Math 7 NOTES – Part B: Percent

Prep for 7.RP.A.3

Percents are special fractions whose denominators are 100. The number in front of the percent symbol (%) is the numerator. The denominator is not written, but understood to be 100.

Examples $7\% = \frac{7}{100}$ $14\% = \frac{14}{100}$ $87\% = \frac{87}{100}$

Because a percent is a special fraction, then, just like with decimals, all the rules for percents come from the rules for fractions. That should make you feel pretty good. It's not like we are learning brand new stuff you're not familiar with. To add or subtract percents, you add the numerators and bring down the denominator just like with fractions.

Adding & Subtracting Percents

Example: $34\% + 15\% = 49\%$

Notice I added the numbers in front of the percent symbol, the numerators, and then I brought down the common denominator, the percent symbol.

Oh, yes, this is really, really, really good stuff. Don't you wish that - just sometimes - you could make math difficult. As long as you see the patterns develop and you know your definitions and algorithms, math is just plain easy.

Multiplying Percents

If I wanted to multiply percents, again I would go back to my rules for multiplying fractions. To multiply fractions, you multiplied the numerators, then the denominators. To multiply percents, you do the same thing. Multiply the numerators, then the denominators.

Examples $5\% \times 12\%$

Multiplying the numerators, $5 \times 12 = 60$. Remember, the denominators are not written. They are defined to be 100. Therefore we multiply 100×100, that equals 10,000.

$$5\% \times 12\% = \frac{5}{100} \times \frac{12}{100} = \frac{60}{10,000}$$

Prep for 7.RP.A.3 Converting Percents, Fractions and Decimals

To convert a percent to a fraction, we just use the definition. The number in front of the percent symbol is the numerator, the denominator is 100, and then simplify.

Example Convert 53% to a fraction

$$\frac{53}{100}$$

What if someone asked you to convert percents to decimals, would you do it the same way? Of course.

Example Convert 53% to a decimal ,

$\frac{53}{100}$ but that's a fraction.

How do you divide by 100? Move the decimal point 2 places to the left. So, $53\% = 0.53$.

If we did enough of these, we'd soon realize to convert a percent to a decimal, you move the decimal point 2 places to the left.

Example Convert 3% to a decimal.

Moving the decimal point 2 places to the left, we have 0.03.

Knowing that you convert a percent to a decimal by moving the decimal point 2 places to the left, how would you convert a decimal to a percent? That's right, you'd do just the opposite, move the decimal 2 places to the right and put the percent symbol at the end.

Example Convert 0.34 to a percent.

Move the decimal point 2 places to the right and put a percent symbol at the end. The answer is 34%.

Now, why are we moving the decimal point 2 places? Because the denominator for a percent is 100, two zeros, and we learned shortcuts for multiplying and dividing by powers of 10.

$$0.34 = \frac{34}{100} = 34\%$$

When students first learn these types of problems and try to apply the shortcuts, they get confused as to which direction to move the decimal point. So here's a hint that might help you remember the rules.

To convert a percent to a decimal, the loop on the "d" in "decimal" is on the left, so move the decimal point to the left 2 places.

To convert a decimal to a percent, the loop on the "p" in "percent" is on the right, so move the decimal point to the right 2 places.

Again, those two hints came from patterns we recognized.

Example Convert 63% to a decimal.

The loop on the "d" is on the left, move the decimal point 2 places in that direction. The answer is 0.63.

That's the shortcut; the reason why that works is because 63% means 63/100. Simplifying 63/100 in decimal form is 0.63

Example Convert 0.427 to a percent.

The loop on the "p" is on the right, move the decimal point 2 places in that direction. The answer is 42.7%.

That's the shortcut that allows you to compute the answer quickly. But, shortcuts are soon forgotten, so it's important that you understand why the shortcut works.

Let's see what that would look like if we did not use the shortcut. To convert that to a percent, I have to rewrite that fraction with a denominator of 100.

$$\frac{427}{1,000} = \frac{42.7}{100} = 42.7\%$$

One nice thing about mathematics is the rules don't change. Problems might look a little different, but they are often done the same way. The first example we discussed was converting 6% to a fraction. We said the number in front of the percent symbol was the numerator, the denominator was 100.

$6\% = \frac{6}{100}$ Simplifying, the answer would be $\frac{3}{50}$.

What if I asked you to convert $\frac{1}{4}\%$ to a fraction?

You would do exactly what you did to convert 6% to a fraction. The numerator is the number in front of the percent symbol, the denominator is 100.

By converting to a fraction by the definition of percent, we have

$$\frac{1}{4}\% = \frac{\frac{1}{4}}{100}$$

Simplifying that complex fraction, I'd invert, multiply, and then simplify.

$$\frac{\frac{1}{4}}{100} = \frac{1}{4} \div 100 = \frac{1}{4} \times \frac{1}{100} = \frac{1}{400}$$

Notice, the problems looked different, but we used the same strategy: put the numerator over 100 and simplify. Piece of cake! If you simplified a number of fractional percents, you'd probably see a nice pattern develop that would allow you to simplify them in your head.

Must memorize:

$$33\frac{1}{3}\% = \frac{1}{3} = 0.33\frac{1}{3} \text{ or } 0.\bar{3} \quad \text{and} \quad 66\frac{2}{3}\% = \frac{2}{3} = 0.66\frac{2}{3} \text{ or } 0.\bar{6}$$

Beware: Holt textbooks include converting and ordering fractions, decimals and percents but ordering them is missing in this unit. McDougal Littel only minimally addresses this skill. Teachers will need to supplement for mastery of this skill.

Example Order the numbers from greatest to least.

$$33\frac{1}{3}\%, \quad \frac{3}{10}, \quad 0.34, \quad \frac{7}{20}, \quad 37\%$$

Change each entry to the same form (all fractions, or all decimals or all percents).

$$33\frac{1}{3}\% = 0.33\frac{1}{3} \qquad \frac{3}{10} = 0.3 \qquad 0.34 = 0.34 \qquad \frac{7}{20} = 0.35 \qquad 37\% = 0.37$$

Ordering them from greatest $0.37, \quad 0.35, \quad 0.34, \quad 0.33\frac{1}{3}, \quad 0.30$

Putting them back in the original form

$37\%, \quad \frac{7}{20}, \quad 0.34, \quad 33\frac{1}{3}\%, \quad \frac{3}{10}$

OR, as fractions with a denominator of 100

$$33\frac{1}{3}\% = \frac{33.\bar{3}}{100} \text{ or } \frac{33\frac{1}{3}}{100} \qquad \frac{3}{10}\bullet\frac{10}{10} = \frac{30}{100} \qquad 0.34 = \frac{34}{100} \qquad \frac{7}{20}\bullet\frac{5}{5} = \frac{35}{100} \qquad 37\% = \frac{37}{100}$$

Ordering them from greatest to least $\frac{37}{100}, \quad \frac{35}{100}, \quad \frac{34}{100}, \quad \frac{33\frac{1}{3}}{100}, \quad \frac{30}{100}$

Putting them back in the original form

$37\%, \quad \frac{7}{20}, \quad 0.34, \quad 33\frac{1}{3}\%, \quad \frac{3}{10}$

7.EE.B.3 Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form: convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

As teachers begin their instruction of Percent Application problems, it is imperative that they take the time to teach students to estimate to anticipate/check their answers.

Students should quickly compute 100% of any amount.

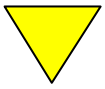
Example 100% of 80 is what?
Since 100% means 1 whole, students should quickly know it is 80.
100% of 80 is 80.

Example 100% of what is 45?
Since 100% means 1 whole, again students should quickly know it is 45.
100% of 45 is 45.

Example What % of 37 is 37?
Since $\frac{37}{37} = 1$, it must be 100%. 100% of 37 is 37.

Once students understand 100%, begin working with 50%. Begin with friendly numbers that students can compute mentally, then gradually build to more difficult numbers.

Example 50% of 80 is what?
Since 50% means $\frac{1}{2}$, students should quickly know $\frac{1}{2}$ of 80 is 40.
50% of 80 is 40.



Work on this skill (50% of a number) until students become proficient, but also remember to include the next 2 forms.

Example 50% of what is 9?
Since 50% means $\frac{1}{2}$, students should figure $\frac{1}{2}$ of what is 9, so $\frac{1}{2}$ of 18 is 9.
50% of 18 is 9.

Example What % of 12 is 6?
Since $\frac{6}{12} = \frac{1}{2}$ or 6 is ½ of 12 students should know $\frac{1}{2} = 50\%$

Students should also be taught how to find 10% mentally.

Example 10% of 400 is what?

Since 10% means 0.1, students should quickly know 0.1 of 400 is 40.

10% of 400 is 40.

10% of $15.00 is what?

Since 10% means 0.1, students should know 0.1 of $15.00 is $1.50.

10% of $15.00 is $1.50.

From here students can then mentally compute problems with 20% by finding 10% and doubling it.

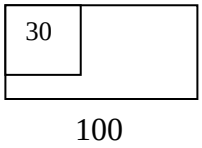
Finding 15% could be computed by finding 10% and adding 1/2 of 10% or 5%.

Estimation is a powerful tool. When students are asked to compute 37% of 597, they can at least estimate what the answer should be. They could quickly estimate this as 1/3 of 600 to get about 200 or 40% of 600, by finding 10% of 600 which is 60, then 60 times 4 to get about 240.

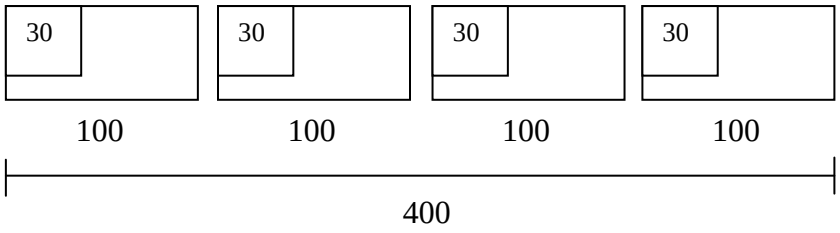
Students should also be given models to help develop their **conceptual understanding** of percent problems. The following type of model helps students visualize percent problems.

Example *30% of 400 is what?*

Since 30% means 30 out of every 100 begin modeling with

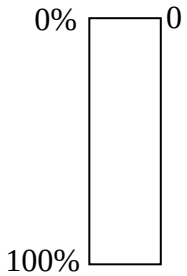


The problem is out of 400 so iterate the model 4 times.



30 + 30 + 30 + 30 = or 30 x 4 = 120 *30% of 400 is 120.*

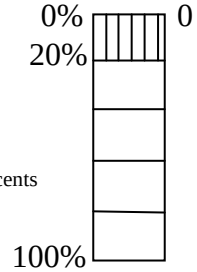
Another approach may be to use percent **bar models**.



Example *20% of what is 5?*

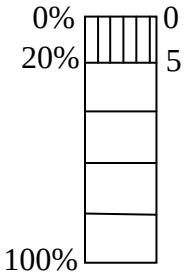
Students begin by creating a rectangle and mark on one side 0% to 100%.

Since 20% is given we subdivide the rectangle to indicate 20%.



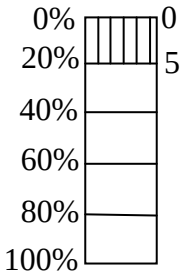
Since we were given 20% of something is 5, we indicate that on our model.

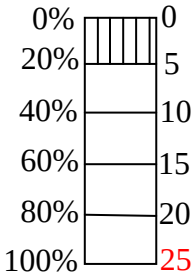
In this case, we are trying to find the number value that corresponds to 100%.



Looking at the model on the percent side, we see the divisions are increments of 20. On the number side the increments are of 5. Counting down we get that 100% is 25.

So, 20% of 25 = 5.

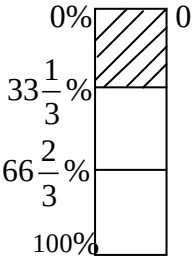




Example $33\frac{1}{3}\%\ of\ 90\ is\ what?$

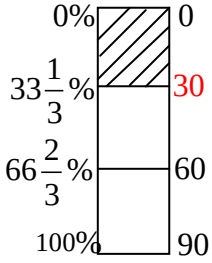
Begin with the basic model.

Since % give is $33\frac{1}{3}\%$ divide the rectangle into thirds.



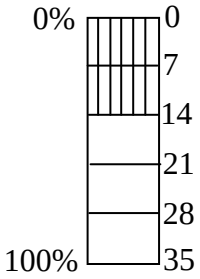
We are given the total is 90. We need to find increments that total 90 that can be divided into 3 equal parts, so 30's.

The shaded region tells us that $33\frac{1}{3}\%$ of 90 is 30.

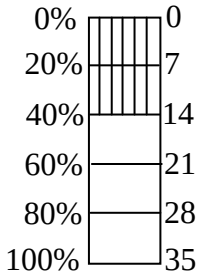


Example *What % of 35 is 14?*

Beginning with the basic model, I must determine the increments to divide my model into. Since both 35 and 14 are divisible by 7, I will make increments of 7. Since $35 \div 7 = 5$, I will divide the rectangle into fifths. I will also shade my model to represent 14 out of the 35.



Finally I need to determine the increments from 0% - 100% with 5 equal parts …so by 20's. As I label my increments I can see, 40% of 35 is 14.



Additional Models/Methods

(Arizona) ***Example***: In 2013, gas prices were projected to increase 124% by April 2015. At the time, a gallon of gas was selling for $4.17. What is the projected cost of a gallon of gas for April 2015?

A student might say "The original cost of a gallon of gas is $4.17. An increase of 100% means that the cost will double. I will also need to add another 24% to get to the projected cost."

100%	100%	24%
$4.17	$4.17	?

Since 25% of $4.17 is about $1.04, the projected cost of a gallon of gas should be around $9.40.

(Arizona) ***Example***: A sweater is marked down 33%. Its original price was $37.50. What is the price of the sweater before sales tax?

37.50 Original Price of Sweater	
33% of 37.50	67% of 37.50 Sale price of sweater

A student might say "The original price of the sweater is 37.50. The sale price is the original price minus the discount or 67% of the original price of the sweater, or Sale Price $=0.67 \times$ original price.

7.RP.A.3 ***Use proportional relationships to solve multistep ratio and percent problems. Examples include simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.***

Percent Proportion

For many of us, a percent is nothing more than a way of interpreting information. We have worked with percents since grade school. In reality, all we are doing is looking at information in terms of a ratio, and then rewriting the ratio so the denominator is 100.

For instance, let's say you got 8 correct out of 10 problems on your quiz. To determine your grade, your teacher would typically want to know how well you have performed if there were 100 questions.

In other words, they would set up a proportion like this: $\frac{\#\,correct}{total} = \frac{?}{100}$

Filling in the numbers, I have $\frac{8}{10} = \frac{\quad}{100} \rightarrow \frac{8}{10} = \frac{80}{100}$

Getting 8 out of 10, I'd expect to get 80 out of 100.
Notice the right side is a fraction whose denominator is 100, just as we defined a percent.

Example Let's say you made 23 out of 25 free throws playing basketball. I might wonder how many shots I would expect to make at that rate if I tried 100 shots.

Again, I have a ratio $\frac{attempts}{total} = \frac{\quad}{100}$

$$\frac{23}{25} = \frac{\quad}{100} \rightarrow \frac{23}{25} = \frac{92}{100}$$

Now I could solve that by making equivalent fractions (and reducing if possible)or by cross-multiplying. Either way, the missing numerator is 92. I would expect to make 92 free throws out of 100 tries.

These problems are just like the ratio and proportion problems we have done before. **Be sure to link/make the connection to all the proportion work you just did in Unit 5.** The only difference is the denominator on the right side will always be 100 because we are working with percents.

A proportion that always has the denominator of 100 on the right side is called the **Percent Proportion.**

PERCENT PROPORTION

$$\frac{part}{total} = \frac{\%}{100}$$

Remembering that you have to describe the ratios the same way on each side of a proportion, we might think this should read.

$$\frac{part}{total} = \frac{part}{total}$$

Well, the percent ratio actually does compare parts to total on both sides. For a percent, the total is always 100 and the percent is always the part you got.

The point I want to make is we have consistency with the math we have already learned. Now for the good news: we can use the percent proportion to solve just about any problem involving percents. **Memorize it!**

$$\frac{part}{total} = \frac{\%}{100}$$

Speaking mathematically, the 100 always goes on the bottom right side. That's a constant. The only things that can change is the part, total or percent. You get that information by reading the problem and placing the numbers in the correct spot, and then solving.

There are only 3 different problems, we can look for a part, a total or a percent. Let's go for it.

Example 1 Bob got 17 correct on his history exam that had 20 questions. What percent grade did he receive?

$\frac{part}{total} = \frac{\%}{100}$ filling in the numbers, $\frac{17}{20} = \frac{\quad}{100}$ $\overset{\times 5}{\frac{17}{20}} = \frac{\quad}{100}$ $\underset{\times 5}{}$

Solving, either by equivalent fractions or by cross-multiplying, we find he made an 85%. In this problem we found a **percent**.

Example 2 A company bought a used typewriter for $320, which was 80% of the original cost. What was the original cost?

Now does the $320 represent the total or part? $\frac{320}{n} = \frac{80}{100}$

The original cost of the typewriter is $400. In this problem we found the **total**.

Example 3 A real estate broker receives 4% commission on an $80,000 sale. How much would he receive?

Does the $80,000 represent the part or total?

$\frac{n}{80,000} = \frac{4}{100} \rightarrow \overset{\times 800}{\frac{n}{80,000}} = \frac{4}{100} \rightarrow 4 \times 800 \rightarrow n = 3,200$ $\underset{\times 800}{}$

He would receive $3,200 in commission. Here, we found the **part**.

While the first three examples were all percent problems and we used the percent proportion to solve them, in each case we were looking for something different. That's the beauty of the percent proportion.
In this next example, everything we learned stays the same, but there is a slight variation in how the problem is written. To do this problem, you must understand how proportion problems are set up.

Example 4 Ted got an 88% on his science test. If there were 50 questions, how many did he get wrong?

This problem gives us the % correct on a test, but asks us to find the number wrong or incorrect. Although this can be solved in other ways, let's see if we can set and solve the proportion for the # wrong. This technique will help on the following examples that CANNOT be done in other ways.

If Ted got an 88% on the test he got

100%
- 88% correct
12% wrong

Setting up the proportion, you put $\frac{the\ \%\ of\ \ wrong\ answers}{100\%} = \frac{the\ number\ of\ \ wrong\ answers}{total\ questions}$

Setting up the proportion $\frac{12}{100} = \frac{n}{50}$

$\div 2$

Solving $\frac{12}{100} = \frac{n}{50}$ $n = 6$ Ted got 6 questions wrong out of 50.

$\div 2$

Example 5 After a person receives a 20% raise, his salary is \$9,600. What was his old salary?

Again the trick to this question is that we are told a person gets a raise and then we are given the salary with the raise included. The only way to approach this problem correctly is to realize we must use the % that includes the raise.

100% original salary
+20% raise
120% original salary w/ raise

Setting up the proportion $\frac{120}{100} = \frac{9600}{n}$ $\rightarrow$ $\frac{120}{100} = \frac{9600}{n}$ OR $\frac{6}{5} = \frac{9600}{n}$

$\times 80$ $\times 80$ $\times 1,600$ $\times 1,600$

$n = 8,000$ His old salary was \$8,000.

Example 6 Dad purchased a radio that was marked down 20% for a price of $68.00. What was the original cost of the radio?

Setting up the proportion, does $68 represent the part or total? It's the part paid.

Filling in the proportion,

$$\frac{paid}{total} = \frac{\%}{100}$$

This is very, very important, the $68 represents the part you paid, what does the 20% represent? That's the part you got off.

We cannot have a proportion with **paid is to total** as **amount off is to total**. If Dad received 20% off, we have to have the same ratio on both sides. That is paid to total as paid to total. If he got 20% off, what percent did he pay? 80%

Now, filling in the numbers, we have $\frac{68}{n} = \frac{80}{100}$ OR $\frac{68}{n} = \frac{4}{5}$ (×17 on the numerator, ×17 on the denominator)

Solving, we have $80n = 6{,}800$

$n = 85$ The original cost was $85.00.

We were able to solve 3 different type problems using the Percent Proportion. We solved for the part, total, and percent by using what we learned in ratios and proportions earlier.

Sales Tax

Sales tax is a *tax* imposed by the government at the point of *sale* on retail goods and services. It is collected by the retailer and passed on to the state. The **sales tax rate** in Clark County, Nevada is 8.1%.

Examples: Compute the tax for each of the following.

	A.	**B.**	**C.**	**D.**	**E.**
Cost	$50.00	$48.00	102.00	$35.50	$7.95
Tax rate	7%	6%	8%	5%	10%
Tip					

A. $\frac{n}{50} = \frac{7}{100}$ (÷2 on the numerator, ÷2 on the denominator) $n = 3.5$ $3.50

B. $\frac{n}{48} = \frac{6}{100}$ $100n = 288$ $n = 2.88$ $2.88

C. $\frac{n}{102} = \frac{8}{100}$ $100n = 816$ $n = 8.16$ $8.16

D. $\frac{n}{35.50} = \frac{5}{100}$ $100n = 177.50$ $n = 1.7750$ $1.78

E. $\frac{n}{7.95} = \frac{10}{100}$ $100n = 79.5$ $n = 0.795$ $0.80

Example: Marco bought a large flat screen TV for his family. The TV sold for $2,400. How much sales tax was he required to pay if the tax rate in the county where he lived was 7%?

$\frac{}{2400} = \frac{7}{100}$ $\frac{}{2400} = \frac{7}{100}$ ($\times 24$ on both sides) $x = 168$ The sales tax on the TV is $168

Example: A new car at the Zyzzx dealer cost $35,800. Compute the sales tax that would be due if the tax rate was 8.1%.

Gratuities/Tips

Gratuity, or **tip**, is a gift of money for services rendered. It is given freely to waitresses, waiters, valets, maids, hair stylists, etc. The **rate of gratuity** for good service is usually 15 – 20% of the cost of the meal or service.

Example: The Jackson family eats out at a restaurant each Friday. The total cost of the meal was $36. Mr. Jackson wants to leave a 20% tip. Compute the tip and the total cost for the meal.

Example: Ava visited her stylist and had the following services:
Cut……$35, Color……$25, Wash……$5, Highlights….$15. If she wants to leave the stylist an additional 20% for a tip, how much was her total bill?

Example: The Lemming family went to Trivoli Restaurant and ordered off the menu below.
Mr. Lemming ordered tilapia, mashed potatoes and broccoli.
Mrs. Lemming ordered fried shrimp, baked potato and green beans.
Katy Lemming ordered a cheeseburger, fries and carrot cake.
Jack Lemming ordered the kid's pizza.
Compute the total bill including an 8% sales tax and a 15% tip on the food only.

Commissions/Royalties/Profits

Commission is the amount paid to a salesperson, often in addition to a regular salary, for selling an item or service. The **rate of commission** is generally a percent of the value of the sales that the person makes.

Royalty is an amount paid to the creator or owner of a musical or literary work, an invention, or a service. The **royalty rate** is a percent of the money earned by the sale of the creation or service.

Example: Ilga sold $4,000 worth of dental supplies to dentists. Her rate of commission is 9%. How much commission did she receive?

$\frac{\quad}{4,000} = \frac{9}{100}$ Solution: $\frac{\quad}{4,000} \overset{\times 40}{=} \frac{9}{100}$ She received $360 in commission.

Example: Mr. Jackson received $200 in commission. Her rate of commission was 25%. What were her total sales?

$\frac{200}{} = \frac{25}{100}$

Solution:

$\frac{200}{} = \frac{25}{100}$ (×8 above, ×8 below)

His total sales were $800

Example: Mr Spalding receives $100 a week salary plus 5% of all sales over $5,000. Last week he sold $9,000 worth of groceries to supermarkets. How much did he earn last week?

$9,000$
$- \ 5,000$
$4,000$

Salary + Commission

$\$100 + \left(\frac{}{4,000} = \frac{5}{100}\right)$ (×40 above)

$100 + \left(\frac{}{4,000} = \frac{5}{100}\right)$ (×40 below)

$100 + 200 = 300$

He earned $300 last week

Example: Nina Boldini receives a royalty of 15% of the selling price of each of her CD's. The recording company receives the remaining portion of the selling price. What is the selling price of each album if the company receives $9.52 for each CD sold?

100%
−15% Nina receives
85% Recording company receives

$\frac{9.52}{x} = \frac{85}{100}$

$9.52 \cdot 100 = 85x$

$952 = 85x$

$\frac{952}{85} = \frac{85x}{85}$

$11.2 = x$

Each CD sells for $11.20.

Example: The author of a paperback book on running marathons receives royalty of 75 cents per copy sold. If the royalty rate is 12% of the single copy price, how much money per copy does *not* go to the author?

$\frac{75}{x} = \frac{12}{100} \rightarrow \frac{75}{x} = \frac{3}{25}$ (×25 above, ×25 below)

$6.25 is the total cost of the book.

6.25 selling price of book
− .75 author's royalty
5.50

$5.50 per book does not go to the author (it goes to the publishing company)

Example: The publisher of *Animals Gone Wild* receives $12.60 from each copy sold. The remaining portion of the $15 selling price goes to the author. What royalty rate does the author earn?

15.00 selling price of the book
−12.60 publisher's portion
2.40 author's portion

$\frac{2.40}{15.00}=\frac{x}{100}$ *or* $\frac{240}{1500}=\frac{x}{100}$ (÷15 above, ÷15 below) $x=16\%$ The author receives a 16% royalty rate.

*****Example:** In May, Sal's bakery had operating costs of $6,630 and made a profit of $1,170. In June, the operating costs are expected to be $6,273. What must the bakery's income be if its profit is to remain the same percent of its income?

$7,380

Percent of Change/Percent Increase or Decrease/Markups or Markdowns

Percent of change is the amount, written as a percent, that a number increases or decreases.

$$\text{Percent of change} = \frac{\text{amount of change}}{\text{original amount}}$$

Example Ted earned $12 per hour this year and will earn $15 per hour next year. What percent increase will he have in pay?

The amount of change is $3.00. His original pay was $12.

$$\text{Percent of change} = \frac{\text{amount of change}}{\text{original amount}} = \frac{\text{percent of change}}{100} \rightarrow \frac{3}{12}=\frac{\quad}{100} \rightarrow \frac{1}{4}=\frac{\quad}{100}$$

(×25 above, ×25 below)

Percent of change = $\frac{3}{12}$ *or* $\frac{1}{4}$ *or* .25 This is a 25% increase in pay.

Example Juan manufactures pants and sells them to department stores for $18.00. The department stores marks them up 20% and sells them. How much does the department stores profit on each pair of pants?

$$\frac{20}{100}=\frac{n}{18}$$

Simplifying $\frac{20}{100}=\frac{1}{5}$, so $\frac{1}{5}=\frac{n}{18}$

$$1\times 18 = 5n$$
$$18 = 5n$$
$$3.6 = n$$

The store profit is $3.60 on each pair of pants.

Example The Pep Club was decreased from 15 members to 12. What percent decrease was there in the club?

The amount of change is $15-12=3$. The original amount was 15.

$\frac{3}{15}=\frac{1}{5}=20\%$ There was a 20% decrease in the club.

Discount/Markup and Markdown

Discount or markdown is a decrease in the price of an item.
Markup is an increase in the price of an item.
Rate of discount or **rate of markdown** is the percent that the item will be reduced by.
Sale Price or discounted price is the cost of the item on sale (when the discount has been deducted).
Original Price is the regular price of an item before it is marked up or marked down.

Example: A shirt originally priced at \$15 is on sale for 20% off. What is the discount on the shirt?

$\frac{n}{15}=\frac{20}{100} \rightarrow \frac{n}{15}=\frac{1}{5}$ ($\times 3$ above, $\div 3$ below) $n=3$ \$3.00 discount

Same problem but looking for different part…
Example: A shirt originally priced at \$15 is on sale for 20% off. What is the sale price of the shirt?

100%
−20% discount
80% sales price

$\frac{n}{15}=\frac{80}{100} \rightarrow \frac{n}{15}=\frac{4}{5}$ $n=12$ \$12

****Although students can find the discount, then subtract it from the original price to get the sale price, NVACS places greater emphasis on multiple representations and approaches. Students will be unable to solve the two-starred examples below without developing this skill,

Example: A pair of pants that sold for \$18 is on sale for \$12. What is the discount rate?

18 original price
−12 sale price
6 discount

$\frac{6}{18}=\frac{n}{100} \rightarrow \frac{1}{3}=\frac{n}{100}$ $n=33\frac{1}{3}$ $33\frac{1}{3}\%$

Example: A warm-up suit that sold for \$42.50 is on sale at a 10% discount. What is the sale price?

100%
− 10% discount
90% sale price

$\frac{n}{42.50}=\frac{90}{100} \rightarrow \frac{n}{42.50}=\frac{9}{10}$ $n=37.40$ \$37.40

***Example:** The price of a new scooter was marked up 6% over the previous year's model. If the previous year's model sold for \$7,800, what was the cost of the new scooter?

100% original rate
+ 6% markup
106% new scooter rate

$$\frac{n}{7{,}800} = \frac{106}{100} \quad (\times 78 \text{ on both sides}) \qquad n = 8{,}268 \qquad \$8{,}268$$

****Example:** A microwave was discounted 30%. The sale price is \$490. What was the original price?

100%
–30% discount
70% sale price

$$\frac{490}{n} = \frac{70}{100} \quad (\times 7 \text{ on both sides}) \qquad n = 700 \qquad \$700$$

****Example:** A restaurant raises its price for the salad bar 20% to \$8.40. What did the salad cost originally?

100%
+ 20% increase
120%

$$\frac{8.40}{n} = \frac{120}{100} \rightarrow \frac{840}{n} = \frac{120}{100} \quad (\times 7 \text{ on both sides}) \qquad n = 700$$

The original price was \$7.00.

Fees

Example: Employers are required to pay 15.8% to the Social Security Administration for each paid employee. If an employee's salary is \$5,000 a month, how much does the employer need to pay Social Security?

$$\frac{n}{5000} = \frac{15.8}{100} \quad (\times 50 \text{ on both sides}) \qquad n = 790 \qquad \$790$$

Example: Lawyer John Doan advertises he will assist you with personal injury cases. His contingency fee is \$1,000 plus 33% of the settlement. Lawyer Mark Hench also advertises he will assist you with personal injury cases. His contingency fee is just 40% of the settlement. If you anticipate being awarded about \$10,000 for your case, which lawyer should you hire?.

Part B: What if you anticipate a settlement of \$100,000?

Percent Error

Example: I thought 70 people would turn up to the concert but in fact 80 did. What is my percent error?

$$\begin{array}{r} 80 \\ -70 \\ \hline 10 \end{array}$$

$$\frac{10}{80}=\frac{n}{100} \rightarrow \frac{1}{8}=\frac{n}{100} \rightarrow \begin{array}{l} 100=8n \\ \frac{100}{8}=\frac{8n}{8} \\ 12.5=n \end{array}$$

I was in error by 12.5%.

Example: The report said the carpark held 240 cars, but we counted only 200 parking spaces.

$$\begin{array}{r} 240 \\ -200 \\ \hline 40 \end{array}$$

$$\frac{40}{200}=\frac{n}{100} \quad (\div 2 \text{ on top and bottom}) \qquad n=20$$

I have a 20% error.

7.RP.A.2c Represent proportional relationships by equations.

Example: In one game a quarterback completed 13 out of 25 passes.

What percent of the passes were completed?

$$x\% \quad \bullet \quad 25 \quad = \quad 13$$

$$x\% \bullet 25 = 13$$

$$\frac{x\% \bullet 25}{25}=\frac{13}{25}$$

$$x\% = 0.52$$

$$x = 52\%$$

$$\frac{13}{25} = 25\overline{)13.00} \quad (0.52)$$

52% of the passes were complete

Example: The Eagles have won 6 out of 8 games they played this season.

What percent of this season's games have the Eagles lost?

$$x\% \quad \bullet \quad 8 \quad = \quad 2$$

8 games played
− 6 games won
2 games lost

$$x\% \cdot 8 = 2$$
$$\frac{x\% \cdot 8}{8} = \frac{2}{8}$$
$$x\% = \frac{2}{8} = \frac{1}{4} = 0.25$$
$$x = 25\%$$

25% of the Eagles games were lost

Example: If the ratio of boys to girls in art class is 5 to 7, how many girls are there in class if there are 60 students enrolled?

5 boys to 7 girls; so there are 7 girls out of every 12 kids

$$\frac{7}{12} = \frac{x}{60}$$
$$7 \cdot 60 = 12x$$
$$\frac{7 \cdot 60}{12} = \frac{12x}{12}$$
$$\frac{7 \cdot 5}{1} = x$$
$$35 = x$$

35 girls in art class

Example: A fruit drink recipe requires fruit juice and milk in the ratio of 3:5. What percent of the drink is milk?
3 parts juice and 5 parts milk = 8 parts fruit juice

$$\frac{3}{8} = \frac{x}{100}$$
$$8x = 300$$
$$\frac{8x}{8} = \frac{300}{8}$$
$$x = 62.5 \text{ or } 62\frac{1}{2}$$

The drink is $62\frac{1}{2}\%$ milk.

Example: In Fall 2011, University of Nevada, Las Vegas (UNLV) admissions reported about 27,000 students were registered in classes. About 80% of that total were undergraduate students.

About how many students were graduate level? about 5,400 students

If enrollment increased at the rate of 8% per year for the next few years, what would the enrollment be in Fall 2014? about 34,000 students

At this same rate, what year would UNLV expect to have more than 40,000 students registered? 2017

Example: In 1913, Henry Ford revolutionized the auto industry when he installed the world's first automated assembly line in his Highland Park factory. Car prices dropped as mass production of cars improved. The 1914 price of a Model T was 53% of the 1908 price. The 1925 price was 59% of the 1914 price. Given that the Model T sold for $260 in 1925, what was the price of a Model T in 1908?

about $831

Example: In 2004, in-state-tuition for Michigan residents to attend Michigan State University was $6,188. Suppose the tuition increased by 10% per year. What would be the first year in which the tuition for Michigan residents is more than $10,000?

2010

Example: Of the 77 billion food and drink cans, bottles and jars Americans throw away each year, about 65% of them are cans. To the nearest billion, how many bottles and jars do Americans throw away each year?

A	12 billion
B	25 billion
C	50 billion
D	65 billion

Example: China's area is about 3.7 million square miles. It is on the continent of Asia, which has an area of about 17.2 million square miles. About what percent of the Asian continent does China cover?

A	about 10%
B	about 15%
C	about 25%
D	about 40%

Simple Interest

Interest is the amount of money you earn when others use your money.
Simple interest is the amount of money you earn based only on the principal. (e.g., savings account)

Principal is the amount of money deposited or borrowed.

Simple Interest Formula

Interest = principal × rate × time (in years)
I = prt

Repayment = Principal + interest
R = Principal + (principal × rate × time)
R= p + prt

I = interest
p = principal

r = rate
t = time (in years)

Example: John borrowed $5000.00 from Maria, agreeing to pay her back in four years at 7% simple interest rate. How much will John **pay back** to Maria at the end of four years?

I = prt

I = 5000(.07)(4)

I = $1400.00

or $R = p + prt$

$= 5,000 + 5000(.07)(4)$

John will need to pay back the original $5000 plus $1400.00 in interest or $6400.00

Note that interest must be added to the principal to determine the amount to be paid back.

Balance is the amount of money in an account after you add the earned interest.

Students must be able to work with parts of a year to determine the time. Be sure students know the following conversions:

12 months = 1 year
52 weeks = 1 year
*365 days = 1 year

*note: most applications that involve days use 360 as a "friendlier" number. So, 180 days = ½ year.

Example: Sarah borrowed $6,000 from her parents to buy a car. They agree to let her pay simple interest. The annual rate they agreed to is 12% for 18 months. How much simple interest would she owe?

Students must be able to convert 18 months to 1.5 years BEFORE they set up the problem.

I = Prt
I = 6,000 (0.12)(1.5)
I = 720(1.5)
I=1,080 She would owe $1,080 in interest.

Example: Give the simple interest on each loan at the given annual rate for 1 year, 3 years, and 6 months.

Principal	Rate	Interest for 1 year	Interest for 3 years	Interest for 6 months
$100	8%			
$200	12%			
$5,000	10%			
$400	5%			
$1,500	6%			
$100	8.4%			

Example: Find the length of time for a $1,200 loan at 6% if the simple interest is $144.

Example: Find the length of time for a $500 loan at 8% if the simple interest is $30.

Example: Find the annual rate of interest for a $1,100 loan for 2 years that yielded a simple interest totaling $220.

Example: Find the annual rate of interest for a $1600 loan for $1\frac{1}{2}$ years that yielded a simple interest totaling $120.

Example: Find the total amount to be repaid on a $600 loan at 9% for 3 years.

(OnCore) Example: Carmelo puts $2,200.00 into savings bonds that pay a simple interest rate of 3.4%. How much money will the bonds be worth at the end of 5.5% years?

A.	$7,136.80
B.	$2,552.20
C.	$2.611.40
D.	$411.40

C

(OnCore) Example: A new house costs $260,000.00. Sara wants to buy the house and needs $35,560.00 for a down payment. Sara currently has $28,000.00 in a CD that earns 9% simple interest. How long must she keep the money in the CD account in order to have enough for the down payment on the house?

A.	92.1 years
B.	14 years
C.	3 years
D.	3 months

C

OnCore examples

1. Carmelo puts $2,200.00 into savings bonds that pay a simple interest rate of 3.4%. How much money will the bonds be worth at the end of 5.5 years?
   a. $7,136.80
   b. $2,552.20
   c. $2,611.40
   d. $411.40

2. A new house costs $260,000.00. Sara wants to buy the house and needs $35,560.00 for a down payment. Sara currently has $28,000.00 in a savings account that earns 9% simple interest. How long must she keep the money in the savings account in order to have enough for the down payment on the house?
   a. 92.1 years
   b. 14 years
   c. 3 years
   d. 3 months

3. Li-ming owns an ice cream shop. On Saturday evening from 6:00 P.M. to 7:00 P.M., 80 people entered her shop. On Tuesday from 3:00 P.M. to 4:00 P.M., 17 people entered her shop. What was the percent decrease in customers from the given hour on Saturday to the given hour on Tuesday? If necessary, round your answer to the nearest hundredth of a percent.
   a. 64.75%
   b. 78.75%
   c. 370.59%
   d. 0.79%

4. The price of a train ticket from Orlando to Atlanta is normally $118.00. However, the train company is offering a special 75% discount to children under the age of 16. What is the sale price of a ticket from Orlando to Atlanta for someone under the age of 16?
   a. $29.50
   b. $41.30
   c. $88.50
   d. $206.50

5. 45 is 26% of what number? If necessary, round your answer to the nearest hundredth.
   a. 11.7
   b. 173.08
   c. 1.73
   d. 0.58

6. Roberto farms a tract of land that typically yields 69.6 kilograms of peas. A fertilizer manufacturer claims that its fertilizer will produce a 60% increase in the yield. What total amount of peas should Roberto expect from this tract of land if he applies the fertilizer? If necessary, round your answer to the nearest hundredth.
   a. 27.84 kilograms
   b. 41.76 kilograms
   c. 111.36 kilograms
   d. 4,245.6 kilograms

7. A bank loans a customer $87,000 for a period of 15 years. The simple interest rate of the loan is 9.2%. What is the total amount that the customer will need to pay the bank over the 15 years?
   a. $1,305,000
   b. $12,093,000
   c. $207,060
   d. $120,060

8. Kayleigh wants to buy a car that costs $17,360.00. She deposits $14,000.00 into a savings account that earns 8% simple interest. How long must Kayleigh leave the money in the savings account to be able to buy the car?
   a. 16 years
   b. 4.5 years
   c. 3 years
   d. 3 months

9. Ms. Kuo buys and sells stocks for investors. Ms. Kuo's salary is based on commissions from her stock sales. She receives 4% commission on each class-A security she sells and 3% on each class-B security she sells. If Ms. Kuo sells $90,100 worth of class-A securities and $56,350 worth of class-B securities in 3 months, what will her total salary be for these 3 months?
   a. $3,604.00
   b. $5,294.50
   c. $1,690.50
   d. $10,251.50

OnCore Examples: Short Answer Questions

1. You work for a company that awards a bonus at the end of the year if your company reaches certain goals. The bonus is paid as a percent of your yearly salary. If your yearly salary is $25,400.00 and your bonus is 3.5% of your salary, what will be your total pay for the year? Show your work.

2. You put $1500.00 in a simple interest savings account. Two years later, the total account balance is $1665.00.

   Part A: What is the interest rate of the savings account? Show your work.
   Part B: What will be the total amount of money in the account after 8 years? Show your work.

3. You earn a base salary of $12,500.00 plus a commission of 6% on every sale you make.
   Part A: If you sold $82,200.00, what would be your total pay for the year? Show your work.
   Part B: If you had to pay 22% in taxes, how much of your total pay would be left after taxes? Show your work.

2014 SBAC Examples:

Task Model 5
 	
Response Type: Equation/Numeric
 	
DOK Level 2
 	
7.RP.3
Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*
 	
Evidence Required: 5. The student computes with percentages in context.
 	
Tools: Calculator
	Prompt Features: The student is prompted to compute with percentages in a real-world context that requires multiple steps to solve.
	
	Stimulus Guidelines:
	• Context of problems should be familiar to students 12 to 14 years old.
	• For items asking for a percentage, the percent symbol (%) should not be required for full credit.
	• For items asking for a dollar amount, the dollar sign ($) should not be required for full credit.
	• Item difficulty can be adjusted via these example methods:
	  ○ 1-3 step(s) problem.
	  ○ Multiplying by a percent which should include benchmark percentages, i.e. 25%, 50%, etc.
	  ○ Divide two numbers or by a percent which should include benchmark percents, 25%, 50%, etc.
	
	TM5
	Stimulus: The student is presented with a real-world context involving adding or subtracting a percent to the whole (simple interest, tax, commission, markup, markdowns, tips, coupons, and discounts).
	
	Example Stem 1: Dave buys a baseball for $15 plus an 8% tax. Mel buys a football for $20 plus an 8% tax. Enter the difference in the amount Dave and Mel paid, including tax. Round your answer to the nearest cent.
	
	Rubric: (1 point) Student gives the correct difference in the amount between David and Mel (e.g., 5.40).
	
	Response Type: Equation/Numeric
	
	Example Stem 2: A bicycle is originally priced at $80. The store owner gives a discount and the bicycle is now priced at $60. Enter the percentage discount for the cost of the bicycle.
	
	Rubric: (1 point) Student gives the correct percentage discount (e.g., 25).
	
	Response Type: Equation/Numeric
	
	Example Stem 3: Dave has a 32 ounce energy drink. He drinks 10 ounces. Enter the percentage of ounces Dave has left from his energy drink. Round your answer to the nearest hundredth.
	
	Rubric: (1 point) Student gives the correct percentage (e.g., 68.75).
	
	Response Type: Equation/Numeric

2014 *SBAC Examples:* *DOK Levels 2, 3*

Example Item 9 (Grade 7):
Primary Target 2A (Content Domain RP), Secondary Target 1A (CCSS 7.RP.3), Tertiary Target 2D

Luke buys a television that is on sale for 25% off the original price. The original price is $120 more than the sale price. What is the original price of the television?

Rubric: (1 point) The student enters the correct full price (e.g., 480).

Response Type: Equation/Numeric

Example Item 1 (Grade 7):
Primary Target 3E (Content Domain RP), Secondary Target 1A (CCSS 6.RP.3), Tertiary Target 3C

Jane wants to buy the following items at a store.

- Jeans, $32.99
- Earrings, $29.99
- T-shirt, $9.99
- Shoes, $23.99

Jane will either use coupon A or coupon B to reduce the cost of her purchase. She sees some socks that cost $4.99. Jane thinks that adding socks to her purchase will cost her less than making the purchase without socks.

Coupon A

$25 off purchase of $100 or more

Coupon B

20% off your entire purchase

Which option should Jane choose to spend the **least** amount of money?

A. Jane should add the socks to her purchase and then use Coupon A.
B. Jane should add the socks to her purchase and then use Coupon B.
C. Jane should make the purchase without socks and use Coupon A.
D. Jane should make the purchase without socks and use Coupon B.

Rubric: (1 point) The student selects the statement that represents correct reasoning (e.g., A).

Response Type: Multiple choice, single correct response

2013 *SBAC Example:* Standard: 7.RP.3 DOK 2 Difficulty: Medium Item Type: SR

The tires Mary wants to buy for her car cost $200 per tire. A store is offering the following deal.

Buy 3 tires and get the 4^{th} tire for 75% off!

Mary will buy 4 tires using the deal. The sales tax is 8%. How much money will Mary save by using the deal versus paying the full price for all 4 tires?

(A) $150
(B) $162
(C) $185
(D) $216

Key and Distractor Analysis:

A. student uses 200×0.75
B. Key
C. student uses $200 \div 1.08$ and rounds
D. student uses $4 \times 200 \times 0.75 \times 1.08$