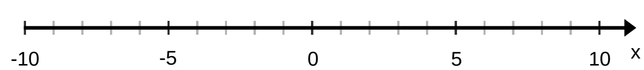
RULE: When you multiply or divide an inequality by a negative number, you reverse the sign of the inequality.

We could have done the same problem without multiplying or dividing by a negative number by keeping the variable positive. That would result in not having to reverse the inequality. Let's look at the same problem.

Example: Find the solution set for $-3x-2>10$ and graph the result.

Rather than having the variables on the left side and have the coefficient negative, I could put the variables on the right side (add 3*x* to both sides). That would result in a positive coefficient.

$$\begin{aligned} -3x-2&>10 && \text{add } 3x \text{ to both sides} \\ +3x\quad&=+3x \\ -2&>3x+10 && \text{subtract 10 from both sides} \\ -10&=-10 \\ -12&>3x && \text{divide both sides by 3} \\ \frac{-12}{3}&>\frac{3x}{3} && or\ x<-4. \\ -4&>x \end{aligned}$$



In both cases, we have the same answer. In the second example, since I kept my variable positive, I did not have to reverse the order of the inequality when I divided. Bottom line, you solve linear inequalities the same way you solve linear equations – use the Order of Operations in reverse using the opposite operation.

Remember, if you don't keep the coefficients of the variable positive, then you may have to multiply or divide by a negative number that will require you to change the direction of the inequality sign.

Please note that students often have difficulty recognizing that the answer “ $-4 > x$ ” is the same answer as “ $x < -4$ ”. The conventional way of writing an inequality is with the variable on the left. Have students practice rewriting inequalities in this way.

Example: Rewrite $5 \geq y$ in the conventional way.

If 5 is greater than or equal to all the y values, then y must be less than or equal to 5; i.e., $y \leq 5$.

Example:

$$\begin{aligned}
2x - \cancel{3\tfrac{1}{2}} &= \frac{5}{12} \\
+\cancel{3\tfrac{1}{2}} &= +3\frac{1}{2} \\
\hline
2x &= \frac{5}{12} \\
&\ +3\frac{6}{12} \\
\hline
2x &= 3\frac{11}{12} \\
2x &= \frac{47}{12} \\
\frac{\cancel{2}x}{\cancel{2}} &= \frac{\frac{47}{12}}{2} \\
x &= \frac{47}{12} \bullet \frac{1}{2} \\
x &= \frac{47}{24} = 1\frac{23}{24}
\end{aligned}$$

or

$$\begin{aligned}
2x - 3\frac{1}{2} &= \frac{5}{12} \\
12\left(2x - 3\frac{1}{2}\right) &= 12\left(\frac{5}{12}\right) \\
12(2x) - 12\left(3\frac{1}{2}\right) &= \frac{12}{1} \bullet \frac{5}{12} \\
24x - 42 &= 5 \\
+42 &= +42 \\
\hline
24x &= 47 \\
\frac{24x}{24} &= \frac{47}{24} \\
x &= \frac{47}{24} = 1\frac{23}{24}
\end{aligned}$$

Example: A volleyball team scored 17 more points in its first game than in its third game. In the second game, the team scored 23 points. The total number of points scored was less than 60. What is the greatest number of points the team could have scored in its first game?

Let **x** = the number of points scored in game 3
x +17 = the number of points scored in game 1
23 = the number of points scored in game 2 (given)

$$\boxed{\text{Pts scored in game 1}} + \boxed{\text{Pts scored in game 2}} + \boxed{\text{Pts scored in game 3}} < 60$$

$$\mathbf{x + 17} \quad + \quad \mathbf{23} \quad + \quad \mathbf{x} \quad < \quad 60$$

$$x + 17 + 23 + x < 60$$
$$2x + 40 < 60$$
$$-40 \quad -40$$
$$2x < 20$$
$$\frac{2x}{2} < \frac{20}{2}$$
$$x < 10$$

The greatest number of points the team could score in game 1 is $9 + 17 = 26$ points

Example: The length of a rectangle is 99 centimeters. Find all the possible values for the width of the rectangle if the perimeter is at least 240 centimeters.

OnCore Examples

Example: Which phrase is represented by the inequality 2n+6<10?

A.	Twice the sum of a number and six is less than 10.
B.	Twice the sum of a number and six is greater than 10.
C.	Two times a number plus six is less than ten.
D.	Two times a number plus six is greater than ten.

Example: Which expression is equivalent to $8g + 2 + 4g - 2 - 2g$ **?**

A.	**16g+4**
B.	**10g**
C.	**16g**
D.	**10g+4**

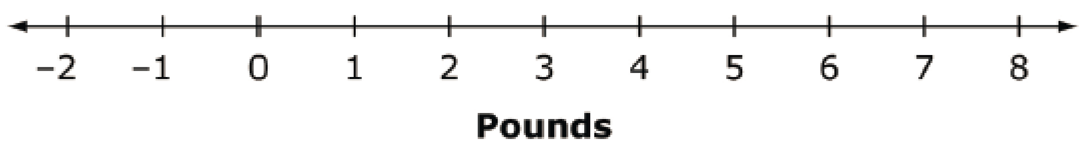
2013 SBAC Examples

David wants to buy 2 pineapples and some bananas.

- The price of 1 pineapple is \$2.99.
- The price of bananas is \$0.67 per pound.

David wants to spend less than \$10.00. Write an inequality that represents the number of pounds of bananas, b, David can buy.

On the number line below, draw a graph that represents the number of pounds of bananas David can buy.



Sample Top-Score Response:

$b < 6$

The graph should be a line segment with an open or closed circle at 0 and an open circle at 6.

Scoring Rubric:

Responses to this item will receive 0–3 points, based on the following:

3 points: The student has thorough understanding of how to solve a real-life problem involving inequalities and how to graph inequalities on a number line. This is shown by the student determining and graphing the solution.

2 points: The student has thorough understanding of how to solve a real-life problem involving inequalities and partial understanding of how to graph inequalities on a number line. This is shown by the student correctly determining the solution but having incorrect endpoint(s) on the graph.

1 point: The student has an understanding of how to solve a real-life problem but limited understanding of how to graph the solution. This is shown by the student determining the solution but making two or more errors in graphing the solution. **OR** The student has an understanding of how to graph inequalities but limited understanding of how to solve a real-life problem involving inequalities. This is shown by the student correctly graphing an incorrect solution to the real-life problem.

0 points: The student shows little or no understanding of how to solve a real-life problem involving inequalities or how to graph inequalities.

2014 SBAC Examples

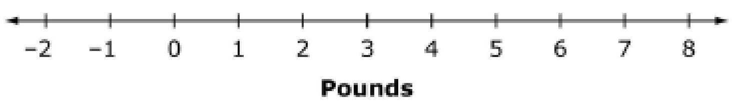
Seen again in 2014…

Example Item 2 (Grade 7):
Primary Target 2C (Content Domain EE), Secondary Target 1D (CCSS 7.EE.4b)

David wants to buy 2 pineapples and some bananas and spend no more than \$10.00.

- The price of 1 pineapple is \$2.99.
- The price of bananas is \$0.67 per pound.

Use the Connect Line or Add Arrow tool to draw a graph that represents all possible values for the number of pounds of bananas, *b*, David can buy.



Interaction: The student is given both the Connect Line and Add Arrow tools to draw an inequality.

Rubric: (1 point) The student is able to correctly graph the inequality defined by the problem (e.g., see below).

Response Type: Graphing

Example Item 2 (Grade 7):
Primary Target 3E (Content Domain EE), Secondary Target 1D (CCSS 7.EE.3)

Shelly incorrectly solves the equation $\frac{1}{2}(c+6)=7$. Her work is shown.

A. Select all the steps that show an error based on the equation in the previous step.

B. Use the Add Point tool to show the correct solution of the given equation.

Delete Add Point

A. $\frac{1}{2}(c+6)=7$

Step 1: $\frac{1}{2}c+6=7$

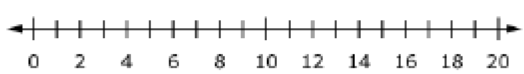
Step 2: $\frac{1}{2}c=7+6$

Step 3: $\frac{1}{2}c=13$

Step 4: $c=13\div 2$

Step 5: $c=6\frac{1}{2}$

B. Correct solution



Rubric: (2 points) The student selects the correct steps and plots the correct point on the number line. (e.g., Part A: 1, 2, 4; Part B: 8).
(1 point) The student either selects the correct steps or plots the correct point on the number line.

Response Type: Hot Spot and Graphing