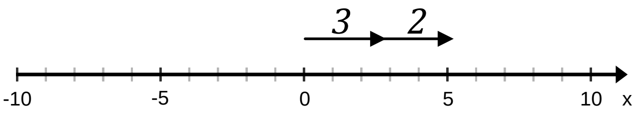
Math 7 Notes – Part B: Rational Numbers

Operations with Rational Numbers

NVACS 7.NS.A. 1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

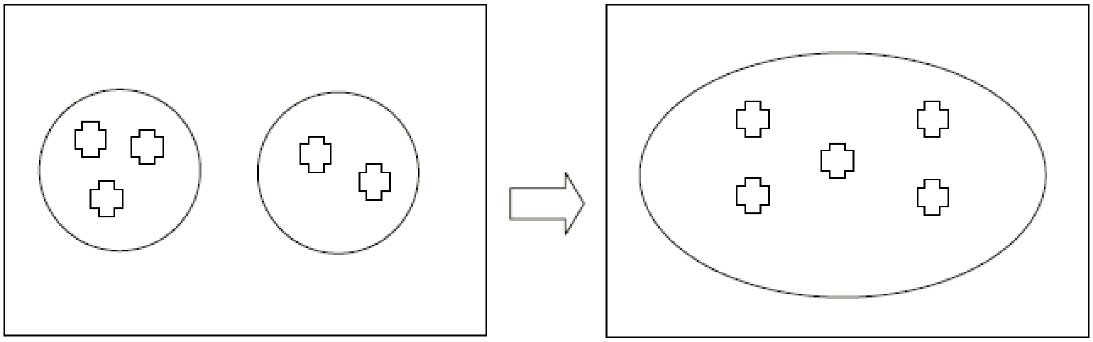
Review with integers then add rationals

With the expression 3 + 2, have students model the expression. One method could be modeling on a number line.

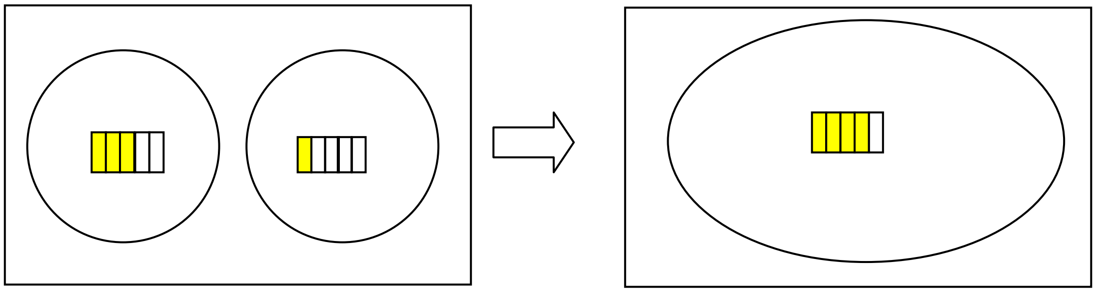


Or using manipulatives or a drawing, students could show three positives and then, two more positives. Establish the meaning of addition as combining, joining or 'putting together' terms. When the student puts the three positives and the two negatives together, it is simple to see the result is positive five. Repeat this process as needed.

$(+3) + (+2) = +5$



We could use similar methods to show $\frac{+3}{5} + \frac{+1}{5} = \frac{+4}{5}$



Math 7 Notes – Part B: Rational Numbers

Remind students of the rules for signed numbers and begin to mix in some rational numbers.

Rule 1: **When adding two positive numbers,**
find the sum of their absolute values, and the answer is positive.

Example: (+8) + (+7)= +15 ***is the same as*** 8 + 7 = 15

To add with other rationals (not just integers), students will need to blend the rules they have learned for operating with integers with the rules for operating with fractions and decimals. Now is a great time to review those skills with positives and begin to infuse the negative rational numbers.

Review

Algorithm for Addition / Subtraction of Decimals

1. Rewrite the problems vertically, lining up the decimal points.
2. Fill in spaces with zeros.
3. Add or subtract the numbers.
4. Bring the decimal point straight down.

Example: $(+3.1)+(+1.2)=+4.3$ ***is the same as*** $3.1+1.2=4.3$

Students may need to set the problem vertically. (Habits are hard to break!)

$$\begin{array}{r} 3.1 \\ \underline{+1.2} \\ 4.3 \end{array}$$

Example: $(+1.23)+(+.4)+(+12.375)$

$$\begin{array}{r} 1.23 \\ .4 \\ \underline{+12.375} \end{array}$$

Rewrite vertically lining up the decimal points.

$$\begin{array}{r} 1.230 \\ .400 \\ \underline{+12.375} \\ 14.005 \end{array}$$

Fill in zeros where necessary so that all three numbers have the same decimal place value.
Same signs - Add.

Review

Algorithm For Adding/Subtracting Fractions

Step 1. Find a common denominator.
Step 2. Make equivalent fractions.
Step 3. Add or subtract the numerators, keep the common denominator, then simplify if possible.

Math 7 Notes – Part B: Rational Numbers

Example: $\left(+\frac{2}{5}\right)+\left(+\frac{1}{5}\right)=+\frac{3}{5}$ ***is the same as*** $\frac{2}{5}+\frac{1}{5}=\frac{3}{5}$

Example: $\left(+\frac{2}{9}\right)+\left(+\frac{2}{9}\right)=+\frac{4}{9}$ ***is the same as*** $\frac{2}{9}+\frac{2}{9}=\frac{4}{9}$

Find a LCD and equivalent fractions

Same signs - Add

Example: $\left(+\frac{1}{2}\right)+\left(+\frac{1}{3}\right) = \left(+\frac{3}{6}\right)+\left(+\frac{2}{6}\right)= +\frac{5}{6}$ ***is the same as*** $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$

Find a LCD and equivalent fractions

Same signs - Add

Example: $\frac{1}{6}+\frac{3}{8}= \frac{4}{24}+\frac{9}{24} = \frac{13}{24}$

Find a LCD and equivalent fractions

Same signs - Add

Example: $3\frac{1}{3}+2\frac{1}{5} = 3\frac{5}{15}+2\frac{3}{15} = 5\frac{8}{15}$

Example: $1.4+1.3=2.7$

Example: $7.6+4.9=12.5$

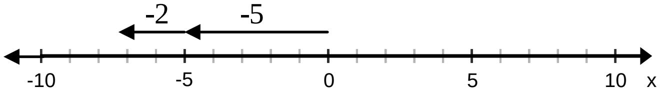
Math 7 Notes – Part B: Rational Numbers

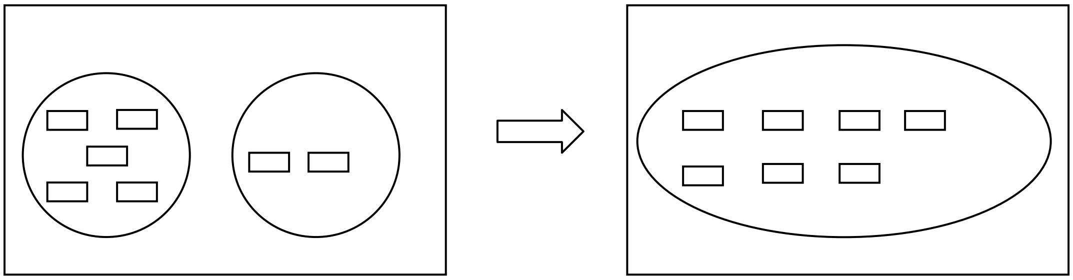
Review with integers then add rationals

Rule 2: **When adding two negative numbers,**

find the sum of their absolute values, and the answer is positive.

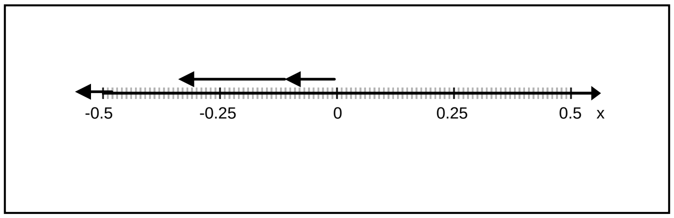
Modeling Examples: –5 + –2= –7





$$\begin{array}{r} -.11 \\ \underline{+\ -.23} \\ -.34 \end{array}$$

Example: $(-.11)+(-.23)=-.34$ ***or***



Same signs – Add
Answer is negative

Example: $-0.3+-0.6=-0.9$

Same signs – Add
Answer is negative

Example: $\frac{-3}{7}+\frac{-2}{7}=\frac{-5}{7}$

Same signs – Add
Answer is negative

Example: $\left(-\frac{5}{10}\right)+\left(-\frac{2}{10}\right)=-\frac{7}{10}$

Math 7 Notes – Part B: Rational Numbers

Same signs – Add
Answer is negative

Example: $\frac{-6}{12}+\frac{-1}{12}=\frac{-7}{12}$

Find a LCD and
equivalent fractions

Same signs – Add
Answer is negative

Example: $\frac{-1}{2}+\frac{-1}{3}= \quad \frac{-3}{6}+\frac{-2}{6}= \quad \frac{-5}{6}$

Example: $-\frac{3}{4} + -\frac{2}{5} = \quad -\frac{15}{20} + -\frac{8}{20} = -\frac{23}{20}$ *or* $-1\frac{3}{20}$

Example: $-4\frac{4}{7} + -5 = -9\frac{4}{7}$

Example: $-1\frac{3}{4} + -2\frac{1}{4} = -3\frac{4}{4} = -4$

Example: $-2\frac{7}{11} + -4\frac{8}{11} = -6\frac{15}{11} = -7\frac{4}{11}$

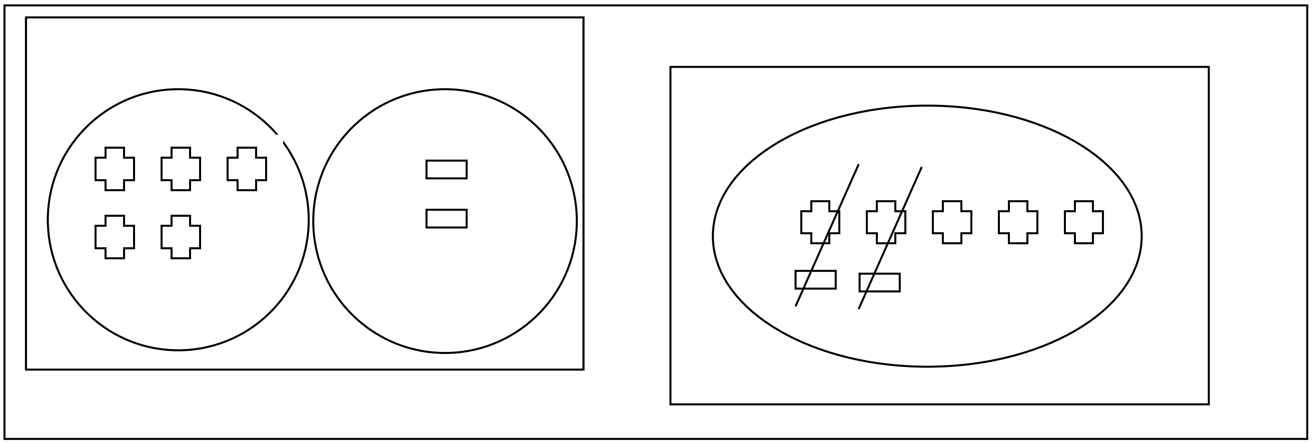
Example: $-3\frac{5}{6} + -4\frac{2}{3} = \quad -3\frac{5}{6} + -4\frac{4}{6} = -7\frac{9}{6} = -8\frac{3}{6} = -8\frac{1}{2}$

Rule 3: When adding one positive and one negative number,

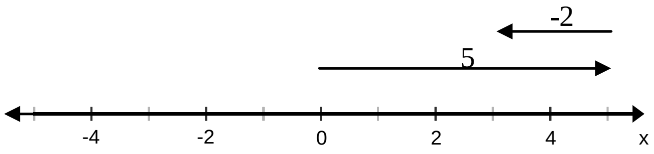
Rule 3: When adding one positive and one negative number,

find the difference of their absolute values, and keep the signe of the the number with the greater absolute value.

Modeling Example: $(+5)+(-2)=+3$



Math 7 Notes – Part B: Rational Numbers



$$(-7)+(+5)=-2$$

$$-3+7=4$$

Different signs – Subtract

Take sign of greater $|\;|$

$$-0.9+0.4= \quad -0.5$$

Different signs – Subtract

Take sign of greater $|\;|$

$$1.6+-0.9= \quad 0.7$$

Different signs – Subtract

Take sign of greater $|\;|$

$$-4.5+8.6= \quad 4.1$$

Different signs – Subtract

Take sign of greater $|\;|$

$$-9.8+8.4= \quad -1.4$$

Different signs – Subtract

Take sign of greater $|\;|$

$$\left(+\frac{2}{5}\right)+\left(-\frac{3}{5}\right)= \quad -\frac{1}{5}$$

Different signs – Subtract

Take sign of greater $|\;|$

Simplify

$$\left(\frac{-7}{10}\right)+\left(\frac{3}{10}\right)= \quad \frac{-4}{10} \quad = \quad \frac{-2}{5}$$

Different signs – Subtract

Take sign of greater $|\;|$

$$\frac{4}{9}+\frac{-2}{9}= \quad \frac{2}{9}$$

Different signs – Subtract

Take sign of greater $|\;|$

Simplify

$$\frac{5}{12}+\frac{-1}{12}= \quad \frac{4}{12} \quad = \quad \frac{1}{3}$$

Find a LCD and equivalent fractions

Different signs – Subtract

Take sign of greater $|\;|$

$$\left(\frac{-3}{4}\right)+\left(\frac{1}{2}\right)= \quad \left(\frac{-3}{4}\right)+\left(\frac{2}{4}\right) \quad = \quad \frac{-1}{4}$$

Find a LCD and equivalent fractions

Different signs – Subtract

Take sign of greater $|\;|$

$$\left(\frac{1}{2}\right)+\left(\frac{-1}{3}\right)= \quad \left(\frac{3}{6}\right)+\left(\frac{-2}{6}\right) \quad = \quad \frac{1}{6}$$

Math 7 Notes – Part B: Rational Numbers

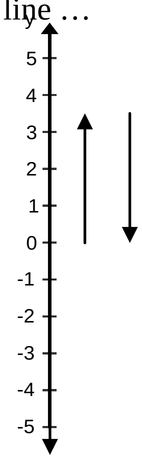
NVACS 7.NS.A.1b – *Understand p + q as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is a positive or negative.* ***Show that a number and its opposite have a sum of 0 (are additive inverses).*** *Interpret sums of rational numbers by describing real-world contexts.*

Example: Show $8 + -8 = 0$

We know … if you only have \$8 and you spend \$8 that you are broke, you have zero dollars .

… that a gain of $3\frac{1}{2}$ yards and a loss of $3\frac{1}{2}$ yards is zero.

SHOW….We can show this on a number line …



After several examples (using a variety of methods like those above) students should begin to see that adding opposites equals 0. They need to be introduced to the formal property name for this concept at this time…**Additive Inverses.**

Example: For any real number b, $b+(-b)$ has what value?

Example: Write a numeric example, using rational numbers, to illustrate the Additive Inverses Property.

Example: Complete each of the following.

$\frac{3}{4} + \frac{-3}{4} = \boxed{}$

$-2.57 + \boxed{} = 0$

$3\frac{4}{5} + \boxed{} = 0$

$\boxed{} + 23.896 = 0$

Math 7 Notes – Part B: Rational Numbers

Examples:

A submarine descends to a depth of 480 meters below sea level. Write an integer to represent this situation. Then find the additive inverse of the integer, and tell what it represents.

a. Integer: +480
Additive Inverse: –480
The additive inverse represents the distance, in meters, that the submarine would need to rise to return to sea level.

b. Integer: –480
Additive Inverse: +480
The additive inverse represents the distance, in meters, that the submarine would need to descend to return to sea level.

c. Integer: –480
Additive Inverse: +480
The additive inverse represents the distance, in meters, that the submarine would need to rise to return to sea level.

d. Integer: +480
Additive Inverse: –480
The additive inverse represents the distance, in meters, that the submarine would need to descend to return to sea level.

An investment lost $975. Write an integer to represent this situation. Then find the additive inverse of the integer, and tell what it represents.

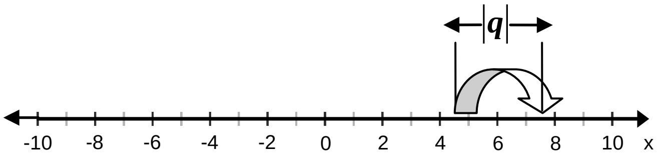
a. Integer: –975
Additive Inverse: +975
The additive inverse represents the amount, in dollars, that the investment would have to earn to return to its starting value.

b. Integer: –975
Additive Inverse: +975
The additive inverse represents the amount, in dollars, that the investment would have to lose to return to its starting value.

c. Integer: +975
Additive Inverse: –975
The additive inverse represents the amount, in dollars, that the investment would have to earn to return to its starting value.

d. Integer: +975
Additive Inverse: –975
The additive inverse represents the amount, in dollars, that the investment would have to lose to return to its starting value.

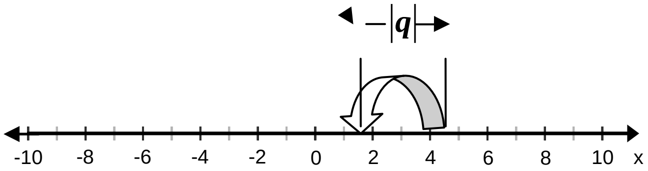
NVACS 7.NS.A.1b- Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. *Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.*



Let $p = 4$, then looking at the problems such as

$$4\frac{1}{2}+(-3) \text{ and } 4\frac{1}{2}+3,$$

we start at $4\frac{1}{2}$ and move "q" units (in these examples 3 units). If q is positive we move q units right; if q is negative we move q units left.



Math 7 Notes – Part B: Rational Numbers

Subtracting Rational Numbers

NVACS 7.NS.A.1c – Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

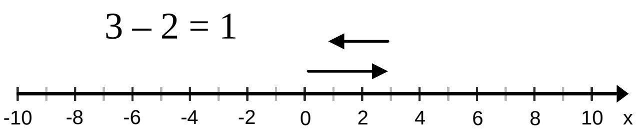
Subtraction – is defined as **adding the opposite.**

Remind students by reviewing some integer problems such as:

Examples: 3 – 2 is the same as 3 plus the opposite of 2 OR 3 + (–2)

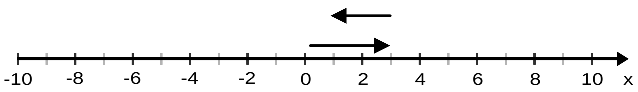
Let's review this on a number line. We show both 3–2 and 3+(–2)

3 – 2 = 1

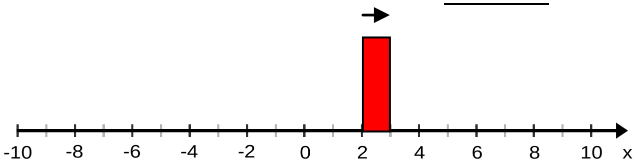


Looking at 3+ (–2) graphically, we see (Start at 0, move 3 units right, then 2 units left)

3+(–2) = 1

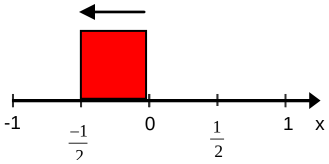


Keep in mind here what we found was the difference or <u>distance</u> between 2 to 3 graphically.



Example: $\frac{-1}{2} - 0 = \frac{-1}{2}$

We found the distance between 0 and $\frac{-1}{2}$.



Examples: $4 - 3 = 4 + (-3) = 1$

$(-8) - (-6) = -8 + 6 = -2$

$\frac{2}{3} - \frac{2}{3} = \frac{2}{3} + \left(\frac{-2}{3}\right) = 0$

$0 - (-0.4) = 0 + 0.4 = 0.4$

Math 7 Notes – Part B: Rational Numbers

$\frac{4}{7}-\frac{3}{7}=\frac{4}{7}+\frac{-3}{7}=\frac{1}{7}$ $\frac{-8}{9}-\frac{-6}{9}=\frac{-8}{9}+\frac{6}{9}=\frac{-2}{9}$

$-0.5-0.3=-0.5+(-0.3)=-0.8$ $3.8-4.8=3.8+(-4.8)=-1$

Rule 4: When subtracting signed numbers,

(A) ***change the sign of the subtrahend (second number) and add using rule 1, 2, or 3, whichever applies.***

OR

(B) ***add the opposite.***

Example**:** 6 – (+13) ⟶ 6 + (–13) Change sign and add, according to rule 3.

–7

Example**:** (–4) – (+2) ⟶ (–4) + (–2) Change sign and add, according to rule 2.

–6

Alternative Rules for Adding and Subtracting Numbers

1. When the signs are the SAME, find the SUM of their absolute values, and keep the common sign.

Examples: $5+2=7$ $(-5)+(-3)=-8$ $\frac{-3}{11}+\frac{-1}{11}=\frac{-4}{11}$ $0.2+0.4=0.6$

2. When the signs are DIFFERENT, find the DIFFERENCE of their absolute values, and use the sign of the number with the greater absolute value.

Examples: 5 + (–12) = –7 (–4) + 10 = +6 $\frac{-5}{7}+\frac{2}{7}=\frac{-3}{7}$ $0.9+(-0.6)=0.3$

Remember to change each subtraction problem to an addition problem, since subtracting is ADDING THE OPPOSITE; then use the rules listed above.

Examples: (–2) – 8 = (–2) + (–8) = –10 3 – (–6) = 3 + 6 = 9 $\left(\frac{1}{10}\right)-\left(\frac{-8}{10}\right)=\frac{1}{10}+\frac{8}{10}=\frac{9}{10}$

Identify the number(s) that makes each statement true. You may select more than one number for each statement.

$-\frac{\mathbf{3}}{\mathbf{8}} + \square$ = a positive number ○ $\frac{\mathbf{-5}}{\mathbf{8}}$ ○ $\frac{5}{8}$

$\square$ **– .04** = a negative number ○ **–3** ○ **0**

–2 – $\square$ = a negative number ○ **–0.7** ○ **3.1**

$\square - \frac{-4}{5}$ = *zero* $-\frac{4}{5}$ ○ $\frac{4}{5}$

NVACS 7.NS.A.1b – *Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses).* ***Interpret sums of rational numbers by describing real-world contexts.***

Example: A ship set sail from Hawaii, whose latitude is 20,35°N. The ship sailed 10.5°S, then 8.2°N then 5.9°S. What was the latitude of the ship at that point?

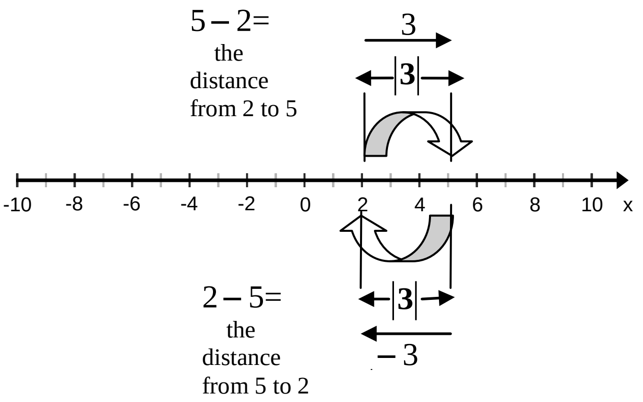
Example: Stock in LUK Corporation was issued at $100 per share. In the next three years it went up $21.75 per share, down $15.25 and down $17.81. What was the price of a share of stock at the end of the 3 year period?

Example: On Monday the level of water in the spa fell $\frac{1}{2}$ inch. On Tuesday it fell $\frac{1}{3}$ inch. What was the total change in the water level?

NVACS 7.NS.A.1c – *Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q).* ***Show that the distance between two rational numbers on the number line is the absolute value of their difference,*** and *apply this principle in real-world contexts****.***

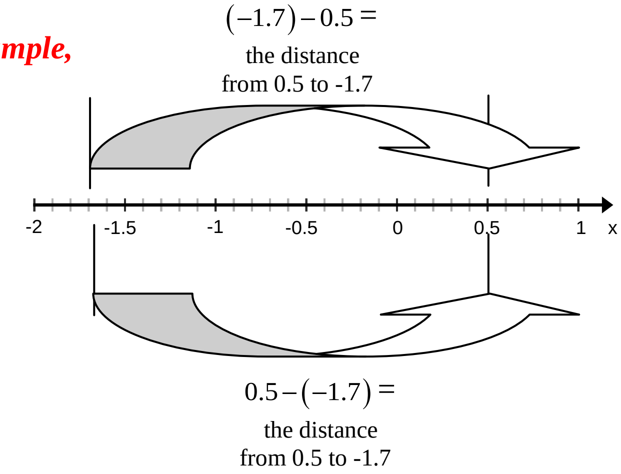
Math 7 Notes – Part B: Rational Numbers

Notice the following:



In both the examples $5-2$ and $2-5$, we notice the distance between these numbers is $|\mathbf{3}|$. If we subtract $5-2$, we get 3. If we subtract $2-5$, we get -3.

In the example,



In both the examples $(-1.7)-0.5$ and $0.5-(-1.7)$, we notice the distance between these numbers is $|\mathbf{2.2}|$. If we subtract $(-1.7)-0.5$, we get –2.2. If we subtract $0.5-(-1.7)$, we get 2.2.

NVACS 7.NS.A.1c – *Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference,* and ***apply this principle in real-world contexts.***

Example: Kent bought $15\frac{3}{4}$ yd of drapery material for $63 at a sale. If she used all except $1\frac{1}{16}$ yd, how much material did she actually use?

Example: Jeff and Jo vacationed on an island. Jeff choose to cliff dive and Jo went scuba diving. If Jeff climbed to a height of 15.2 m and Jo dove to a depth of 37.2 m, how far apart were they?

Example: These announcements were heard at a rocket launch: “Minus 45 seconds and counting” and “We have second-stage ignition at plus 110 seconds”. How much time past between the announcements?

Math 7 Notes – Part B: Rational Numbers

Example: A plane flew from Las Vegas, NV latitude 36.1215° N, to Sydney, Australia latitude 33.86° S. What was the plane's change in latitude?

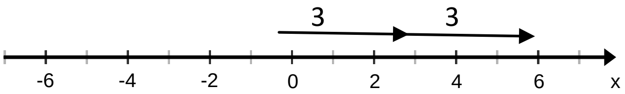
Example: Paul Garcia's first-of-the-month balance was $250.50. During the month he withdrew $65.50 and $258, and made one deposit. How large was the deposit if his end-of-the-month balance was $526?

Multiplying Rational Numbers

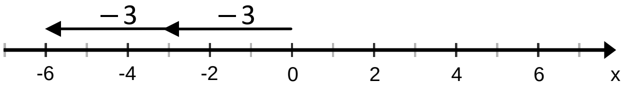
NVACS 7.NS.A.2 – Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.

Review

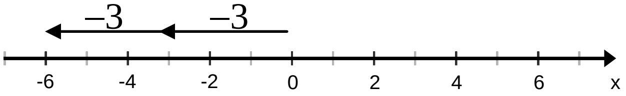
Example: Using a number line, show 2(3). Remind students this means 2 groups of 3 units.



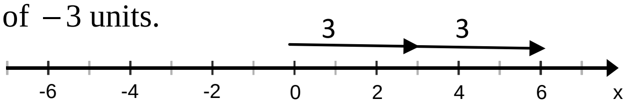
Example: Using a number line, show $2(-3)$. Remind students this means 2 groups of -3 units.



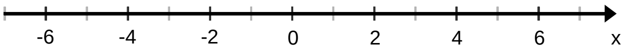
Example: Using a number line, show $-2(3)$. Remind students this means <u>the opposite of</u> 2 groups of +3 units.



Example: Using a number line, show $-2(-3)$. Remind students this means <u>the opposite of</u> 2 groups of -3 units.

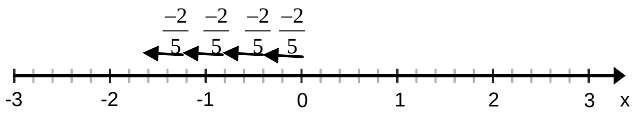


Example: Using a number line, show $2\left(3\frac{1}{2}\right)$ = 7



Math 7 Notes – Part B: Rational Numbers

Example: Using a number line, show $4\left(\frac{-2}{5}\right)$ $= \frac{-8}{5} = -1\frac{3}{5}$



NVACS 7.NS.A.2a– Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. *Interpret products of rational numbers by describing real-world contexts.*

Rule 5: When multiplying or dividing two numbers with the same sign, the answer is positive.

Rule 6: When multiplying or dividing two numbers with different signs, the answer is negative.

Examples: $(+5)\bullet(+4) = +20$ $(-3)\bullet(-12) = (+36)$ $12 \div 3 = 4$ $-15 \div -3 = 5$

Examples: $(+7)\bullet(-6) = (-42)$ $(-8)\bullet(+7) = (-56)$ $-16 \div 2 = -8$ $30 \div -5 = -6$

To work with other rational numbers (not just integers), students will need to blend the rules they have learned for operating with integers with the rules for operating with fractions and decimals. Now is a great time to review those skills and begin to infuse the negative rational numbers.

Review

Algorithm for Multiplying Decimals:

Step 1. Rewrite the numbers vertically (if needed)
Step 2. Multiply normally, ignoring the decimal point
Step 3. Count the number of digits to the right of the decimal points
Step 4. Count and then place the decimal point that same number of places from right to left in the product (answer)

$(-0.3)(-0.2) = \ +0.06$

$(1.1)(-7) = \ -7.7$

$(5)(-1.4) = \ -7.0 = -7$

$$(-3.4)(-5.12) = \quad \begin{array}{r} -5.12 \\ x\ -3.4 \\ \hline 2048 \\ +1536 \\ \hline +17.408 \end{array} \qquad +17.408$$

Review

Algorithm for Multiplying Fractions:

Step 1. Make sure you have proper or improper fractions
Step 2. Cancel if possible
Step 3. Multiply numerators
Step 4. Multiply denominators
Step 5. Simplify if needed

Different signs are negative

Example: $\frac{-1}{2} \bullet \frac{3}{5} = \frac{-3}{10}$

Same signs are positive

Example: $\frac{-3}{4} \bullet \frac{-1}{3} = \frac{\overset{-1}{\cancel{-3}}}{4} \bullet \frac{-1}{\underset{1}{\cancel{3}}} = \frac{1}{4}$

Different signs are negative

Example: $-1\frac{1}{2} \bullet \frac{3}{8} = \frac{-3}{2} \bullet \frac{3}{8} = \frac{-9}{16}$

Same signs are positive

$\left(\frac{2}{5}\right)4 = \frac{2}{5} \bullet \frac{4}{1} = \frac{8}{5} = 1\frac{3}{5}$

Math 7 Notes – Part B: Rational Numbers

Different signs are negative

$$3\left(-\frac{3}{4}\right)=\frac{3}{1}\bullet\frac{-3}{4}=\frac{-9}{4}=-2\frac{1}{4}$$

Same signs are positive

$$\left(-3\frac{1}{2}\right)\left(-\frac{3}{7}\right)=\frac{\overset{-1}{\cancel{-7}}}{2}\bullet\frac{-3}{\underset{1}{\cancel{7}}}=\frac{3}{2}=1\frac{1}{2}$$

Same signs are positive

$$\left(-3\frac{1}{3}\right)\left(-1\frac{1}{5}\right)=\frac{-10}{3}\bullet\frac{-6}{5}=\frac{\overset{-2}{\cancel{-10}}}{\underset{1}{\cancel{3}}}\bullet\frac{\overset{-2}{\cancel{-6}}}{\underset{1}{\cancel{5}}}=\frac{+4}{1}=4$$

NVACS 7.NS.A.2b – Understand that integers can be divided, provided that the divisor is not zero, *and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then* $-(p/q)=(-p)/q=p/(-q)$. *Interpret quotients of rational numbers by describing real world contexts.*

Students should be able to build on knowledge from work with whole numbers and integers that division by 0 is undefined. Teachers must explicitly review that division by 0 is undefined with rational numbers also. Be sure to show them:

$0\overline{)-3.25}$ and $\frac{1}{2}\div 0$ are undefined (cannot be done)   **but also show** $-1.5\overset{0}{\overline{)0}}$ *and* $0\div\frac{1}{2}=0$

Algorithm for Dividing Decimals

1. **In the divisor, move the decimal point as far to the right as possible.**
2. **In the dividend, move the decimal point the same number places to the right.**
3. **Bring the decimal point straight up into the quotient.**
4. **Divide in the normal way.**

Math 7 Notes – Part B: Rational Numbers

***Example*:** $.12\overline{)\,.456}$ Move the decimal point 2 places to the right in the divisor.

$\underset{\curvearrowright\curvearrowright}{12}\overline{)\,.456}$ Move the decimal point 2 places to the right in the dividend.

$12\overline{)\,\underset{\curvearrowright\curvearrowright}{45.6}}$

$$\begin{array}{r} 3.8 \\ 12\overline{)\,45.6} \\ -\underline{36} \\ 96 \\ -\underline{96} \\ 0 \end{array}$$

Divide as usual –
- Divide
- Multiply
- Subtract
- COMPARE
- Bring down
- Repeat if needed

Example: Compute: $7.5 \div -.15 =$

$$-.15\overline{)\,7.5} \rightarrow \begin{array}{r} -50. \\ -15\overline{)\,750.} \\ -\underline{75} \\ 00 \\ -\underline{00} \\ 0 \end{array}$$

Move decimal point two places to the right in the divisor to make it a whole number.
Move decimal point two places to the right in the dividend.
Bring decimal point straight up into the quotient.
Divide as usual
Determine the sign.

Why does this work? By moving the decimal the same number of places to the right in the divisor and the dividend, we are essentially multiplying our original expression by one. We are making equivalent fractions by multiplying the numerator and denominator by the same number. If we move the decimal point one place, we are multiplying the numerator and denominator by 10. By moving it two places, we are multiplying the numerator and denominator by 100, etc.

$$\frac{.456}{.12}\times\frac{100}{100}=\frac{45.6}{12}$$

Math 7 Notes – Part B: Rational Numbers

Dividing by Powers of 10

Examining the patterns discovered when dividing by powers of 10 (10, 100, 1,000 …) we can do many division problems mentally. What pattern do you see from these problems?

67.89 ÷ 10 = 6.789 2398.6 ÷100 = 23.986 2468 ÷ 1000 = 2.468
654 ÷ 10 = 65.4 78 ÷ 100 = .78 8 ÷ 1000 = .008

We can generalize a rule from the above observed pattern: when dividing by positive powers of 10, move the decimal point to the left, the same number of places as there are zeros.

Example: 5.6 ÷ 10

Since there is *one* zero in 10, move the decimal point *one* place to the left.
5.6 ÷ 10 = .56

Example: 9832 ÷ 100

Since there are *two* zeros in 100, move the decimal point *two* places to the left.
9832 ÷ 100 = 98.32

Example: 92 ÷ 1000

There are *three* zeros in 1,000, so move the decimal point *three* places to the left.
9.2 ÷ 1000 = .0092 Notice in this problem we had to fill in a couple of placeholders to move it three places to the left.

Dividing Rational Numbers

Before we learn how to divide fractions, let’s revisit the concept of division using whole numbers. When I ask, how many 2’s are there in 8, I can write that mathematically three ways.

$$2\overline{)8} \qquad \frac{8}{2} \qquad 8 \div 2$$

To find out how many 2’s there are in 8, we will use the subtraction model:

$$\begin{array}{r} 8 \\ \underline{-2} \\ 6 \\ \underline{-2} \\ 4 \\ \underline{-2} \\ 2 \\ \underline{-2} \\ 0 \end{array}$$

Now, how many times did we subtract 2? Count them: there are 4 subtractions. So there are 4 twos in eight.

Mathematically, we say 8 ÷ 2 = 4.

Math 7 Notes – Part B: Rational Numbers

Division is defined as repeated subtraction. That won't change because we are using a different number set. In other words, to divide fractions, I could also do repeated subtraction.

Example: $1\frac{1}{2} \div \frac{1}{4}$

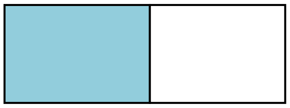
Another way to look at this problem is using your experiences with money. How many quarters are there in $1.50? Using repeated subtraction we have:

$$\begin{array}{r} 1\frac{1}{2}=1\frac{2}{4} \\ -\frac{1}{4} \\ \hline 1\frac{1}{4} \\ -\frac{1}{4} \\ \hline 1 \end{array} \longrightarrow \begin{array}{r} 1=\frac{4}{4} \\ -\frac{1}{4} \\ \hline \frac{3}{4} \\ -\frac{1}{4} \\ \hline \frac{2}{4} \end{array} \longrightarrow \begin{array}{r} \frac{2}{4} \\ -\frac{1}{4} \\ \hline \frac{1}{4} \\ -\frac{1}{4} \\ \hline 0 \end{array}$$

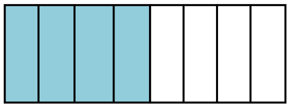
How many times did we subtract $\frac{1}{4}$? Six. But this took a lot of time and space.
A visual representation of division of fractions would look like the following.

Example: $\frac{1}{2} \div \frac{1}{8} =$

We have $\frac{1}{2}$. Representing that would be



Since the question we need to answer is how many $\frac{1}{8}$'s are there in $\frac{1}{2}$, we need to cut this entire diagram into eighths. Then count each of the shaded one-eighths.

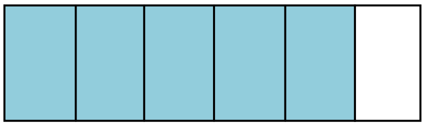


As you can see there are four. So $\frac{1}{2} \div \frac{1}{8} = 4$.

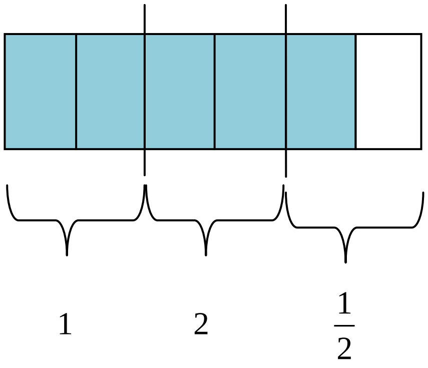
Math 7 Notes – Part B: Rational Numbers

Example: $\frac{5}{6} \div \frac{1}{3} =$

We have $\frac{5}{6}$. Representing that would be



Since the question we need to answer is how many $\frac{1}{3}'s$ are there in $\frac{5}{6}$, we need to use the cuts for thirds only. Then count each of the one thirds.



As you can see there are $2\frac{1}{2}$. So $\frac{5}{6} \div \frac{1}{3} = 2\frac{1}{2}$.

Be careful to choose division examples that are easy to represent in visual form.

Well, because some enjoy playing with numbers, they found a quick way of dividing fractions. They did this by looking at fractions that were to be divided and they noticed a pattern. And here is what they noticed.

Algorithm for Dividing Fractions and Mixed Numbers

1. **Make sure you have proper or improper fractions.**
2. **Invert the divisor (2nd number).**
3. **Cancel, if possible.**
4. **Multiply numerators.**
5. **Multiply denominators.**
6. **Determine your sign and simplify.**

The very simple reason we tip the divisor upside-down (use the reciprocal), then multiply (for division of fractions), is because it works. And it works faster than if we did repeated subtractions, not to mention it takes less time and less space.

Example: $-\frac{3}{4} \div \frac{2}{5} \longrightarrow -\frac{3}{4} \bullet \frac{5}{2} \longrightarrow -\frac{15}{8} = -1\frac{7}{8}$

(Invert the divisor.)

Multiply numerators and denominators, and simplify.

Let's look at this same problem in the format of complex fractions. In this format we need to get the denominator to be equal to 1. To do that we multiply the top and bottom by $\frac{5}{2}$, remember this is another name for 1 whole $\left(\frac{\frac{5}{2}}{\frac{5}{2}}\right)$. This is a much better explanation of why we invert the divisor and multiply.

Example: $-\frac{3}{4} \div \frac{2}{5} = \frac{-\frac{3}{4}}{\frac{2}{5}} = \frac{-\frac{3}{4} \bullet \frac{5}{2}}{\frac{2}{5} \bullet \frac{5}{2}} = \frac{-\frac{15}{8}}{1} = -\frac{15}{8}$ *or* $-1\frac{7}{8}$

Example: $3\frac{1}{3} \div \frac{4}{9} \longrightarrow \frac{10}{3} \div \frac{4}{9}$ Make sure you have proper or improper fractions.

$\frac{10}{3} \bullet \frac{9}{4}$ Invert the divisor.

$\frac{\overset{5}{\cancel{10}}}{\underset{1}{\cancel{3}}} \bullet \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{4}}}$ Cancel 10 and 4 by 2, and cancel 9 and 3 by 3.

$\frac{5}{1} \bullet \frac{3}{2} = \frac{15}{2}$ Multiply numerators and denominators.

$\frac{15}{2} = 7\frac{1}{2}$ Simplify.

Math 7 Notes – Part B: Rational Numbers

7.EE.3-3 Convert between forms of rational numbers as appropriate.

Example: Karen worked $12\frac{1}{2}$ hours last week and earned \$100. What was her hourly rate of pay?

$100 \div 12\frac{1}{2}$ *or* $100 \div 12.5$ \$8 per hour

Example: A radio station released 300 balloons at an outdoor celebration. Of these, $\frac{3}{4}$ were orange. How many were orange balloons?

$\frac{3}{4} \bullet 300$ *or* $.75(300)$ 225 orange balloons

Example: A town has raised $\frac{3}{8}$ of the \$12,000 it needs to furnish its new library. How much more is it hoping to raise?

$1-\frac{3}{8}=\frac{5}{8}$ *so* $\frac{5}{8}(12,000)$ *or* $.625(12,000)$ \$7,500

CAUTION: Be sure to include absolute values with simple expressions once operations have been taught.

Examples: $|5-(-3)|=|8|=8$

$$\left|\frac{3}{4}+\left(-\frac{2}{4}\right)\right|=\left|\frac{1}{4}\right|=\frac{1}{4}$$

$$|0.5(-8)|=|-4.0|=4$$

$$|.25-.27|=|-.02|=.02$$

$$\mathbf{3\left|-\frac{2}{3}-\frac{1}{3}\right|=3\left|-\frac{3}{3}\right|=3(1)=3}$$

Math 7 Notes – Part B: Rational Numbers

Once again students need to review powers but this time including rational numbers.

$(-8)^2 = (-8)(-8) = 64$

$(-0.5)^2 = (-0.5)(-0.5) = 0.25$

$(-5)^3 = (-5)(-5)(-5) = -125$

$\left(-\frac{3}{4}\right)^3 = \left(\frac{-3}{4}\right)\left(\frac{-3}{4}\right)\left(\frac{-3}{4}\right) = -\frac{27}{64}$

Notice the difference in these two problems. In $-(2)^4$ you are taking the opposite of (2) to the fourth power, whereas in $(-2)^4$ you are taking (−2)to the fourth power.

Examples: $-(2)^4 \, or -2^4 = -(2)(2)(2)(2) = -16$

$(-2)^4 = (-2)(-2)(-2)(-2) = 16$

$-\left(\frac{1}{3}\right)^2 = -\left(\frac{1}{3}\right)\left(\frac{1}{3}\right) = -\left(\frac{1}{9}\right) = -\frac{1}{9}$

$\left(\frac{-1}{3}\right)^2 = \left(\frac{-1}{3}\right)\left(\frac{-1}{3}\right) = \frac{1}{9}$

$-(0.4)^2 = -(0.4)(0.4) = -(0.16) = -0.16$

$(-0.4)^2 = (-0.4)(-0.4) = 0.16$

Example: $-6^2 \bullet 2 = -36 \bullet 2 = -72$

Example: $(-6 \bullet 2)^2 = (-12)^2 = 144$

Example: $8(-1)^3 - 7(-1)^2 + 2(-1) + 4 = 8(-1) - 7(1) - 2 + 4 = -8 - 7 - 2 + 4 = -13$

Example: $\frac{3 \cdot 10^2 - 4(-15+6)}{-4^2 + 3(3+2) + 2} = \frac{300 - 4(-9)}{-16 + 15 + 2} = \frac{300 + 36}{1} = \frac{336}{1} = 336$

Examples: Identify the following statements as true or false. If the statement is false give a reason or an example to show it is false.

The square of every negative rational number is positive.
True - because a $- \bullet - = +$

Every rational number and its opposite have equal squares. True

The cube of every negative rational number is positive. False
The cube of every negative integer is negative.

$$\left(\frac{-1}{3}\right)^3 = \left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right) = \frac{-1}{27}$$

$(-0.2)^3 = (-0.2)(-0.2)(-0.2) = -0.008$

Math 7 Notes – Part B: Rational Numbers

Every rational number and its opposite have equal cubes. False

$$(0.3)^3 = 0.027,\ (-0.3)^3 = -0.027$$

$$0.027 \neq -0.027$$

The greater of two rational numbers has the greater square. False

$$\frac{1}{2} > \frac{-1}{3}$$

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\left(-\frac{1}{3}\right)^2 = \frac{1}{9}$$

$$\frac{1}{4} > \frac{1}{9} \quad \textit{True}$$

$$-0.3 > -0.5$$

$$(-0.3)^2 = 0.09$$

$$(-0.5)^2 = 0.25$$

$$0.09 > 0.25 \quad \textit{False}$$

The greater of two rational numbers has the greater cube. True

$$\frac{1}{2} > \frac{1}{3}$$

$$\left(\frac{1}{2}\right)^3 = \frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} = \frac{1}{8}$$

$$\left(\frac{1}{3}\right)^3 = \frac{1}{3} \bullet \frac{1}{3} \bullet \frac{1}{3} = \frac{1}{27}$$

$$\frac{1}{8} > \frac{1}{27} \quad \textit{True}$$

$$\frac{-1}{3} > \frac{-1}{2}$$

$$\left(\frac{-1}{3}\right)^3 = \frac{-1}{3} \bullet \frac{-1}{3} \bullet \frac{-1}{3} = \frac{-1}{27}$$

$$\left(\frac{-1}{2}\right)^3 = \frac{-1}{2} \bullet \frac{-1}{2} \bullet \frac{-1}{2} = \frac{-1}{8}$$

$$\frac{-1}{27} > \frac{-1}{8} \quad \textit{True}$$

$$0.5 > 0.4$$

$$(0.5)^3 = (0.5)(0.5)(0.5) = 0.125$$

$$(0.4)^3 = (0.4)(0.4)(0.4) = 0.064$$

$$0.125 > 0.064 \quad \textit{True}$$

$$-0.4 > -0.5$$

$$(-0.4)^3 = (-0.4)(-0.4)(-0.4) = -0.064$$

$$(-0.5)^3 = (-0.5)(-0.5)(-0.5) = -0.125$$

$$-0.064 > -0.125 \quad \textit{True}$$

$$0.4 > -0.5$$

$$(0.4)^3 = (0.4)(0.4)(0.4) = -0.064$$

$$(-0.5)^3 = (-0.5)(-0.5)(-0.5) = -0.125$$

$$0.064 > -0.125 \quad \textit{True}$$

Example: Which statement is true?

A	$8 \div \left(\frac{-1}{2}\right) = -4$
B	$-20 \div \frac{1}{4} = 80$

C	$-14 \div \left(\frac{-1}{7}\right) = -2$
D	$15 \div \left(\frac{-1}{5}\right) = -75$

NVACS 7.NS.A.2d – Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0's or eventually repeats.

A decimal is equivalent to a fraction whose denominator is a power of 10. The numerator is the number to the right of the decimal point.

Strategy 1

If the denominator will divide evenly into a power of 10, then make equivalent fractions and use the definition of a decimal to write the decimal numeral.

Examples: $\frac{3}{4} = \frac{75}{100} \to 0.75$ $\qquad$ $\frac{2}{5} = \frac{4}{10} \to 0.4$

Strategy 2

If the denominator has prime factors other than 2 or 5, the decimal will be non-terminating (will not end), and the best way to determine the decimal equivalent is by dividing the numerator by the denominator.

Example: Convert : $\frac{1}{3}$ to a decimal.

$$\begin{array}{r} 0.3333 \\ 3\overline{\smash{)}1.0000} \end{array} = 0.333... = 0.\overline{3}$$

The bar (called a **vinculum**) over the digit 3 means that digit is repeating over and over and over…to infinity.

Example: Convert : $\frac{7}{12}$ to a decimal.

Since 12 has prime factors other than 2 or 5, the decimal equivalent will be non-terminating.

Divide the numerator 7, by the denominator 12.

$$\begin{array}{r} 0.58333 \\ 12\overline{)7.00000} \end{array} = 0.58\overline{3}$$

Notice here the 5 and the 8 are NOT repeating, therefore the vinculum does not go over the digits 5 and 8.

Example: Convert : $\frac{2}{7}$ to a decimal.

Since 7 is prime and contains factors other than 2 or 5, the decimal equivalent will be non-terminating.
Divide the numerator 2, by the denominator 7.

$$\begin{array}{r} .2857142.... \\ 7\overline{)2.000000} \end{array} = 0.\overline{285714}$$

Notice this time all 6 digits repeat so the vinculum is over all the digits. Sometimes we must divide out a number of places before we see the pattern.

NVACS 7.NS.A.2a – Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. ***Interpret products of rational numbers by describing real-world contexts.***

Example: Your cell phone bill of $52.75 is automatically deducted from your bank account each month. How much will the deductions total for the year?

$$\mathbf{12(-52.75) = -633}$$

The bank will deduct $633 dollars from your account.

Example: Josh made four withdrawals of $100.50 from his bank account. How much did he withdraw in total?

$$\mathbf{4(-100.50) = 402.00}$$

Josh withdrew $402.

Example: The price of one share of Arcan Company declined $32.21 per month for six consecutive months. How much did the price of one share decline in total for that period of six months?

$$\mathbf{6(-32.21) = -193.26}$$

Each share decreased $193.26.

Math 7 Notes – Part B: Rational Numbers

NVACS 7.NS.A.2b – *Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number.* ***If p and q are integers, then*** $-(p/q)=(-p)/q=p/(-q)$**.** *Interpret quotients of rational numbers by describing real world contexts.*

Students should understand that $-\dfrac{\mathbf{15}}{\mathbf{3}}=\dfrac{\mathbf{-15}}{\mathbf{3}}=\dfrac{\mathbf{15}}{\mathbf{-3}}$ each have the same value.

Examples: If p=any integer and q= any integer except 0, then their quotient is a rational number.

1. Let $p=-7$ and $q=3$

then $\frac{p}{q}=\frac{-7}{3}$

$\dfrac{-7}{3}$ is rational

2. Let $p=0$ and q = -5

$$\frac{p}{q}=\frac{0}{-5}=0$$

0 is rational

3. Let $p=-6$ and q = -3

$$\frac{p}{q}=\frac{-6}{-3}=2$$

2 is rational

Note: The quotient of any two integers is rational and this IS or can be used as a definition.

NVACS 7.NS.A.2b– *Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then* $-(p/q)=(-p)/q=p/(-q)$. ***Interpret quotients of rational numbers by describing real world contexts.***

Examples: It took a submarine 20 seconds to drop to 100 feet below sea level from the surface. What was the rate of the descent?

$\mathbf{-100\div 20=-5}$ **–5** *feet per second*

NVACS 7.NS.A.3 Solve real-world and mathematical problems involving the four operations with rational numbers.

Example: Mary is baking cookies for the holidays. The recipe calls for $1\frac{3}{4}$ cups of flour, $\frac{2}{3}$ cup of sugar and $\frac{1}{2}$ pound of butter. If she wants to cut the recipe in half, how much of each ingredient will she need? *Solution:* $\frac{7}{8}$ cup flour, $\frac{1}{3}$ cup sugar and $\frac{1}{4}$ pound butter

Math 7 Notes – Part B: Rational Numbers

NVACS 7.EE.B.3 - Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form *(whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and* ***assess*** **the reasonableness of answers using mental computation and estimation strategies.** ***Of course estimation should be an on-going skill throughout the year.

Example: A book has pages that are 7 inches wide and 9 inches high. The printed area measures $5\frac{3}{8}$ inches by $7\frac{3}{4}$ inches. The left margin is $\frac{5}{16}$ in. and the top margin is $\frac{9}{16}$ in. How wide are the margins at the right and the bottom of each page?

Solution: Right: $1\frac{5}{16}$ in.; bottom: $\frac{11}{16}$ in.

Example: A gasoline tank with a capacity of 15 gallons is $\frac{3}{4}$ full. How many gallons will it take to fill the tank? *Solution:* $3\frac{3}{4}$ gallons

***Example*:** Kevin's regular rate of pay is $8.00 per hour. When he worked overtime, he earns $1\frac{1}{2}$ times as much per hour. How much will Kevin earn for $5\frac{1}{2}$ hours of overtime work? *Solution:* $63.00

***Example*:** An advertising sign is to have six lines of printing. The letters are to be $1\frac{1}{2}$ inches high with $\frac{1}{4}$ inch between the lines.

a. How much vertical space is required for the printing? *Solution:* $10\frac{1}{4}$ in.

b. If there is a top margin of 3 inches and a lower margin of $3\frac{1}{2}$ inches, what will be the total height of the sign? *Solution:* $16\frac{3}{4}$ in.

Example: Mary's cell phone bill is automatically deducting $54.95 from her bank account every month. How much will the deductions total for the year?

Solution: –$659.40

Math 7 Notes – Part B: Rational Numbers

Example: Metro Taxi Cab Company charges its customers as follows:

\$3.00 for the 1^{st} $\frac{1}{9}$ mile
\$0.30 for each additional $\frac{1}{9}$ mile

How much would he pay for a 2 mile taxi ride?
Solution: \$8.10

Example: Mike paid \$53.85 for 15 gallons of gasoline. What was the price per gallon? *Solution:* \$53.85

Example: Mr. McMillan plans to interview 18 applicants for a job. If he spends 3 hours interviewing each day and spends $\frac{3}{4}$ hour interviewing each applicant, how many days will she need for the interviews? *Solution:* 5 days

Example: Estimate $8.35-27.516+18.814-4.16-73.8+45.6$ by rounding to the nearest integer.

A.	33
B.	–34
C.	–32
D.	–33

Example: A train stops at Buxton, then Caliente and then Dogwood. The distance from Buxton to Dogwood is 13 miles and the distance from Buxton to Caliente is $10\frac{2}{3}$ mile. Estimate the distance from Buxton to Dogwood.

A.	About 1 mile
B.	About 2 miles
C.	About 3 miles
D.	About 4 miles

NVACS 7.EE.B.3 – *Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically.* ***Apply properties of operations to calculate with numbers in any form;*** *convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.*

Example: Karen worked $12\frac{1}{2}$ hours last week and earned \$100. What was her hourly rate of pay? $100 \div 12\frac{1}{2}$ *or* $100 \div 12.5$ \$8 per hour

Example: A radio station released 300 balloons at an outdoor celebration. Of these, $\frac{3}{4}$ were orange. How many were orange balloons?

$\frac{3}{4} \bullet 300$ *or* $.75(300)$ 225 orange balloons

Example: A town has raised $\frac{3}{8}$ of the \$12,000 it needs to furnish its new library. How much more is it hoping to raise?

$1-\frac{3}{8}=\frac{5}{8}$ *so* $\frac{5}{8}(12{,}000)$ *or* $.625(12{,}000)$ \$7,500

NVACS 7.NS.A.1d – Apply properties of operations as strategies to add and subtract rational numbers.

NVACS 7.NS.A.2c – Apply properties of operations as strategies to multiply and divide rational numbers.

NVACS 7.EE.A.1 Apply properties of operations as strategies to add, subtract, *factor and expand linear expressions* ***with rational coefficients.***

Example: $1\frac{1}{5} \bullet 5 = (1 \bullet 5) + \left(\frac{1}{5} \bullet 5\right) = 5 + 1 = 6$ *Using the Distributive Property*

Example: $\frac{1}{5}+\frac{2}{3}+\frac{4}{5}=$ $\frac{1}{5}+\frac{4}{5}+\frac{2}{3}=$ $1+\frac{2}{3}=$ $1\frac{2}{3}$ *Using the Commutative Property*

Math 7 Notes – Part B: Rational Numbers

Example: $\frac{3}{4}+\frac{-1}{5}+\frac{-1}{4}+\frac{3}{5}=$

$\frac{3}{4}+\frac{-1}{4}+\frac{-1}{5}+\frac{3}{5}=$ *Using the Commutative Property*

$\left(\frac{3}{4}+\frac{-1}{4}\right)+\left(\frac{-1}{5}+\frac{3}{5}\right)=$ *Using the Associative Property*

$\frac{2}{4} \quad + \quad \frac{2}{5}=$

$\frac{10}{20} \quad + \quad \frac{8}{20}=$

$\frac{18}{20}=\frac{9}{10}$

Example: $\mathbf{3+(2-5)=}$

$\mathbf{3+(2-5)=(3+2)-5}$

Using the Associative Property of Addition $\mathbf{5 \; -5=}$

$\mathbf{0}$

Example: $\mathbf{3(-5)(-2)(0)=}$

$\mathbf{3(-5)(-2)(0)=0(3)(-5)(-2)}$ ***Using the Commutative Property of Multiplication then the Property of Zero for Multiplication***

$\mathbf{0}$

Example: $\mathbf{5.5(-2+-8)=}$

Using the Distributive Property ***or*** ***Order of Operations***

$\mathbf{5.5(-2+-8)=5.5(-2)+5.5(-8)=}$ ***or*** $\mathbf{5.5\left(-10\right)=}$

$\mathbf{-11+\;-44=}$ $\mathbf{-55}$

$\mathbf{-55}$

Example: $\mathbf{\left|-5\right|(x+2.1)=}$ ***Using the Distributive Property***

$\mathbf{\left|-5\right|(x+2.1)=5(x+2.1)=}$

$\mathbf{5(x)+5(2.1)=}$

$\mathbf{5x+10.5}$

Example: $7.3+5.2+(-7.3)=$ $7.3+(-7.3)+5.2=$ ***Commutative Property of Addition***

$0 \qquad +5.2=$ ***Additive Inverses***

5.2 ***Addition***

Math 7 Notes – Part B: Rational Numbers

***NVACS** 7.EE.A.2*

Understand that writing an expression in different forms is a problem context can shed light on the problem and how the quantities in it are related

Example: A town has raised $\frac{3}{8}$ of the $12,000 it needs to furnish its new library. How much more is it hoping to raise?

$$1-\frac{3}{8}=\frac{5}{8} \quad \textit{so} \quad \frac{5}{8}(12{,}000) \quad \textit{or} \quad .625(12{,}000) \qquad \$7{,}500$$

Yet some students may only see

$$\frac{3}{8}\bullet 12{,}000=\$4{,}500 \qquad 12{,}000-4{,}500=\$7{,}500$$

NVACS 7.EE.A.3 *Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form: convert between forms as appropriate; and* ***assess the reasonableness of answers using mental computation and estimation strategies.*** **** Estimation should be an ongoing skill.***

Estimation with Fractions

<table>
<tr><th colspan="3">Benchmarks for Rounding Fractions</th></tr>
<tr>
<td>Round to 0 if the numerator is much smaller than the denominator.
Examples: $\frac{3}{20}, \frac{1}{8}, \frac{7}{100}$</td>
<td>Round to ½ if the numerator is about half the denominator.
Examples: $\frac{11}{20}, \frac{5}{9}, \frac{47}{100}$</td>
<td>Round to 1 if the numerator is nearly equal to the denominator.
Examples: $\frac{17}{20}, \frac{7}{8}, \frac{87}{100}$</td>
</tr>
</table>

Examples: Round to 0, ½ or 1

1. $\frac{7}{9}\approx 1$
2. $\frac{27}{50}\approx\frac{1}{2}$
3. $\frac{2}{25}\approx 0$

Math 7 Notes – Part B: Rational Numbers

Estimating Sums and Differences

Round each fraction or mixed number to the nearest half, and then simplify using the rules for signed numbers.

Example: Estimate $\frac{7}{8}+\frac{1}{9}$.

$$\begin{array}{r l}
\frac{7}{8} \approx 1 & \\
+\frac{1}{9} \approx 0 & \\
\hline
1 & \text{is our estimate}
\end{array}$$

Example: Estimate $5\frac{3}{4}-8\frac{1}{7}$

$$\begin{array}{r l}
5\frac{3}{4} \approx \;\; 6 & \\
-8\frac{1}{7} \approx -8 & \\
\hline
-2 & \text{is our estimate}
\end{array}$$

Estimating Products and Quotients

Round each mixed number to the nearest integer, and then simplify.

Example:

$$-7\frac{5}{6}\cdot 11\frac{1}{5} \approx -8\cdot 11$$

$$= -88 \quad \text{is our estimate}$$

Example:

$$17\frac{5}{6} \div (-2\frac{4}{5}) \approx 18 \div (-3)$$

$$= -6 \quad \text{is our estimate}$$

Estimating with Decimals

An estimation strategy for adding and subtracting decimals is to round each number to the nearest integer and then perform the operation.

Examples:

$$\begin{array}{rcr}
25.8 & \rightarrow & 26 \\
\underline{-14.2} & \rightarrow & \underline{-14} \\
 & & 12
\end{array}
\qquad
\begin{array}{rcr}
-5.2 & \rightarrow & -5 \\
\underline{+\ 3.7} & \rightarrow & \underline{+\ 4} \\
 & & -1
\end{array}
\qquad
\begin{array}{rcr}
-8.98 & \rightarrow & -9 \\
\underline{-9.2} & \rightarrow & \underline{-9} \\
 & & -18
\end{array}$$

12 would be our estimate $\qquad$ -1 would be our estimate $\qquad$ -18 would be our estimate

An estimation strategy for multiplying and dividing decimals:

When multiplying, round numbers to the nearest non-zero number or to numbers that are easy to multiply.

- When dividing, round to numbers that divide evenly, leaving no remainders.
- Remember, *the goal of estimating is to create a problem that can easily be done mentally*.

Examples:

$$\begin{array}{rcr} 38.2 & \rightarrow & 40 \\ \underline{\times\ 6.7} & \rightarrow & \underline{\times 7} \\ & & 280 \end{array} \qquad -33.6 \div 4.2 \;\rightarrow\; -32 \div 4 = -8$$

280 would be our estimate -8 would be our estimate

Estimating can be used as a test-taking strategy. Use estimating to calculate your answer first, so you can eliminate any obviously wrong answers.

Example: A shopper buys 3 items weighing 4.1 ounces, 7.89 ounces and 3.125 ounces. What is the total weight?

A. 0.00395

B. 3.995

C. 14.0

D. 15.115

Round the individual values:

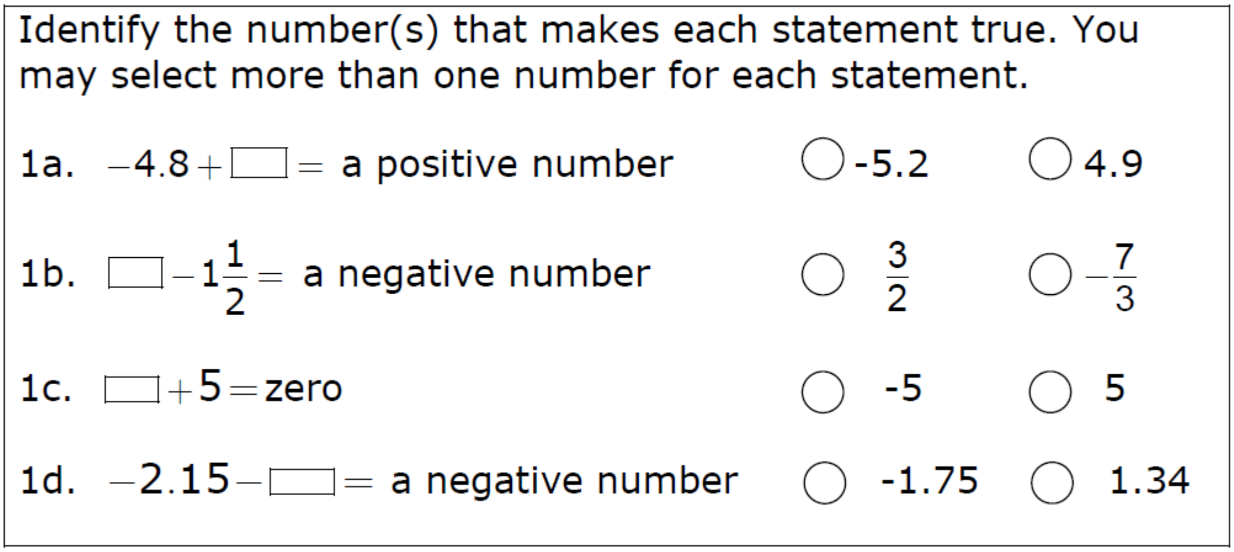
$$\begin{array}{rcr} 4.1 & \rightarrow & 4 \\ 7.89 & \rightarrow & 8 \\ 3.125 & \rightarrow & \underline{+3} \\ & & 15 \end{array}$$

The only two reasonable answers are C) and D). Eliminating unreasonable answers can help students to avoid making careless errors.

Math 7 Notes – Part B: Rational Numbers

SBAC Samples

Standard: *7.NS.A.1* ***DOK:*** *2* ***Difficulty:*** *M* ***Question Type:*** *SR (Selected Response)*



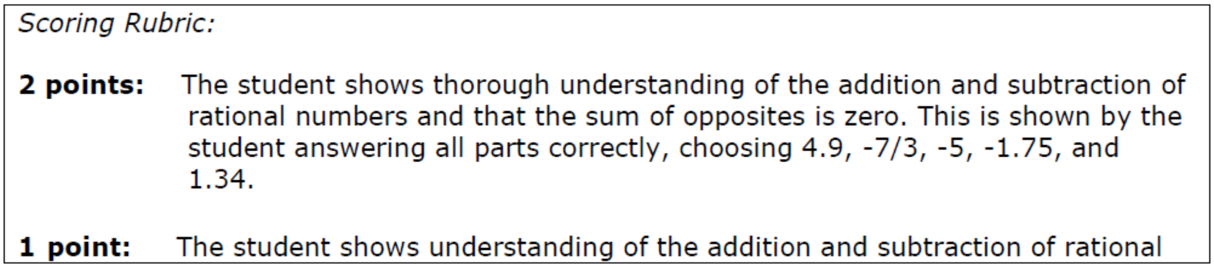
Identify the number(s) that makes each statement true. You may select more than one number for each statement.

1a. $-4.8+\square=$ a positive number ○ -5.2 ○ 4.9

1b. $\square-1\frac{1}{2}=$ a negative number ○ $\frac{3}{2}$ ○ $-\frac{7}{3}$

1c. $\square+5=$zero ○ -5 ○ 5

1d. $-2.15-\square=$ a negative number ○ -1.75 ○ 1.34



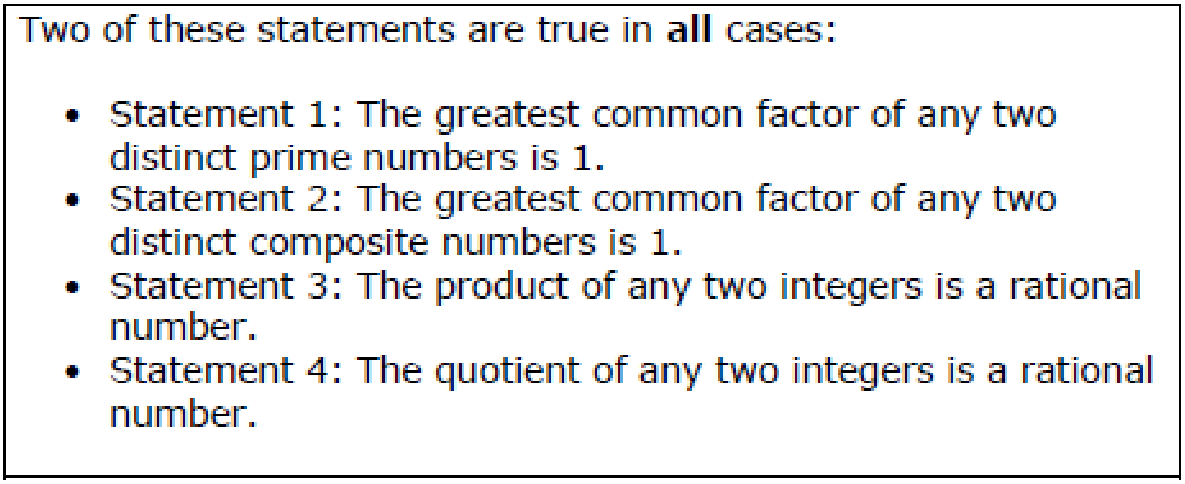
Scoring Rubric:

2 points: The student shows thorough understanding of the addition and subtraction of rational numbers and that the sum of opposites is zero. This is shown by the student answering all parts correctly, choosing 4.9, -7/3, -5, -1.75, and 1.34.

1 point: The student shows understanding of the addition and subtraction of rational

Math 7 Notes – Part B: Rational Numbers

Standard: *7.NS.A.2* ***DOK:*** *3* ***Difficulty:*** *High* ***Question Type:*** *ER (Extended Response)*



Two of these statements are true in **all** cases:

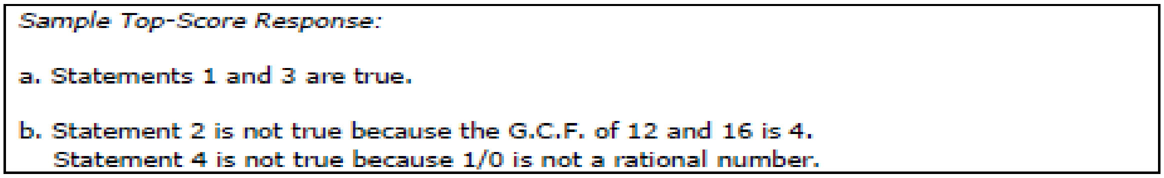
- Statement 1: The greatest common factor of any two distinct prime numbers is 1.
- Statement 2: The greatest common factor of any two distinct composite numbers is 1.
- Statement 3: The product of any two integers is a rational number.
- Statement 4: The quotient of any two integers is a rational number.

Part A: Which two statements are true in all cases?

Part B: For both statements that you did not choose in *Part A*, provide one clear reason and/or example for each statement that proves the statement can be false.

Statement ☐ Reason/example

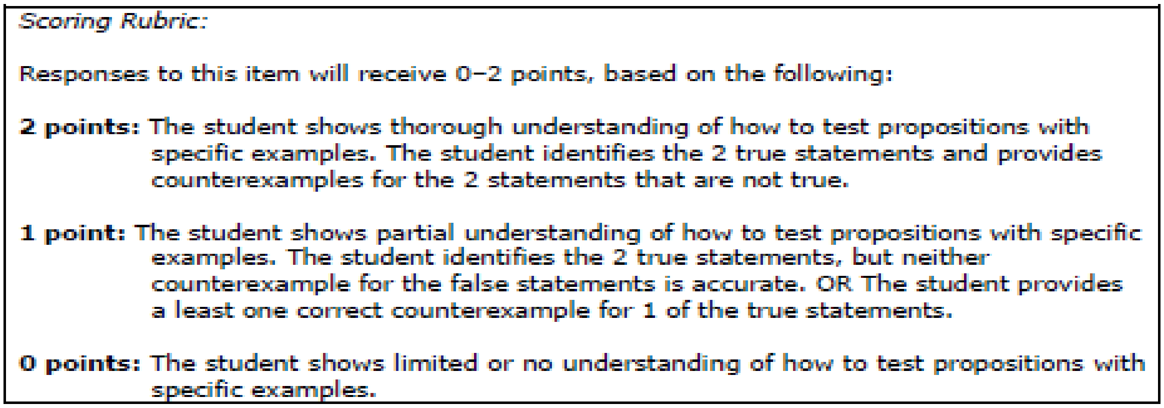
Statement ☐ Reason/example



Sample Top-Score Response:

a. Statements 1 and 3 are true.

b. Statement 2 is not true because the G.C.F. of 12 and 16 is 4.
Statement 4 is not true because 1/0 is not a rational number.



Scoring Rubric:

Responses to this item will receive 0–2 points, based on the following:

2 points: The student shows thorough understanding of how to test propositions with specific examples. The student identifies the 2 true statements and provides counterexamples for the 2 statements that are not true.

1 point: The student shows partial understanding of how to test propositions with specific examples. The student identifies the 2 true statements, but neither counterexample for the false statements is accurate. OR The student provides a least one correct counterexample for 1 of the true statements.

0 points: The student shows limited or no understanding of how to test propositions with specific examples.