Math 7 Notes - Part A: Ratio and Proportional Relationships

***CCSS 7.RP.A.2:* Recognize and represent proportional relationships between quantities.**

RATIO & PROPORTION

Beginning middle school students typically can reason with one variable (called univariate reasoning), but working with two quantities (bivariate reasoning) requires some attention/exposure/experience. For example, given a series of numbers or geometric shapes, students can examine the pattern and identify the next (number/figure in the series). Working with two quantities, as we do in ratios, creates a new challenge for students.

For example, students were shown a container of orange juice and were told it was made from orange concentrate and water. Two glasses – one large glass and one small glass – were filled with the orange juice from the container. The students were then asked if they thought the orange juice from the two glasses would taste equally orangey, or if they thought that the juice in one glass would taste more orangey than the juice in the other.

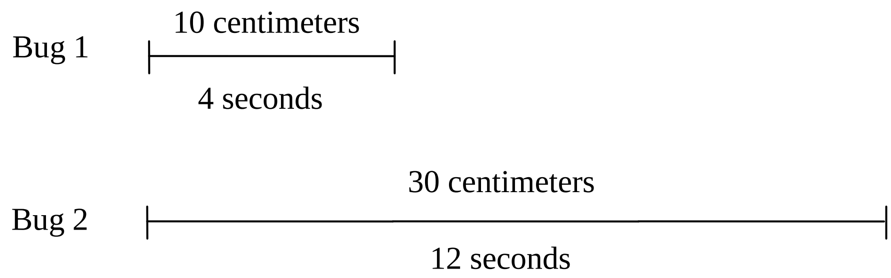
As adults we see this kind of situation so simply, we don't even recognize its importance until we hear the students thinking. Student responses were interesting – nearly half the class responded incorrectly. Approximately half of these student said that the juice in the large glass would taste more orangey, and the other half chose the smaller glass as more orangey. Their explanations suggest they focused on one quantity – the water or the orange concentrate – or they did not coordinate both quantities. Some students explained their thinking that "the larger glass is bigger, so it would hold more orange concentrate". Others explained that the juice in the small glass would taste more orangey because the smaller volume would allow less water to get in, which would leave more room for the orange concentrate.

The goal is to get students to understand that since the ratio of water to orange concentrate is the same within that container, the two glasses would taste equally orangey.

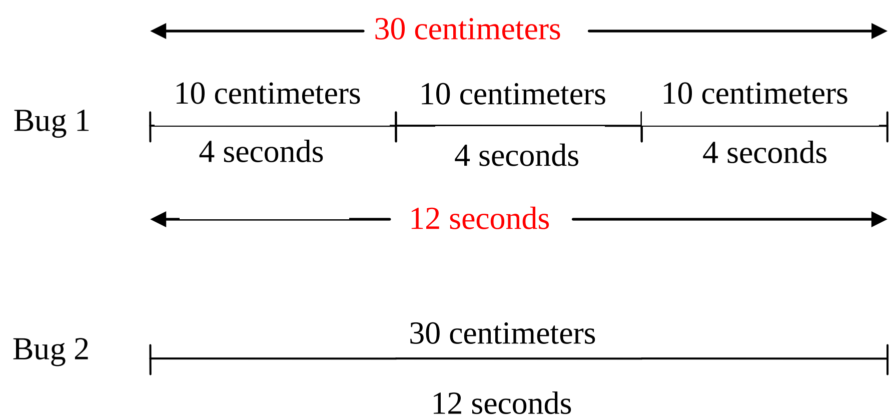
What happens when we give a situation such as: Bug 1 walks at the rate of 10 centimeters in 4 seconds. Bug 2 walks 30 centimeters in 12 seconds. Which bug is faster?

As adults and as mathematics teachers we jump either right into setting up ratios and then a proportion and we solve it or we mentally reason our way through the problem. With student learners we need to scaffold this thought process so our students truly understand how to work with ratios, proportions and rates.

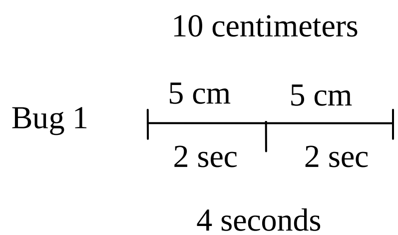
It would help students to begin with a visual representation something like this:



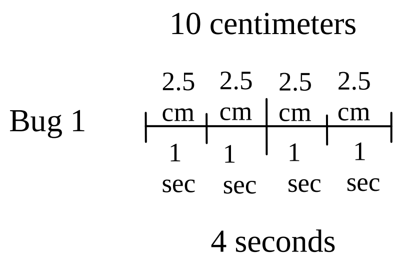
Some students will repeat (iterate) the composed unit until they find a match (or not). Below they will see that it is like Bug 1 walking the distance three times.



Once students see this visually they will realize that both bugs can walk 30 centimeters in 12 seconds so they are traveling at the same rate. Ask students to create other "same speed" values. Hopefully they will see in the graph above that 20 centimeters in 8 seconds is the same. They may continue repeating this "joining" or they may begin to "partition" (break apart into equal sized sections).



Here we see another same rate value of 5 cm in 2 seconds.



Once again, we see another same rate value of 2.5 cm in 1 second.
These ***equivalent ratios*** arise by multiplying each measurement in a ratio pair by the same positive number. Such pairs are said to be in the same ratio.
This can be described as:
2.5 cm *for each 1* second
2.5 cm *for each* second
2.5 cm *per* second
2.5 cm *for every* second

For this reason we must get students to attend to and coordinate <u>two</u> quantities.

A **RATIO** in our textbooks is commonly defined as a comparison between two quantities. We use ratios everyday; one Pepsi which costs 50 cents describes a ratio. On a map, the legend might tell us one inch is equivalent to 50 miles or we might notice one hand has five fingers. In our classrooms we are concerned with the student/teacher ratio and the ratio of boys to girls in a particular class. Other examples could include, the number of red M & M's to green M & M's in a bag of M&M candies, ratios found in recipes, etc. Those are all examples of comparisons – ratios. Students should have some background knowledge here, so this is a good place to begin. A ratio can be written three different ways. If we wanted to show the comparison of one inch representing 50 miles on a map, we could write that as:

Using the word "to"	1 to 50	or
Using a colon	1:50	or
Using a fraction	$\frac{1}{50}$	

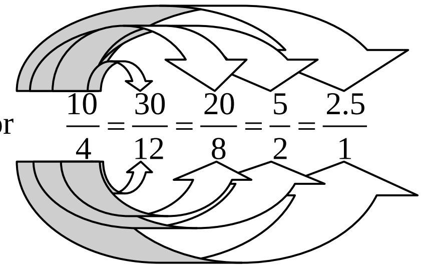
Does $\frac{3}{150}$ represent the same comparison as $\frac{1}{50}$?
The answer is yes – and if we looked at other ratios, we would see that reducing ratios does not affect those comparisons.
We noticed that $\frac{3}{150}$, 3 inches represents 150 miles, could be reduced to $\frac{1}{50}$, meaning 1 inch represents 50 miles. Mathematically, by setting the ratios equal, we could write $\frac{3}{150}=\frac{1}{50}$.

Because we are going to learn to solve problems, it's easier to write the ratios using fractional notation. If we looked at the ratio of one inch representing 50 miles, $\frac{1}{50}$, we might determine 2 inches represents 100 miles, 3 inches represents 150 miles by repeating (iterating). These ***equivalent ratios*** arise by multiplying each measurement in a ratio pair by the same positive number. Such pairs are said to be in the same ratio.

With CCSS we need to dig deeper into the understanding of ratio, proportion and proportional reasoning so a clearer definition would be – **a ratio** *is a multiplicative comparision of two* *quantities, or it is a joining or composing two quantities in a way that preserves a multiplicative relationship.*

Referring back to Bugs 1 and 2, we can put this in numerical form.

10:4 ⇨ 30:12 ⇨ 20:8 ⇨ 5:2 ⇨ 2.5:1 or



$$\frac{10}{4}=\frac{30}{12}=\frac{20}{8}=\frac{5}{2}=\frac{2.5}{1}$$

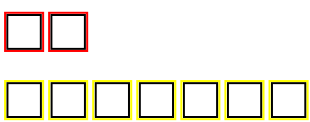
Point out to students that we began by tripling the original distance and tripling the time. Then we doubled the original distance and doubled the original time. Next we cut the original distance in half and the original time in half. Finally, we cut the original distance in fourths and the original time in fourths.

Remind students they must attend to <u>both</u> quantities equally.

Example: Suppose that you have a batch of orange paint by mixing 2 cans of red paint with 7 cans of yellow paint. What are some other combinations of numbers of can of red paint and yellow paint that you can mix to make the same shade of orange? Solve the problem in two different ways – first by using a multiplicative comparison and then by using a composed unit.

Red

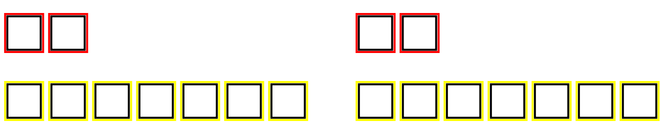
Yellow



<u>Sample solution</u>
Doubling would yield:

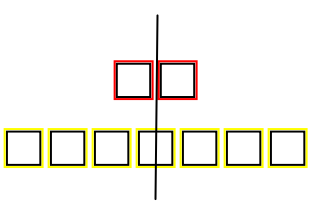
Red

Yellow



$$\frac{2\,red}{7\,yellow}=\frac{4\,red}{14\,yellow}$$

Partitioning would yield:



$$\frac{2\,red}{7\,yellow}=\frac{1\,red}{3\frac{1}{2}\,yellow}$$

Rate in the CCSS refers to a ratio that compares two quantities measured in the same units or different units. For example, cups to cups or meters to seconds.

Example: *Mike travels 300 miles in 5 hours, find Mike's rate.*

$$\text{Mike's } rate = \frac{300 \text{ miles}}{5 \text{ hours}}$$

Unit rate is a rate whose denominator is 1. To convert a rate to a unit rate, divide both the numerator and denominator by the denominator. (Remember to demonstrate and allow models for students who need them.) The problems below **show typical examples** of what we have been doing.

Example: *Find the unit rate of Mike's travel above.*

$$\frac{300 \text{ miles} \div 5}{5 \text{ hours} \div 5} = \frac{60 \text{ miles}}{1 \text{ hour}}; \quad \text{read 60 miles for each hour}$$

Example: *Stan's heart beats 520 times every four minutes, find Stan's heartbeat per minute.*

$$\frac{520 \text{ beats} \div 4}{4 \text{ minute} \div 4} = \frac{130 \text{ beats}}{1 \text{ minute}}; \quad \text{read 130 beats per 1 minute}$$

Example: *Find the rate of pay if you earn $52 for 8 hours of work.*

$$\frac{\$52.00 \div 8}{8\,hours \div 8} = \frac{\$6.50}{1\,hour}; \quad \text{read \$6.50 per hour or \$6.50 for each hour}$$

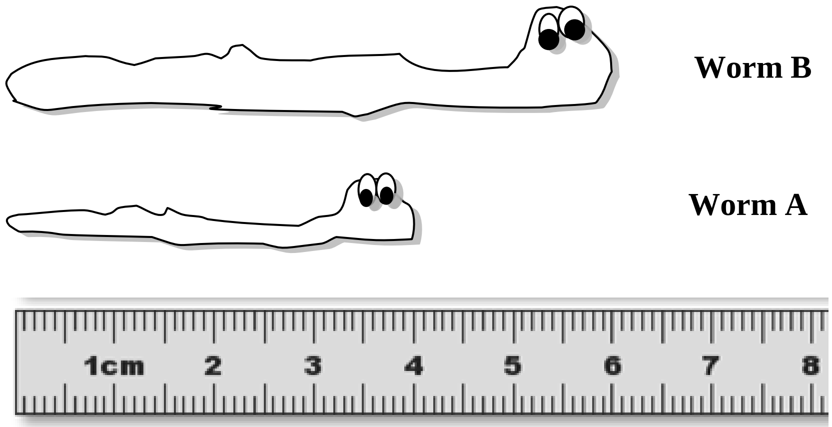
Example: *An area of 3 acres measure 14,520 square yards. How many square yards are there in one acre?*

$$\frac{14{,}520\,sq.\,yd.}{3\,acres} = \frac{4{,}840}{1}; \quad \textit{4,840 sq yd for every acre}$$

CCSS 7.RP.A.1: ***Compute unit rates associated with ratios of fractions, ratios of lengths, areas and other quantities measured in like or different terms.***

Expectations for this grade level <u>include</u> complex fractions. Students should be able to demonstrate a variety of "modeling" techniques such as the use of the tape diagrams and double number line diagrams shown here, in addition to procedural techniques.

Example: Consider comparing the lengths of the two worms below. Worm A is 4 centimeters long and Worm B is 6 centimeters long.



Writing the ratio of the length of worm A to the length of worm B we could write 4 to 6, 4:6 or $\frac{4}{6}$.

The worms can be compared using a multiplicative comparison by asking question such as "How many times greater is one thing than another?" or "What part or fraction is one thing of another?" So we need to ask students to compare them.

How many times longer is worm B than worm A?

(Worm B is $1\frac{1}{2}$ times the length of worm A.)

$$\frac{\text{worm } B}{\text{worm } A} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$$

The length of worm A is what part, or fraction, of the length of worm B?

(Worm A is $\frac{2}{3}$ the length of worm B.)

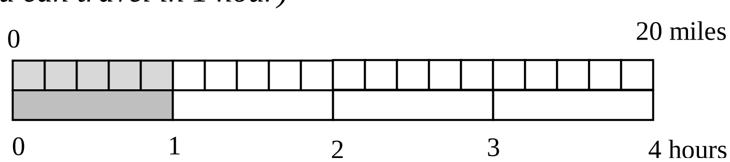
$$\frac{\text{worm } A}{\text{worm } B} = \frac{4}{6} = \frac{2}{3}$$

Student must be able to write and understand comparative statements like Worm B is 1.5 times the length of Worm A or Worm A is $\frac{2}{3}$ the size of Worm B.

Example: *On a bicycle you can travel 20 miles in 4 hours. What are the unit rates in this situation (the distance you can travel in 1 hour and the amount of time required to travel 1 mile)?*

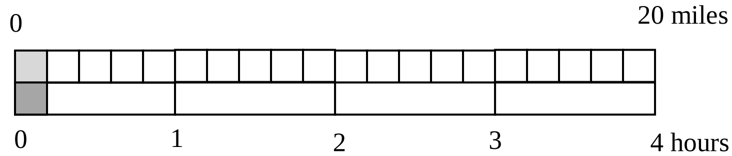
Using a model we could show….

Solution 1: *(the distance you can travel in 1 hour)*



5 miles in 1 hour or 5 mph

Solution 2: (*the amount of time required to travel 1 mile)*



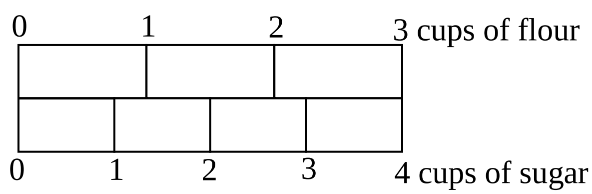
1/5 hour per mile

The first example above using a **tape diagram**, is a relatively simple one. The graphic is easily read. Below the use of the tape diagram graphic becomes more difficult to read so it was solved both graphically and procedurally.

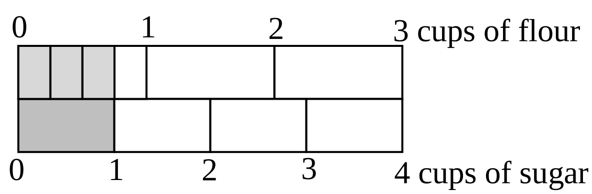
Note: When you are asked for both unit rates within a problem the unit rates will always be reciprocal of each other.

Example: *A recipe has a ratio of 3 cups of flour to 4 cups of sugar. Find the per unit rate in terms of <u>each</u> ingredient.*

Begin with



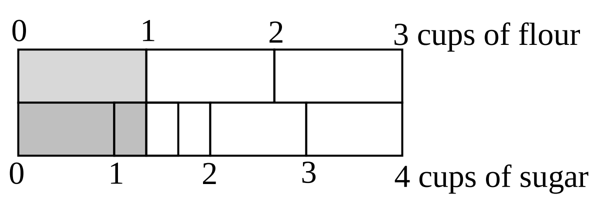
<u>Solution 1</u>:



$$\frac{3\,cups\ of\ \ flour}{4\,cups\ of\ \ sugar} = \frac{3 \div 4}{4 \div 4} = \frac{\frac{3}{4}}{1}$$

¾ cup of flour to each cup of sugar

<u>Solution 2</u>:

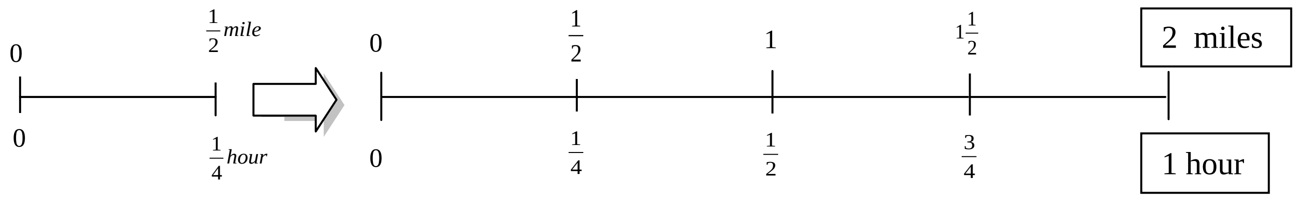


$$\frac{4\,cups\ of\ \ sugar}{3\,cups\ of\ \ flour} = \frac{4 \div 3}{3 \div 3} = \frac{\frac{4}{3}}{1}$$

4/3 cups of sugar for each cup of flour
or
$1\frac{1}{3}$ **cups of sugar for each cup of flour**

Example: *If a person walks ½ mile in each ¼ hour, compute the unit rates.*

Students may use a ***double number line diagram*** *while learning to work with complex fractions.*



Solution 1: $\dfrac{\frac{1}{2}\,mile}{\frac{1}{4}\,hour} = \dfrac{1}{2} \div \dfrac{1}{4} = \dfrac{1}{2} \bullet \dfrac{4}{1} = \dfrac{2\,miles}{1\,hour}$ **or** $\dfrac{\frac{1}{2}\,mile}{\frac{1}{4}\,hour} = \dfrac{\frac{1}{2} \bullet \frac{4}{1}}{\frac{1}{4} \bullet \frac{4}{1}} = \dfrac{\frac{4}{2}}{\frac{4}{4}} = \dfrac{2\,miles}{1\,hour}$ **2 miles per hour**

Solution 2: $\dfrac{\frac{1}{4}\,hour}{\frac{1}{2}\,mile} = \dfrac{1}{4} \div \dfrac{1}{2} = \dfrac{1}{4} \bullet \dfrac{2}{1} = \dfrac{1}{2}$ **or** $\dfrac{\frac{1}{4}\,hour}{\frac{1}{2}\,mile} = \dfrac{\frac{1}{4} \bullet \frac{2}{1}}{\frac{1}{2} \bullet \frac{2}{1}} = \dfrac{\frac{2}{4}}{\frac{2}{2}} = \dfrac{\frac{1}{2}}{1} = \dfrac{\frac{1}{2}\,hour}{1\,mile}$ $\dfrac{1}{2}$ **hour per mile**

Notice also in the double number line diagram that the reciprocal unit rate is shown. For each ½ hour the distance walked is 1 mile or ½ hour per mile.

CCSS 7.RP.A.2: ***Recognize and represent proportional relationships between quantities.***

Proportional relationships involve collections of pairs of measurements in equivalent ratios. In contrast, a proportion is an equation stating that two ratios are equivalent. Equivalent ratios have the same unit rate.

So in the bug example (the first example given) we have several pairs of measurements we can write in equivalent ratios. We see $\dfrac{10}{4} = \dfrac{30}{12} = \dfrac{20}{8} = \dfrac{5}{2} = \dfrac{2.5}{1}$

CCSS 7.RP.A.2a: ***Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.***

CCSS 7.RP.A.2b: ***Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.***

Example:

Inches on map	***Distance in miles***
1	***5***
2	***10***
3	***15***
4	***20***
5	***25***

Solution:

Since $\frac{1}{5}=\frac{2}{10}=\frac{3}{15}=\frac{4}{20}=\frac{5}{25}$ yes, this is a proportional relationship.

The constant of proportionality or unit rate is 5 miles per inch.

Example:

Cups grape	***Cups peach***
5	***2***
10	***4***
15	***6***
20	***8***
25	***10***

Solution:

Since $\frac{2}{5}=\frac{4}{10}=\frac{6}{15}=\frac{8}{20}=\frac{10}{25}$

yes, this is a proportional relationship.

The constant of proportionality or unit rate is $\frac{2}{5}$ cup of peach to 1 cup of grape.

Example:

meters	***3***	***6***	***9***	***12***	***15***	***18***
seconds	***2***	***4***	***6***	***8***	***10***	***12***

Solution:

Since

$\frac{3}{2}=\frac{6}{4}=\frac{9}{6}=\frac{12}{8}=\frac{15}{8}=\frac{18}{12}$

The constant of proportionality or unit rate is $\frac{3}{2}$ meter s per second.

Example:

x	4	5	6	7
y	5	6	7	8

Solution:

Since

$\frac{4}{5}\neq\frac{5}{6}\neq\frac{6}{7}\neq\frac{7}{8}$

no, this is a NOT a proportional relationship.

CCSS 7.RP.A.2a: ***Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.***

CCSS 7.RP.A.2b: ***Identify the constant of proportionality (unit rate) in graphs,*** *equations,* ***diagrams,*** *and verbal descriptions of proportional relationships.*

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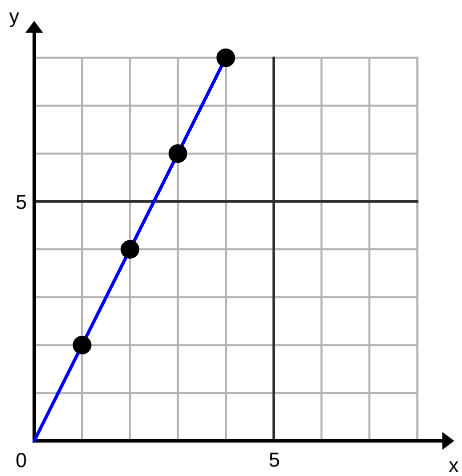
Example:

x	1	2	3	4
y	2	4	6	8

Solution:

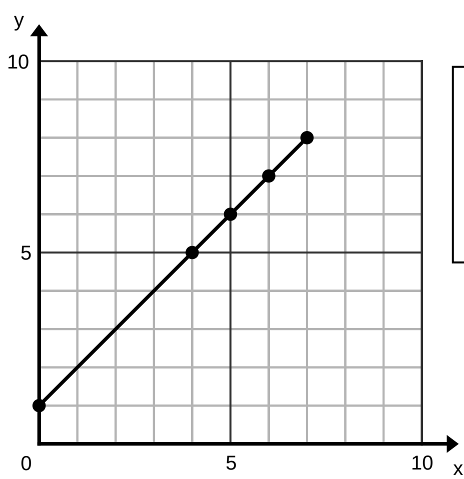


We can see visually that the graph is a straight line through the origin, and so it is a proportional relationship. The **constant of proportionality (**unit rate) is $\frac{2}{1}$.

Example:

x	4	5	6	7
y	5	6	7	8

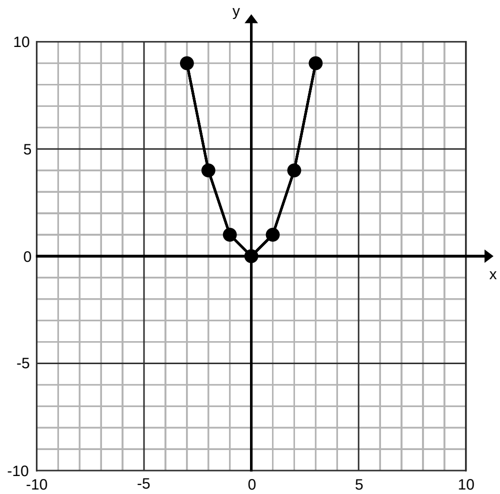
Solution:



Not a proportional relationship, the graph is NOT a straight line through the origin.

Example: ***Solution:***

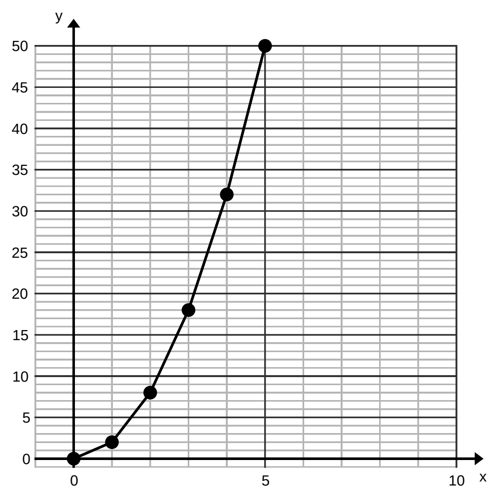
x	*y*
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



Not a proportional relationship, the graph is NOT a straight line through the origin.

Example: ***Solution:***

x	*y*
0	0
1	2
2	8
3	18
4	32
5	50

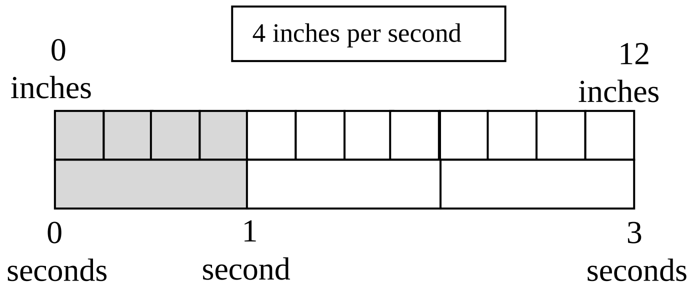


Not a proportional relationship, the graph is NOT a straight line through the origin.

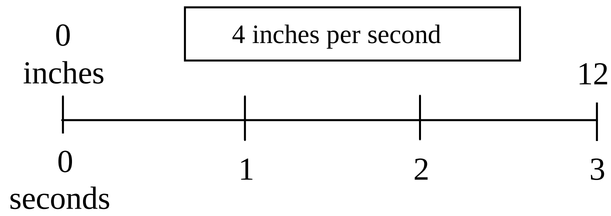
***CCSS 7.RP.A. 2b:* Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.**

Example: A giant tortoise moves at a slow but steady pace. It takes the giant tortoise 3 seconds to travel 12 inches. What is the unit rate for the giant tortoise?

Solution(s):



OR



OR

Time (sec)	*1*	2	3
Distance (in)	4		12

$\frac{12}{3} = 4$

4 inches per second

Example: Susan types 50 words per minute. Is the relationship between the number of words and the number of minutes a proportional relationship? Why or why not?

Solution:
Begin with

Time (min)	***1***	***2***	***3***	***4***	***5***
Number of words	***50***				

Complete the table

Time (min)	***1***	***2***	***3***	***4***	***5***
Number of words	***50***	***100***	***150***	***200***	***250***

$\frac{\textit{Number of words}}{\textit{Time}} \quad \frac{50}{1} = \frac{100}{2} = \frac{150}{3} = \frac{200}{4} = \frac{250}{5}$ *so it is a proportional relationship since each fraction is equivalent to* $\frac{50}{1}$**.** The unit rate or constant of proportionality is $\frac{50}{1}$ or 50 words per minute.

Example: John recorded his distance from home each hour on the first day of his vacation. Using the information below, determine if the relationship between the distance and the time is a proportional relationship? Why or why not?

Time (h)	***1***	***2***	***3***	***4***	***5***
Distance (mi)	***75***	***140***	***210***	***240***	***300***

Solution:

$$\frac{\text{Distance}}{\text{Time}} = \frac{75}{1} \neq \frac{140}{2} \neq \frac{215}{3} \neq \frac{240}{4} \neq \frac{300}{6}$$ *so this is not a proportional relationship.*

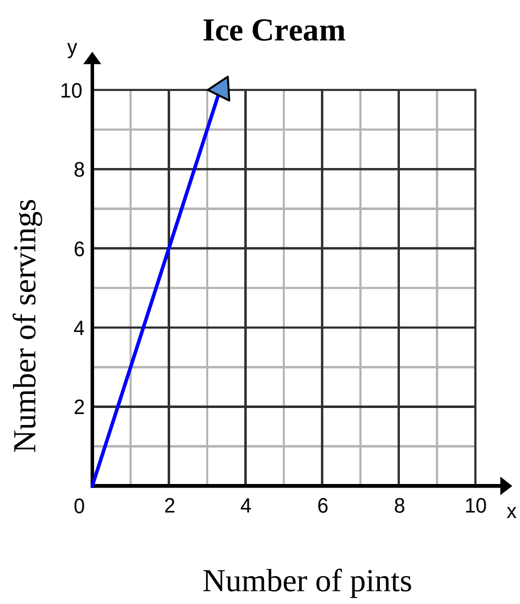
There is no common ratio.

Follow up question – Do you think John drove at a constant rate for the entire trip? Why or why not?

CCSS 7.RP.A.2d: ***Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0,0) and (1, r) where r is the unit rate.***
CCSS 7.RP.A.2c: ***Represent proportional relationships by equations.***

Students need numerous exposures to tables and graphs of proportional relationships in a variety of situations. Below are some examples of where CCSS may expect students to demonstrate their understanding.

Example: *The graph shows the number of servings in different amounts of ice cream. Explain what the point (3,9) means on this graph. Identify the unit rate and explain how to find the unit rate using the graph.*



Solution: The point (3,9) on this graph represents 9 servings in 3 pints of ice cream. The unit rate is 3 servings in 1 pint of ice cream. This is represented in the ordered pair (1, 3) found on the graph.

Example: *Using the above graph, write an equation that gives the number of servings,* ***y****, in* ***x*** *pints of ice cream.*

Solution: $y = 3x$

CCSS 7.RP.A.2b: Identify the constant of proportionality (unit rate) in graphs, equations, diagrams, and verbal descriptions of proportional relationships.

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Students should be given ample opportunities to identify the constant of proportionality in different forms and different situations. Besides just identifying the constant of proportionality (the "*m*" in *y=mx)* students need to know and understand what that means. The next example highlights this point.

Example: A gallon of gasoline costs $3.56. A table shows the number of gallons of gasoline and the total cost of gas. Which of the following must be true about the data in the table?

# of gallons of gasoline	1	2	3	4	5
Total Cost (in dollars)	$3.56	$7.12	$10.68	$14.24	$17.80

A. The ratio of the total cost to the number of gallons is 3.56
B. The ratio of the number of gallons to the total cost is always 3.56.
C. The total cost is always 3.56 greater than the number of gallons.
D. The number of gallons is always 3.56 times the total cost.

CCSS 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems.

Example: Johanna sells pizza sauce and charges $3.00 for a 7-ounce jar or $16.00 for two jars that hold a total of $37\frac{1}{2}$ ounces. Is buying a 7-ounce jar a better deal than buying two jars that hold $37\frac{1}{3}$ ounces? How do you know?

Solution: The 7ounce jar $\frac{\$3.00}{7oz} = \$0.\overline{428571}$ *per ounce*

The $37\frac{1}{2}$ ounce jars $\frac{\$16.00}{37\frac{1}{2}} = \$0.42\overline{6}$ *per ounce*

Buying the two jars that hold $37\frac{1}{2}$ ounces is cheaper because $\$0.42\overline{6}$ is less than $\$0.\overline{428571}$.

Example: Mark hikes $\frac{1}{2}$ mile every $\frac{1}{4}$ hour. Cheryl hikes $\frac{1}{3}$ mile every $\frac{1}{6}$ hour. Who hikes faster? How do you know?

Solution: Mark $\frac{\frac{1}{2}}{\frac{1}{4}} = \frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = 2 \;\; 2\,miles\,an\,hour$

Cheryl $\frac{\frac{1}{3}}{\frac{1}{6}} = \frac{1}{3} \div \frac{1}{6} = \frac{1}{3} \times \frac{6}{1} = 2 \;\; 2\,miles\,an\,hour$

They hike at the same rate (2 miles per hour) so they tie.

Proportions

A ***PROPORTION*** is a statement of equality between 2 ratios.

Looking at a proportion like $\frac{1}{2} = \frac{3}{6}$, we might see some relationships that exist if we take time and manipulate the numbers.

For instance, what would happen if we tipped both ratios ***up-side down***?

$\frac{2}{1}$ and $\frac{6}{3}$, notice they are also equal, so $\frac{2}{1} = \frac{6}{3}$

How about writing the original proportion ***sideways***, will we get another equality?

$\frac{1}{3}$ and $\frac{2}{6}$, notice they are equal also, so $\frac{1}{3} = \frac{2}{6}$

If we continued looking at the original proportion, we might also notice we **could cross multiply** and retain an equality. In other words $1x6 = 2x3$.

Makes you wonder whether tipping ratios up-side down, writing them sideways or cross multiplying only works for our original proportion?

Well, to make that determination, we would have to play with some more proportions. Try some, if our observation holds up, we'll be able to generalize what we saw.

Let's try these observations with the proportion $\frac{2}{3}=\frac{4}{6}$ Can I tip them upside down and still retain an equality? In other words, does $\frac{3}{2}=\frac{6}{4}$?

How about writing them sideways, does $\frac{2}{4}=\frac{3}{6}$?

How about cross multiplying in the original proportion, does $2x6=3x4$?

The answer to all three questions is yes.

Since everything seems to be working, we will generalize our observations using letters instead of numbers. If $\frac{a}{b}=\frac{c}{d}$, then…

1) $\boldsymbol{\frac{b}{a}=\frac{d}{c}}$ 2) $\frac{a}{c}=\frac{b}{d}$ 3) $ac=bd$

Those 3 observations are referred to as ***Properties of Proportions***. Those properties can be used to help us solve problems.

To solve problems, most people use either equivalent fractions or cross multiplying to solve proportions.

Generally you use equivalent fractions when either the numerator or denominator of a fraction is a multiple of the numerator or denominator of the other fraction. If that is not immediately obvious, then cross multiply.

Example: *Find the value of n.* $\frac{6}{10}=\frac{36}{n}$

This problem can be done by equivalent fractions or by cross multiplying.

$\frac{6}{10}=\frac{36}{n}$

x 6

$\frac{6}{10}=\frac{36}{n}$

x 6

$n=60$

$\frac{6}{10}=\frac{36}{n}$

$6n = 360$

$n = 60$

Example: *If a turtle travels 3 inches every 10 seconds, how far will it travel in 50 seconds?*

What we are going to do is set up a proportion. How surprising? The way we'll do this is to identify the comparison we are making. In this case we are saying 3 inches every 10 seconds. Therefore, and this is very important, we are going to set up our proportion by saying inches is to seconds.

On one side we have $\frac{3}{10}$ describing inches to seconds. On the other side we have to again use the same comparison, inches to seconds. We don't know the inches, so we'll call it "n". Where will the 50 go in the ratio, top or bottom? Bottom, because it describes seconds – good deal. So now we have,

$$\frac{\text{inches}}{\text{seconds}} = \frac{3}{10} = \frac{n}{50}$$

Now, we can find n by equivalent fractions or we could use property 3 and cross multiply.

$$\frac{3}{10} = \frac{n}{50}$$

x 5

$$\frac{3}{10} = \frac{n}{50}$$

x 5

$n = 15$

$$\frac{3}{10} = \frac{n}{50}$$
$$10n = 3 \times 50$$
$$10n = 150$$
$$n = 15$$

The turtle will travel 15 inches.

It is very important to write the same comparisons on both sides of the equal signs. In other words, if we had a ratio on one side comparing inches to seconds, then we must write inches to seconds on the other side.

If we compared the number of boys to girls on one side, we would have to write the same comparison on the other side, boys to girls. We could also write it as girls to boys on one side as long as we wrote girls to boys on the other side. The first Property of Proportion, tipping the ratios upside down, permits this to happen.

In the above examples, I could have simplified the fractions before cross multiplying. By simplifying first, that keeps the numbers smaller. You get the same answers.

***CCSS 7.G.A.1:* Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from scale drawing and reproducing a scale drawing at a different scale.**

Another application of proportions is in the use of scale drawings. A **scale drawing** is a two-dimensional drawing that is similar to the object it represents. A **scale model** is a three-dimensional model that is similar to the object it represents. The **scale** of a scale drawing or scale model gives the relationship between the drawing or model's dimensions and the actual dimensions. For example, if a map shows a scale of 1 cm : 5 m, it means that 1 centimeter on the scale drawing represents an actual distance of 5 meters. The scale of a scale drawing or scale model can be written without units if the measurements have the same unit. To write the scale from our example without units, write 5 meters as 500 centimeters.

So, $$1\text{ cm} : 5\text{ m} \rightarrow \frac{1\text{ cm}}{5\text{ m}} \rightarrow \frac{1\text{ cm}}{500\text{ cm}} \rightarrow 1:500$$ we can write the scale without units as 1: 500.

***Example*:** *On a map, the distance from your house to school is 5 centimeters. The scale is 1 cm : 500 m. What is the actual distance from your house to school?*

$$\frac{\text{map distance}}{\text{actual distance}} \rightarrow \frac{1\text{ cm}}{500\text{ m}} = \frac{5\text{ cm}}{d\text{ m}}$$

$$1d = 500 \cdot 5$$

$$d = 2500\ m$$

The distance from your house to school is 2500 meters.

***Example*:** *You have a scale model of an airplane, scale of 1:90. The length of the model airplane from nose to tail is 1.8 feet. Determine the length (from nose to tail) of the actual airplane.*

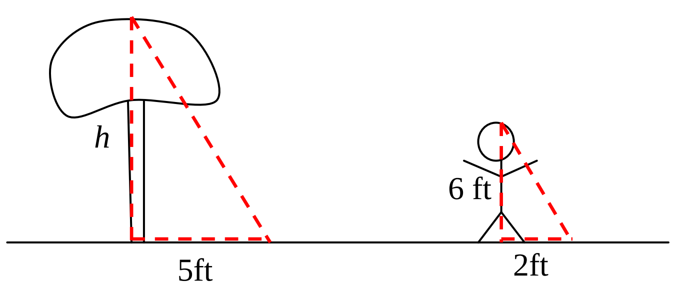
$$\frac{\text{model length}}{\text{airplane length}} \rightarrow \frac{1}{90} = \frac{1.8}{x}$$

$$1x = 162\ ft$$

The length of the airplane is 162 feet.

***Example*:** *A student whose height is 6 feet is standing near a tree. The length of the student's shadow is 2 feet. If the tree casts a shadow of 5 feet, how tall is the tree?*

Since the student and the tree are perpendicular to the ground, the sun's rays strike the student and the tree at the same angle, creating two similar figures. A sketch will help us to see the similar triangles.



So,

$$\frac{\text{height of the tree}}{\text{height of the student}} = \frac{\text{length of the tree's shadow}}{\text{length of the student's shadow}}$$

$$\frac{h}{6} = \frac{5}{2}$$

$$2h = 6 \cdot 5$$

$$2h = 30$$

$$h = 15$$

The height of the tree is 15 feet.

Example: *The height of a tower on a scale drawing is 18 centimeter. The scale is 2 cm: 19 m. What is the actual height of the tower?*

Example: *A drawing of a hummingbird has the scale 5 cm : 1 cm. The actual distance from the tip of the hummingbird's beak to the end of its tail feathers is 6.4 cm. What is this length in the drawing?*

Example: *A crystal that is 0.12 millimeter long appears to be 60 millimeters long under a microscope. What is the power of the microscope?*

A.	*500:1*
B.	*200:1*
C.	*50:1*
D.	*400:1*

Example: *A scale drawing of a desk uses the scale 1 in : 2.5 in. Find the actual measurement for the given measurement in the drawing.*

1. *The width is 16 in*
2. *The height is 15 in*
3. *The depth is 9 in*
4. *The depth of the lid is 6.4 in*
5. *A leg is 0.8 in thick*
6. *A drawer is 7 in wide*

Example: *If $\frac{1}{4}$ in represents 5 yd, how long must a drawing be to represent a football field that is 100 yards long?*

Example: *The scale on a map is 1 cm: 100 km. The distance between Court City and Southbridge is 4.8 cm on the map. What is the approximate distance between the cities?*

Here is a perfect opportunity for students to be given a project to (create a scale drawing) apply all the skills and concepts taught in this unit.

SBAC example:

Standard: 7.G.1, 7.RP.2 **DOK:** 2 **Difficulty:** M **Question Type:** SR Selected Response

> A company designed two rectangular maps of the same region. These maps are described below.
>
> Map 1: The dimensions are 8 inches by 10 inches.
> The scale is $\frac{3}{4}$ mile to 1 inch.
>
> Map 2: The dimensions are 4 inches by 5 inches.

Which ratio represents the scale on Map 2?

A.	$\frac{1}{2}$ mile to $\frac{3}{4}$ inch
B.	$\frac{3}{4}$ mile to $\frac{1}{2}$ inch
C.	$\frac{1}{4}$ mile to 1 inch
D.	$\frac{3}{8}$ mile to 1 inch

Key and Distractor Analysis:

A. Found correct relationship but reversed order
B. Correct
C. Subtracted the first term of ratio by scale factor
D. Multiplied the first term of ratio by scale factor

SBAC example:

Standard: 7.RP.2 **DOK:** 2 **Difficulty:** M **Question Type:** SR
Selected Response

Helen made a graph that represents the amount of money she earns, *y*, for the numbers of hours she works, *x*. The graph is a straight line that passes through the origin and the point (1, 12.5).

Which statement **must** be true?

A.	The slope of the graph is 1.
B.	Helen earns $12.50 per hour.
C.	Helen works 12.5 hours per day.
D.	The *y*-intercept of the graph is 12.5.

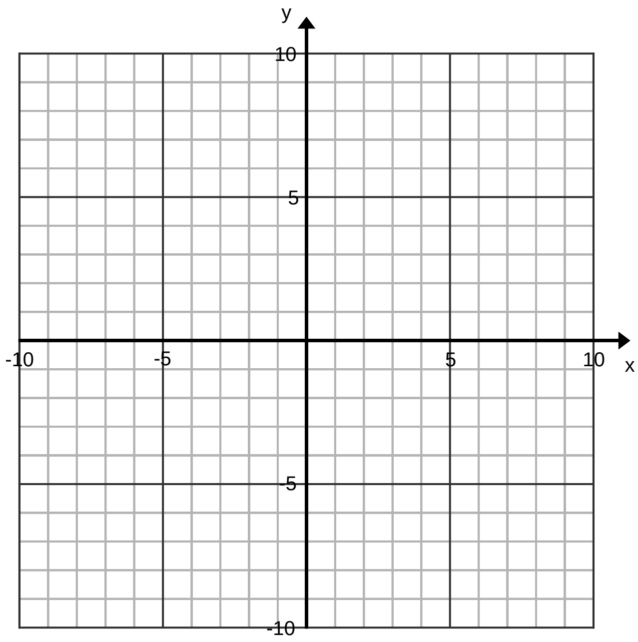
Key and Distractor Analysis:

A. Reverses the meaning of the coordinates.
B. Correct
C. Focuses on the vertical axis.
D. Thinks 12.5 is the initial value.

SBAC example:

Standard: 7.RP.2 **DOK:** 2 **Difficulty:** M **Question Type:** TE
Technology Enhanced

The value of *y* is proportional to the value of *x*. The constant of proportionality for this relationship is 2. On the grid below, graph this proportional relationship.



[Create two points by clicking on the intersections of the gridlines. When you create the second point, a line will automatically be drawn through the two points. If you make a mistake, use the Clear button to begin again.]

Key and Distractor Analysis:

Student must select two of these points: (-4, -8), (-3, -6), (-2, -4), (-1, -2), (0, 0), (1, 2), (2, 4), (3, 6), (4, 8).

SBAC example:

Standard: 7.RP.2 **DOK:** 2 **Difficulty:** Low **Question Type:** SR Selected Response

Roberto is making cakes. The number of cups of flour he uses is proportional to the number of cakes he makes.

Roberto uses $22\frac{1}{2}$ cups of flour to make 10 cakes.

Which equation represents the relationship between *f*, the number of cups of flour Roberto uses, and *c*, the number of cakes he makes?

Ⓐ $f = \frac{4}{9}c$

Ⓑ $f = 2\frac{1}{4}c$

Ⓒ $f = 2\frac{1}{2}c$

Ⓓ $f = 10c$

Key and Distractor Analysis:

A. inverts the ratio

B. Key

C. student thinks $22\frac{1}{2} \div 10 = 2\frac{1}{2}$

D. uses 10 as the coefficient