Math 7 Notes – Part A: Rational Numbers

As we begin this unit it's a good idea to have an overview. When we look at the subsets of the real numbers it helps us organize the groups of numbers students have been exposed to and those that are soon to be exposed.

Natural Numbers (also known as the Counting Numbers) = {1, 2, 3, …}

Whole Numbers = {0, 1, 2, …}

Integers = {..., –3, –2, –1, 0, 1, 2, 3, …} or more simply {…, -1, 0 +1, …} or {all the whole numbers and their opposites}.

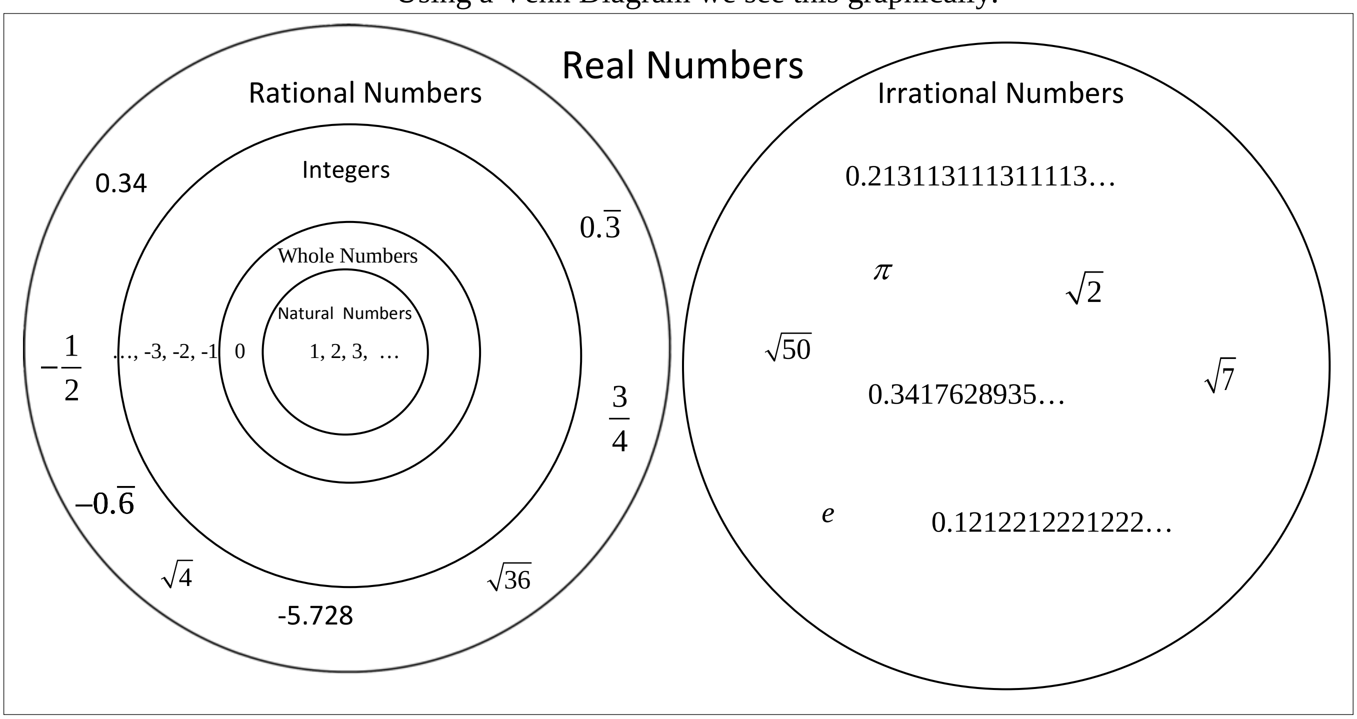
Rational Numbers = {any integers, *a* and *b*, that can be written in the form $\frac{a}{b}$, where $b \neq 0$}

or more simply {any number that can be expressed as a ratio of two integers}

Irrational Numbers = {any integers, *a* and *b*, that cannot be written in the form $\frac{a}{b}$, where $b \neq 0$} or more simply {any number that cannot be expressed as a ratio of two integers or as a repeating or terminating decimal}

Real Numbers = {all the rational and irrational numbers}.

Using a Venn Diagram we see this graphically.



Math 7 Notes – Part A: Rational Numbers

At first glance, one might think that the set of rational numbers and the set of irrational numbers appears sparse. The opposite is true. The sets are both so dense that we cannot name them in a sequence like we do the natural numbers, whole numbers and the integers; that's why we must use a word description or definition to define them. Why? Between any two rational numbers there are an infinite number of rational numbers. Just for a quick example let's look at the example of 0.2 and 0.3.

Between 0.2 =0.20 and 0.3 =0.30 are
0.21, 0.22, 0.23, 0. 24, 0.25, 0.26, 0.27, 0.28, 0.29

Between 0.2 =0.200 and 0.3 =0.300 are
0.201, 0.202, 0.203, 0.204, 0.205, 0.206, …, 0.299

Between 0.2 =0.2000 and 0.3 =0.3000 are
0.2001, 0.2002, 0.2003, 0.2004, 0.2005, …, 0.2999

Between 0.2 =0.20000 and 0.3 =0.3000 are
0.20001, 0.20002, 0.20003, …, 0.29999

WOW, we could go on and on to infinity, but hopefully you see the point. So what about fractions?

Again, for a quick example let's use $\frac{1}{3}$ and $\frac{1}{2}$.

Between $\frac{1}{3}=\frac{2}{6}=\frac{4}{12}$ and $\frac{1}{2}=\frac{3}{6}=\frac{6}{12}$

$$\frac{5}{12}$$

$\frac{1}{3}=\frac{2}{6}=\frac{4}{12}=\frac{400}{1200}$ and $\frac{1}{2}=\frac{3}{6}=\frac{6}{12}=\frac{600}{1200}$

$$\frac{401}{1200},\frac{402}{1200},\frac{403}{1200},\frac{404}{1200},\ ...,\ \frac{599}{1200}\text{ and so on}$$

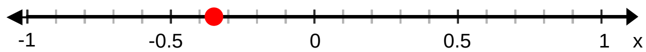
So with just two quick examples, you should see there are an infinite number of numbers between any two rational numbers.

Math 7 Notes – Part A: Rational Numbers

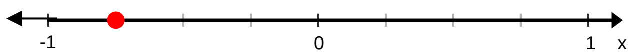
Review from Grade 6 6.NS.C.6 and 6.NS.C.7 and Prep for 7.NS.A.1

Locating integers on the number line was covered in Grade 6 but should be reviewed here and extended to include rational numbers.

Graph -0.35 on the number line.
(graph should be divided into 0.1's)



Graph $-\frac{3}{4}$ on the number line.

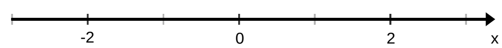


(graph should be divided into $\frac{1}{4}$'s)

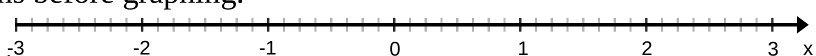
Students will need practice subdividing the number lines between the integers into appropriate - but reasonable – spaces. For example, in the first sample above, although we are graphing a number in the hundredths, a reasonable division would be in tenths.

Example: Graph the following numbers on the number line.

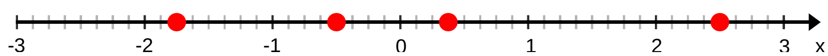
$$\frac{-1}{2},\ \frac{3}{8},\ -1\frac{3}{4},\ \frac{5}{2}$$



Solution: Most students will want to subdivided the units into $\frac{1}{8}$'s. They may need to rename each value in eighths before graphing.

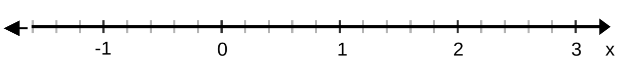


Then they will plot each point.

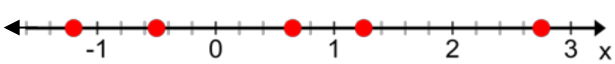


Example: Graph the following rational numbers on the number line. Then, list the numbers in order from greatest to least.

1.25, 2.75, -0.5, 0.65, -1.2



Solution:



2.75, 1.25, 0.65, -0.5, -1.2

Another way to compare decimal numerals:

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1) Write the decimals so that each decimal numeral has the same number of digits (add zeros if needed).
2) Forget about the decimal point; the largest number will be the largest decimal numeral.

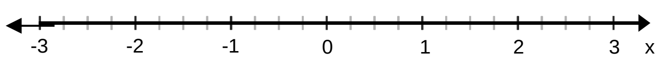
Example: List the following set of numbers from least to greatest.

1.25, 2.75, -0.5, 0.65, -1.2

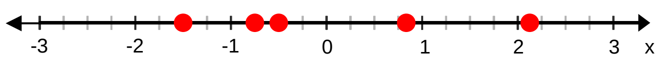
1. (add zeros) 1.25, 2.75, -0.50, 0.65, -1.20
2. (order) $-1.2, -0.5, 0.65, 1.25, 2.75$

Graph the following rational numbers on the number line. Then, list the numbers in order from least to greatest to.

Example: $-\frac{3}{4}, 2\frac{1}{8}, -1\frac{1}{2}, \frac{5}{6}, -\frac{1}{2}$



Solution:



$-1\frac{1}{2}, -\frac{3}{4}, -\frac{1}{2}, \frac{5}{6}, 2\frac{1}{8}$

Example: *Which is larger .8032 or .82?*

Add 2 zeros to .82 so both numbers will have 4 digits to the right of the decimal point.

.8032 and .8200 ⟶ now both have 4 digits to the right of the decimal point (or we can say both are now expressed to the ten-thousandths place).
Since 8200 is larger than 8032, then .82 > .8032

Example: *Compare* .62 *and* .547 *using* <, =, or >.

Although students should see that the 6 in the tenths place is greater than the 5 in the tenths place, some do not. Add one zero to .62 so both numbers will have 3 digits to the right side of the decimal point (both numbers will be expressed to the thousandths place).
.620 and .547
620 is larger than 547, therefore $.62 > .547$

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***Examples**: Use > or < to compare the decimals.*

A) .9 ☐ .235

.900 .235
Since 900 is greater than 235 use >.

B) 3.56 ☐ 9.1

Since 3 is less than 9, there is no need to compare the decimals. <.

C) .007 ☐ .7

.007 .700
since 700 is greater than 007 use <.

Example: In a car race, Elvin's car finished in third place with a time of 56 minutes and 14 seconds. The table shows the times of the other racers as **compared to Elvin's time.**

Racer	Time (min:sec)
Annie	$+0:18$
John	$-0:06$
Roy	$-0:10$
Emily	$+0:22$

Who won the race? How do you know?

a. Emily; Emily's time is furthest from Elvin's.
b. John; John's time is closest to Elvin's.
c. Roy; Roy's time is less than Elvin's and further from Elvin's time.
d. Annie; Annie's time is greater than Elvin's and closer to Elvin's time than Emily's.

Solution: c

Example Which set of numbers is in order from least to greatest?

A.	$\frac{3}{8}$, 0.62, $\frac{1}{9}$, 0.27
B.	0.27, $\frac{1}{9}$, $\frac{3}{8}$, 0.62
C.	$\frac{1}{9}$, $\frac{3}{8}$, 0.27, 0.62
D.	$\frac{1}{9}$, 0.27, $\frac{3}{8}$, 0.62

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Example Which set of numbers is in order from least to greatest?

A.	$\frac{5}{8}, \frac{-2}{5}, \frac{1}{9}, \frac{-1}{3}$
B.	$\frac{-2}{5}, \frac{1}{9}, \frac{-1}{3}, \frac{5}{8}$
C.	$\frac{-2}{5}, \frac{-1}{3}, \frac{1}{9}, \frac{5}{8}$
D.	$\frac{-1}{3}, \frac{-2}{5}, \frac{1}{9}, \frac{5}{8}$

Example Which set of numbers is in order from least to greatest?

A.	$-0.9, -0.5, -1.3, -1.8$
B.	-1.3, -1.8, -0.5, -0.9
C.	-1.8, -1.3, -0.9, -0.5
D.	-0.5, -0.9, -1.3, -1.8

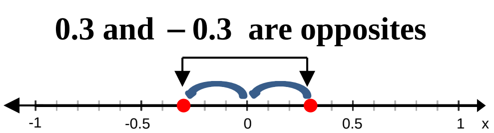
Example: Bill bought some stock for $47. After one month the stock had dropped in value below $30. How would you describe the size of his loss?

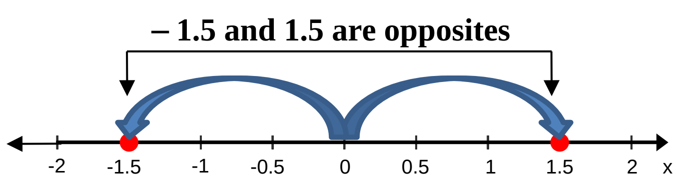
A	Greater than $\lvert -17 \rvert$
B	Less than $\lvert -17 \rvert$
C	$\lvert -17 \rvert$
D	Greater than $\lvert -47 \rvert$

Solution: A

Review with rational #'s

Opposites are numbers that are the same distance from 0 on a number line but on the other side of 0. Students were introduced to opposites while studying integers; now we include rationals.





Example: Name the opposite.

4	-6	2.3
$\frac{-1}{2}$	$\frac{5}{6}$	$-1.2\overline{25}$

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Example: Identify two different examples in which opposite quantities combine to make 0.

The dolphin dove to a depth of 25.6 m below the surface of the water.
Then swam 25.6 m back to the surface.

The temperature rises 5.5 degrees and then falls 5.5 degrees.

The stock shares fell 1.3% and then rose 1.3%.

You earn $15.25 and then you spend $15.25.
A person loses ten and a half pounds then gains ten and a half pounds.

Karl deposited some checks totaling $103.14, then withdrew $103.14.

Example: Sam hiked a trail that runs 4.5 miles west. He then hikes back to where he started from. Which best describes the total change in miles for the entire hike?

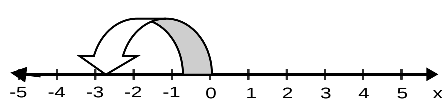
A	**9**
B	**4.5**
C	**0**
D	**-4.5**

Example: Which is a solution of $-\frac{3}{4}+x=0$

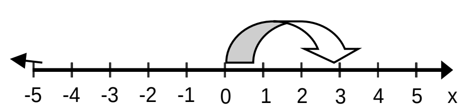
A	$-\frac{3}{4}$
B	0
C	$\frac{3}{4}$
D	$1\frac{1}{2}$

Prep for 7.NS.A.1

Absolute Value – the distance from 0 on a number line. Students were introduced to opposites while studying integers; now we include rationals.



Examples: $|-2.9|=2.9$ since –2.9 is 2.9 units to the left of 0.



$|2.9|=2.9$ since 2.9 is 2.9 units to the right of 0.

So both $|\mathbf{-2.9}|$ ***and*** $|\mathbf{2.9}|$ equal 2.9 because the distance from zero is 2.9 units. It doesn't matter which direction.

$$\left|+\frac{5}{9}\right| = \frac{5}{9} \text{ since } \frac{5}{9} \text{ is } \frac{5}{9} \text{ units from 0.}$$

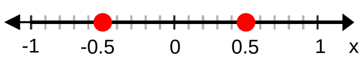
$$|\mathbf{-0.12}| = \mathbf{0.12}$$

$$|0| = 0$$

Students need to understand that if $|x| = 8.7$ then x $= 8.7$ <u>and</u> x $= -8.7$.
We can write that as $x = \pm 8.7$.

NVACS 7.NS.A.1c* – *Understand subtraction of rational numbers as adding the additive inverse, p + q = p + (– q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Example: Find the distance between each set of points on the number line.



$$|\mathbf{-0.5-0.5}| = |\mathbf{-1}| = \mathbf{1}$$

(Some students may still need to count each 0.1)

Example: Cheryl says $|4|\, and\, |-4|$ are the same number. Theresa says $|4|\, and\, |-4|$ are different numbers. Tia says $|4|\, and\, |-4|$ both equal zero. Vicki says $|4|\, and\, |-4|$ combine to make 0. Who is correct?

A	Cheryl
B	Theresa
C	Tia
D	Vicki

Review

Rules of Divisibility

It is beneficial for students to be familiar with rules of divisibility. When familiar with these rules, it allows students to be focused more on the concept and skill of the lesson rather than to be bogged with the arithmetic. Students already know some of these rules.

Math 7 Notes – Part A: Rational Numbers

Chances are students already know if a number is divisible by 2, 5 or 10. For instance, if asked to determine if a number is divisible by two, students can tell you that it has to be even. Divisibility rules for 10 and 5 are also familiar to students.

Before starting to work with rational numbers it is imperative that students have some basic understanding of fractions. Everyone will benefit if you take a few minutes to review or learn the Rules of Divisibility. Employing these rules will make life a lot easier in the future; not to mention it will save you time and allow you to do problems very quickly when others are experiencing difficulty.

In general if you were asked if any given number was divisible by a second number you could divide them and if the remainder is 0, we would say yes the first number is divisible by the second number.

For example, if asked if 132 is divisible by 3 we could divide to see the remainder.

$$\begin{array}{r} 44 \\ 3\overline{)132} \\ \underline{12} \\ 12 \\ \underline{-12} \\ 0 \end{array}$$

Since the remainder is 0, we say that 132 is divisible by 3.

We could say 3 is a factor of 132 and 132 is a multiple of 3.

What if you were asked if 197 was divisible by 14? Again we could divide and see the remainder.

$$\begin{array}{r} 14 \\ 14\overline{)197} \\ \underline{-14} \\ 57 \\ \underline{-56} \\ 1 \end{array}$$

Since the remainder is not 0, we say that 197 is NOT divisible by 14.

We could say 14 is NOT a factor of 197 and 197 is NOT a multiple of 14.

To be quite frank, you already know some of them. For instance, if I asked you to determine if a number is divisible by two, would you know the answer? Sure you do, if the number is even, then it's divisible by two. Can you tell if a number is divisible by 10? How about 5?

Because you are familiar with those numbers, chances are you know if a number is divisible by 2, 5 or 10. We could look at more numbers to see if any other patterns exist that would let you know what they are divisible by, but we don't have that much time or space. So, if you don't mind, I'm just going to share some rules of divisibility with you first, and then I will give the examples.

Rules of Divisibility

A number is divisible:

- by 2, if the number ends in 0, 2, 4, 6 or 8. In other words the number must be even.
- by 5, if the number ends in 0 or 5.
- by 10, if the number ends in 0.
  Notice here we are suggesting you teach these rules grouped together since they all involve just "looking at the one's digit".

- by 3, if the sum of the digits is a multiple of 3.
- by 9, if the sum of the digits is a multiple of 9.
- by 6, if the number ends in 0, 2, 4, 6 or 8 AND the sum of the digits is a multiple of 3. (In other words, if the number is divisible by 2 and 3, then it is by 6.)
  Notice here we are suggesting you teach these 3 rules grouped together since they are similar – they all involve "finding a sum of the digits".
- by 4, if the last 2 digits of the number is divisible by 4. (Remember $4=2^2$)
- by 8, if the last 3 digits of the number is divisible by 8. (Remember $8=2^3$)
  Again we grouped these similar rules together.

Others that are important and quick to teach are the rules for 20, 25, 50 and 100.

- by 20, if the number ends in 00, 20, 40, 60 or 80.
- by 25, if the number ends in 00, 25, 50 or 75.
- by 50, if the number ends in 00 or 50.
- by 100, if the number ends in 00.

Now come the examples…

Divisibility by 3

Example: Is 111 divisible by 3? (We could write this as 111;3 which is read "Is 111 divisible by 3?".)

The rule says to find the sum of the digits and if that sum is a multiple of 3 the number is divisible by 3. 1+1+1=3, 3 is a multiple of 3, so the number 111 is divisible by 3.

Example: 147;3 (Read, "Is 147 divisible by 3?)

Adding 1, 4, and 7 we get 12. Is 12 divisible by 3? If that answer is yes, that means 471 is divisible by 3. If you don't believe it, try dividing 471 by 3.

Example: 1,316;3 (Read, "Is 1,316 divisible by 3?)

The sum is 1+3+1+6=11. Is 11 a multiple of 3? (3, 6, 9, 12, 15, 18, 21, 24, 27, 30, …)
No, then 1,316 is NOT divisible by 3.

Divisibility by 9

Example: Is 111 divisible by 9? (We could write this as 111;9 which is read "Is 111 divisible by 9?".)

The rule says to find the sum of the digits and if that sum is a multiple of 9 the number is divisible by 9. 1+1+1=3, 3 is not a multiple of 9, so the number 111 is NOT divisible by 9.

Example: 5,247;9 (Read, "Is 5,247 divisible by 9?)

5+2+4+7=18 18 is a multiple of 9 so yes, 5,247 is divisible by 9.

Example: 254,145;9 (Read, "Is 254,145 divisible by 9?)

2+5+4+1+4+5=21 21 is NOT a multiple of 9, so the number is NOT divisible by 9.

Divisibility by 6

Example: Is 21,306 divisible by 6? (We could write this as 21,306;6 which is read "Is 21,306 divisible by 6?".)

The rule has two parts - the number ends in 0, 2, 4, 6 or 8 yes
AND
- the sum of the digits is a multiple of 3. 2+1+3+0+6=12 yes
So, yes, 21,306 is divisible by both 2 and 3 so it is divisible by 6.

Example: 746; 6? (This is read "Is 746 divisible by 6?".)

The rule has two parts - the number ends in 0, 2, 4, 6 or 8 yes
AND
- the sum of the digits is a multiple of 3. 7+4+6=17 no
So 746 is divisible by 2 but NOT by3 so it is NOT divisible by 6.

Example: 48,761;6 (This is read "Is 21,306 divisible by 6?".)

The rule has two parts - the number ends in 0, 2, 4, 6 or 8 no
AND
- the sum of the digits is a multiple of 3.
Since it is not divisible by 2, it is NOT divisible by 6.

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Divisibility by 4

Example: Is 12,316 divisible by 4? (We could write this as 12,316;4 which is read "Is 12,316 divisible by 4?".)

Using the rule for 4, we look at the last 2 digits. In this example te last two digits are 16, since 16 is divisible by 4. Then 12,316 is divisible by 4.

Example: 87,502;4 (This is read "Is 12,316divisible by 4?".)

We look at the last 2 digits, in this case 02, which is NOT divisible by 4.

Example: 961,500;4 (This is read "Is 961,500divisible by 4?".)
divisible by 4

Using the rule for 4, we look at the last 2 digits. In this example they are 00, since 4 goes into 00 zero times it is divisible by 4.

Divisibility by 8

Example: Is 456,040 divisible by 8? (We could write this as 456,040;8 which is read "Is 456,040 divisible by 8?".)

Using the rule for 8, we look at the last 3 digits. In this example the last 3 digits are 040, since it is divisible by 8, then 456,040 is divisible by 8.

Example: 85,600;8 (This is read "Is 85,600 divisible by 8?".)

We look at the last 3 digits, in this case 600, and we would probably have to divide (still easier to divide $600 \div 8$ compared to $85{,}600 \div 8$). Once we divide we see $600 \div 8 = 75$ with no remainder. So yes 85,600 is divisible by 8.

Example: 612,513;8 (This is read "Is 612,513 divisible by 8?".)

Using the rule for 8, we look at the last 3 digits. In this example 513, since it is not even an even number we know it is NOT divisible by 8.

Divisibility by 11

A fun rule for testing divisibility is divisibility by 11. Use it to challenge some of your students. The rule is "wordy" but the process is simple once learned.

- by11, if the sum of the 1^{st}, 3^{rd}, 5^{th}, …digits and the sum of the 2^{nd}, 4^{th}, 6^{th}, …digits are equal <u>OR</u> differ by a multiple of 11.

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Example: Is 186,021 divisible by 11?

1. the sum of the 1st, 3rd, 5th, …digits 1+6+2=9
2. the sum of the 2nd, 4th, 6th, …digits 8+0+1=9
3. the sums are equal so ……186,021 is divisible by 11.

Example: Is 806,124 divisible by 11?

1. the sum of the 1st, 3rd, 5th, …digits 8+6+2=16
2. the sum of the 2nd, 4th, 6th, …digits 0+1+4=5
3. the sums are not equal so find the difference between the sums 16-5=11
4. since 11 is a multiple of 11 (11, 22, 33, 44, 55, 66, 77, 88, 99, …)
   806,124 is divisible by 11.

Example: Is 123,456 divisible by 11?

1. the sum of the 1st, 3rd, 5th, …digits 1+3+5=9
2. the sum of the 2nd, 4th, 6th, …digits 2+4+6=12
3. the sums are not equal so find the difference between the sums 12-9=3
4. since 3 is NOT a multiple of 11 (11, 22, 33, 44, 55, 66, 77, 88, 99, …)
   then, 123,456 is NOT divisible by 11.

Example: Using the rules of divisibility, determine if the following numbers are divisible by 2, 3, 4, 5, 6, 8, 9 or 10. Place a check in each box that the number IS divisible by.

#	By 2	By 3	By 4	By 5	By 6	By 8	By 9	By 10
756								
9,045								
16,701								
86,400								
720,000								

Example: Write a 5 digit number that is divisible by 3 and 4.

Example: Write the smallest 6 digit number that is divisible by 3and 5.

Example: What single digit value or values might the "?" represent in each of the following?

a. 12,30? ; 4 and 6 ?=0

b. 145,6?0; 3 and 5 ?=2, 5 or 8

c. 6,1?2: 4 and 9 ?=9

Examples: True or False?

All even numbers are divisible by 2. True

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All odd numbers are divisible by 3. False
Some even numbers are divisible by 5. True
Every number divisible by 2 is also divisible by 4. False
Every number divisible by 8 is also divisible by 4. True
Every number divisible by 9 is also divisible by 3. True

Examples: If a number is divisible by 3 and 5, it must be a multiple of what other number?
15

If a number is divisible by 12, then it is divisible by what other numbers?
2, 3, 4, and 6

Find a number that is divisible by 2, 3, 4, 5 and 6 but not by 9.
Possible answer: 120

Primes and Composites

Prime Number: A number that has **only two** factors - one and itself.

Examples: We could say 2, 3, 5, 7 and 11 have only two factors - 1 and itself - or we can say they are only divisible by 1 and itself.

Composite number: A number that is has **more than two** factors.

Examples: 4, 6, 8, 9, and 10 all have 3 or more factors OR we can say they are divisible by three or more numbers. For instance, 8 is divisible by 1, 2, 4 and 8.

The numbers 0 and 1 are neither prime nor composite numbers.

As students begin to identify primes it may be helpful to use a Sieve of Eratosthenes. Really all it is, is a 100's chart like below.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Math 7 Notes – Part A: Rational Numbers

As you begin to talk about whether a number is prime or composite always **go back to the definitions to verify.**

1. Starting with the number 1, ask students how many factors the number 1 has. Since it has only 1 factor it is neither prime nor composite. Choose a color – I will use aqua - and shade in the one's box.
2. Next we see the number 2. Ask students how many factors the number 2 has. They should state that 2 has only 2 factors – 1 and itself. That makes it prime and we will color the primes boxes yellow.

   Next we think about all the multiples of 2. Since every multiple of 2 will have at least 3 factors - 1, itself, and at least 2 (since they are even), the remaining even numbers in the chart are composites.

   For example: 4 is $1 \bullet 4$ and $2 \bullet 2$, so 3 factors.
   6 is $1 \bullet 6$ and $2 \bullet 3$, so 4 factors.

   Students may skip count 4, 6, 8, 10, 12, … remind them these are multiples of 2.
   We begin to color all the multiples of 2 – I will use gray. Your chart should look like the following.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

3. Next we see the number 3. Ask students how many factors the number 3 has. They should state the number 3 has only 2 factors – 1 and itself. That makes it prime and we will color the 3 box yellow.

   Once again we will see that the multiples of 3 have more than 2 factors.

   For example: 3x2 or 6 has $1 \bullet 6$ and $2 \bullet 3$, so 4 factors.
   3x3 or 9 has $1 \bullet 9$ and $3 \bullet 3$, so 3 factors. Etc.

   Students may skip count, 6, 9, 12, 15, … remind them these are multiples of 3.

Math 7 Notes – Part A: Rational Numbers

We begin to color all the multiples of 3 in gray. Your chart should look like the following. (Students should notice every other multiple is already shaded….WHY?)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

4. 5 is the next number unshaded in the chart. Ask students how many factors the number 5 has. They should state that 5 has only 2 factors – 1 and itself. That makes it prime and we will color the 5 box yellow.

   Again, any multiple of 5 will have more than 2 factors.
   For example: 5x2 or 10 has $1 \bullet 10$ and $2 \bullet 5$, so 4 factors.
   5x3 or 15 has $1 \bullet 15$ and $3 \bullet 5$, so 4 factors. Etc.

   Teacher and students may skip count, 10, 15, 20, 25, … remind them these are multiples of 5 and begin to color all the multiples of 5 gray. Your chart should look like the following. (Students should notice every other multiple is already shaded….WHY?)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

5. 7 is the next unshaded number in the chart. Ask students how many factors the number 7 has. They should state that 7 has only 2 factors – 1 and itself. That makes it prime and we will color the 7 box yellow.

   Again any multiple of 7 will have more than 2 factors.
   For example: 7x2 or 14 has $1 \bullet 14$ and $2 \bullet 7$, so 4 factors.
   7x3 or 21 has $1 \bullet 21$ and $3 \bullet 7$, so 4 factors. Etc.

   Teacher and students may skip count, 14, 21, 28, 35, … remind them these are multiples of 7 and begin to color all the multiples of 7 gray. Your chart should look like the following. (Note: Only 49, 77 and 91 were unshaded and need to be shaded.)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

6. 11 is the next unshaded number in the chart. Ask students how many factors the number 11 has. They should state that 11 has only 2 factors – 1 and itself. That makes it prime and we will color the 11 box yellow.

   Again any multiple of 11 will have more than 2 factors. They are 22, 33, 44, 55, 66, 77, 88, and 99 but they are all already shaded.

7. 13 is the next number we should check – it is prime. Its multiples would be composite and checking those 26, 39, 52, 65, 78, and 91 they are all already shaded too.

   At this point we find the sieve did its job - it filtered out numbers that are composite or not prime. Shade the remaining unshaded boxes in yellow.

Math 7 Notes – Part A: Rational Numbers

Now we see all the prime numbers less than 100.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Take a moment to ask students what they notice. They should be able to state info like:

- 1 is not prime or composite.
- 2 is the ONLY even prime number.
- 5 is the ONLY prime number ending in a 5.
- 97 is the only number in the 90's that is prime.
- There are 25 prime numbers less than 100.
- Patterns - 23 and 29 and (thirty away) 53 and 59 and (thirty away) 83 and 89
- Patterns - 31 and 37 and (thirty away) 61 and 67 and (thirty away) 97 - not 91 since 7 • 13 = 91
- Patterns – 41, 43 and 47 and (thirty away) 71,73, and 79

Most students will know/learn the primes less than 10 or 20, challenge them to extend this knowledge further. Let them talk with a shoulder buddy or pair/share with a close neighbor and have one student state the primes less than 100 to the other. Then reverse roles. Have a contest to see who can name all 25 primes less than 100.

Remember to point out to students when they "think" a number is prime that is not, e.g. 51, that 51 is not prime because 5+1=6 so it is divisible by 3 which tells them it has more than 2 factors. Use the knowledge they learned in divisibility to support or refute if a number is prime or not.

In addition to identifying if specific numbers are prime or composite be sure to ask questions like:

Examples: Explain why 2 is the only even prime number. In addition to the number 1 and the number itself, all other even numbers have 2 as a factor.

Explain why the sum of two prime numbers greater than 2 can never be a prime number. Prime numbers greater than 2 are odd numbers. The sum of two odd numbers is an even number. All even numbers have 2 as a factor.

Examples: True or False?

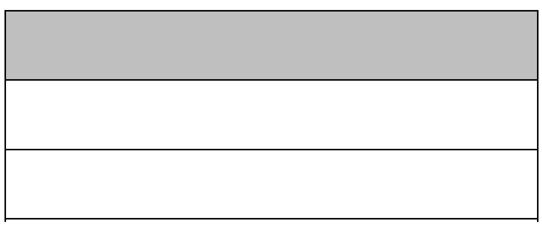
All odd numbers are prime. False

All primes are odd numbers. False

Review

Finding Common Denominators

Let's say we have two cakes, one chocolate and the other vanilla. The chocolate cake was cut into fourths, the vanilla cake into thirds as shown below. You take one piece of each, as shown.



Since you had 2 pieces of cake, can you say you had $\frac{2}{7}$ of a cake?

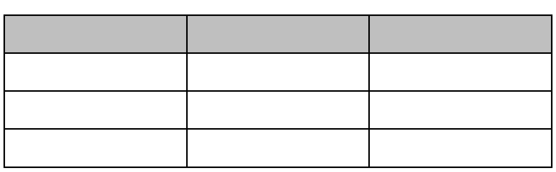
Remember our definition of a fraction:

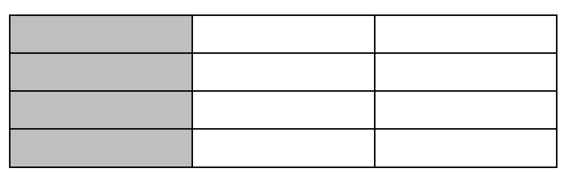
- the numerator indicates the number of equal size pieces you have
- while the denominator indicates how many equal sized pieces make one whole cake

Since your pieces are not equally sized, we can't say you had $\frac{2}{7}$ of a cake.

And clearly 7 pieces does not make one whole cake. We can conclude: $\frac{1}{4}+\frac{1}{3}\neq\frac{2}{7}$.

The key is to cut the cakes into *equally sized* pieces.
We'll cut the first cake (already in fourths) the same way the second cake was cut. And we'll cut the second cake (already in thirds) the same way the first cake was cut. So each cake ends up being cut into twelve *equally sized* pieces.





Math 7 Notes – Part A: Rational Numbers

Cutting the cake into EQUAL size pieces illustrates the idea of common denominator.

Let's look at several different methods of finding a common denominator.

	Methods of Finding a Common Denominator
A ***common denominator*** is a denominator that all other denominators will divide into evenly.	1. Multiply the denominators 2. List multiples of each denominator, use a common multiple. 3. Find the prime factorization of the denominators, and find the Least Common Multiple 1. Use the Simplifying/Reducing Method, especially for larger denominators.

In our cake illustration, the common denominator is the number of pieces that the cakes can be cut so that everyone has the same size piece.

Method 1: To find a common denominator of $\frac{1}{3}$ and $\frac{1}{4}$, *multiply* the denominators, $3 \cdot 4 = 12$.

This technique is very useful when the *denominators are prime or relatively prime (do not share any common factors* other than 1).

Method 2: To find a common denominator of $\frac{5}{6}$ and $\frac{3}{4}$, *list the multiples* of each denominator.

Multiples of 6: 6, **12,** 18, 24…
Mulitples of 4: 4, 8, **12**, …
Since 12 is on each *list of multiples*, it is a common denominator.

This technique is very useful when the *greater denominator is a multiple of the smaller one.*

Such as, find the common denominator for $\frac{3}{4}$ and $\frac{5}{16}$. 16 is a common denominator since it is a multiple of 4.

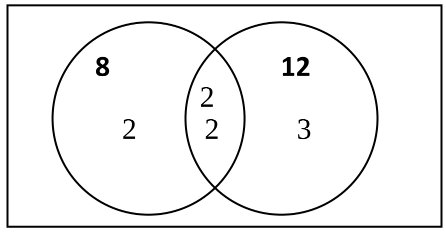
Method 3: To find a common denominator of $\frac{3}{8}$ and $\frac{7}{12}$, find the *prime factorization* of each denominator.

Factoring,

$$8 = 2 \cdot 2 \cdot 2$$
$$12 = 2 \cdot 2 \cdot 3$$

Multiply the prime factors, using overlapping factors only once, we get $2 \cdot 2 \cdot 2 \cdot 3 = 24$.

This can also be done using a Venn Diagram.



This technique can be tedious and may be the least desirable method; although it is good practice *working with prime factors.*

Method 4: To find the common denominator of $\frac{1}{18}$ and $\frac{5}{24}$ using the *Simplifying Method*, create a fraction using the two denominators: $\frac{18}{24}$, and then simplify: $\frac{18}{24} \div \frac{6}{6} = \frac{3}{4}$.

For $\frac{18}{24} = \frac{3}{4}$, cross multiply: $4 \cdot 18 = 72 \ or \ 3 \cdot 24 = 72$

. The common denominator is 72.

Note: It does not matter if you use $\frac{18}{24}$ or $\frac{24}{18}$.

Multiply the numbers in the Venn Diagram.
$2 \bullet 2 \bullet 2 \bullet 3 = 24$

This is an especially good way of finding common denominators for fractions that have large denominators or fractions whose denominators are not that familiar to you (large composite numbers).