



Precision and Accuracy

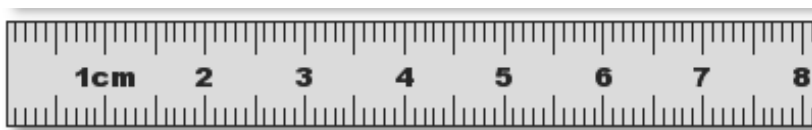
Learning Targets: Unit 2.1 To determine the degree of precision of a measurement.

We often use numbers that are not exact. Measurements are approximate—there is no such thing as a perfect measurement. The *precision* of a number refers to its exactness—to the level of detail to which the tool can measure. Measurements cannot be more precise than the measuring tool. This is very important in science!

Example: To what degree of precision can you measure a length using this ruler?



To the nearest $\frac{1}{8}$ of an inch



To the nearest mm

The smaller the unit of measurement, the more precise the measure. Consider some measures of time, such as 15 seconds and 15 hours. A measure of 15 seconds implies it is precise to the nearest second, or a time interval between 14.5 and 15.5 seconds. The time of 15 hours is far less precise: it suggests a time between 14.5 and 15.5 hours. The potential error in the first interval is 0.5 seconds; the potential error in the 15 hour scenario is 0.5 hour or 1800 seconds. Because the potential for error is greater, the 15-hour-measure is less precise.

Ex 1: Choose the more precise measurement in the given pair.

- (a) 3 m, 35 km **3 m is more precise (meters are smaller than km)**
 (b) 12 inches, 1 foot **12 inches (inches are smaller than a foot)**
 (c) 1 pound, 1 ounce **1 ounce (an ounce is smaller than a pound)**

The number of decimal places in a measurement can also affect precision. Using time again, a measure of 5.1 seconds is more precise than 5 seconds. The 5.1 measurement implies a measure precise to the nearest tenth of a second. The potential error in 5.1 seconds is 0.05 seconds, compared to the potential error of 0.5 seconds with the measure of 5 seconds.

Ex 2: Choose the more precise measurement in the given pair.

- (a) 5.4 m, 5.67 m **5.67 is more precise (hundredths of a meter is smaller than tenths)**
 (b) 3 yards, 3.6 yards **3.6 is more precise (tenths of a yard is more precise than yards)**



Another concept that has to do with measurement is *accuracy*. Many people think precision and accuracy are the same thing. They are not! The *accuracy* of a measurement refers to how close the measured value is to the true or accepted value. For example, if you are in a lab and you obtain a weight measurement of 4.7 kg for an object, but the actual or known weight is 10 kg, then your measurement is not *accurate* (your measurement is not close to the accepted value). However, if you weigh the object five times, and get 4.7 kg each time, then your measurement is precise—each measurement was the same as the previous. Precision is independent of accuracy. In this case, you were very precise, but inaccurate.

Another way to explain it:

Imagine a basketball player shooting baskets. If the player shoots with *precision*, his aim will always take the ball to the same location, which may or may not be close to the basket. If the player shoots with *accuracy*, his aim will always take the ball close to or into the basket. A good player will be both precise and accurate: shoot the ball the same way each time and make the basket. If you are a soccer player, and you always hit the left goal post (instead of scoring), what can you conclude? You are precise, but not accurate!

A dartboard analogy is often used to help us understand the difference between accuracy and precision. Imagine a person throwing darts, trying to hit the bull's eye. There are 4 scenarios:

Not Precise/Not Accurate

It is a random pattern:
darts are not clustered and
are not near bull's eye.



Precise/Not Accurate

Darts are clustered
together but did not hit the
bull's eye.



Not Precise/Accurate
Darts are not clustered
together, but their "average"
hit the bull's eye.



Precise/Accurate
Darts are clustered together
and their "average" position
hit the bull's eye.





It is easy to confuse precision and accuracy. The tool that you use affects both the precision and accuracy of your measurement. Measuring with a millimeter tape allows greater precision than measuring with an inch tape. Because the error using the millimeter tape should be less than the inch tape, accuracy also improves.

Suppose that a tape measure is used to measure the diameter of two circles. Let's suppose you measure the first circle to be 15 cm, and a second circle to be 201 cm. The two measures are equally precise (both measured to the nearest cm). However, their accuracy may be quite different. Let's further suppose that the accepted values for the measurements are 16 cm and 202 cm. The errors for these measurements are $\frac{1}{16} = 0.0625$ or 6.25% and $\frac{1}{202} = 0.0049504$ or about 0.5%. The second measurement is more accurate because the error is smaller.

One more way to think of this: accuracy implies that a measurement is basically right, given a margin of error. Precision is the level of detail; or typically the number of digits after a decimal point. For instance, I ask how far it is to the store. You could tell me "about 3 miles", while a GPS device might tell me "2.85 miles". About 3 miles is pretty accurate, but 2.85 is both accurate and precise. Now, you could have told me "15.345 miles", which would make you very precise, but not accurate.

Most of the time in our everyday life, we want accuracy; precision is not as useful. But in science and engineering, both precision and accuracy are important.

A couple of websites to help clarify accuracy and precision:

<http://www.mathsisfun.com/accuracy-precision.html>

http://commons.bcit.ca/math/competency_testing/testinfo/testsyll11/approxnum/accandprec/accandprec.pdf

<http://mathbitsnotebook.com/Algebra1/Units/UNoutline.html>



SIGNIFICANT FIGURES

Learning Targets: Unit 2.2 To use significant digits to report results of calculations involving measurement.

When rounding to the appropriate significant digit, all the digits that are known with certainty are called *significant digits*. Below are the rules for determining significant digits. The only tricky digits are the zeros:

- All non-zero digits are significant digits.
 - 3 has one significant digit
 - 2.5 has two significant digits
 - 356.491 has six significant digits
- Zeros that occur between significant digits are significant digits.
 - 207 has 3 significant digits
 - 6.005 has 4 significant digits
 - 20.006 has 5 significant digits
- Zeros to the right of the decimal point AND to the right of a non-zero digit are significant digits.
 - 0.10 has 2 significant digits (the 0 before the decimal is not significant while the 0 to the right of the decimal point and the digit 1 are significant)
 - 0.0040 has 2 significant digits (just the last two)
 - 4.60 has 3 significant digits
 - 460 has 2 significant digits (zero is to the left of the decimal point)
 - 46.00 has 4 significant digits
 - 460.00 has 5 significant digits (the two zeros to the right of the decimal point are significant—this makes the zero to the left of the decimal point significant because it lies between significant digits)

Ex 3: Determine the number of significant digits in each measurement.

- | | |
|------------|---|
| (a) 32.75 | 4 significant digits (all nonzero digits) |
| (b) 43.023 | 5 significant digits (zero is between significant digits) |
| (c) 0.0240 | 3 significant digits (zero after the last nonzero digit and to the right of the decimal point is significant) |
| (d) 0.007 | 1 significant digit |

Differing levels of precision can cause us a problem when dealing with arithmetic operations. Suppose I wish to add 11.1 seconds to 13.47 seconds. The answer, 24.57 seconds, is misleading. That is:

11.1 seconds	implies	the time is between 11.05 and 11.15 seconds
13.47 seconds	implies	the time is between 13.465 and 13.475 seconds
The sum	should imply	the time is between 24.515 and 24.625 seconds

But the sum 24.57 seconds implies the time is between 24.565 and 24.575, which is more precise than the actual result.



So it is generally accepted that *when you add or subtract, you report your answer to the same precision as the least precise measure*. In other words, the answer should have the same number of digits to the right of the decimal point as the measurement with the least number of digits to the right of the decimal point. In our example, we would report 24.57 as 24.6 seconds.

Ex 4: Calculate. Use the correct number of significant digits.

(a) $4.5 + 2.17 = 6.67$,
 ≈ 6.7 *Answer rounded to 1 digit to the right of the decimal (tenths)*

(b) $15 - 5.6 = 9.4$ *Answer rounded to no digits to the right of the decimal (units)*
 ≈ 9

Multiplying or dividing measures creates a different type of problem. For instance, I want to find the area of a rectangle that measures 2.7 cm by 4.6 cm. When I multiply, I obtain the answer 12.42 cm^2 . However,

2.7 implies

2.65 cm to 2.75 cm

4.6 implies

4.55 cm to 4.65 cm

The product should imply

12.0575 cm^2 to 12.7875 cm^2

But the product 12.42 cm^2 implies 12.415 cm^2 to 12.425 cm^2 , which is more precise than the actual result.

The accepted practice when *multiplying or dividing is to report the result using the fewest number of significant digits in the original measures given*. Or, when you multiply or divide measurements, the answer should have the same number of significant digits as the measurement with the least number of significant digits. In our example, there are two significant digits in 2.7 cm and 4.6 cm, so the result is rounded to two significant digits, 12 cm^2 .

Ex 5: Calculate. Use the correct number of significant digits.

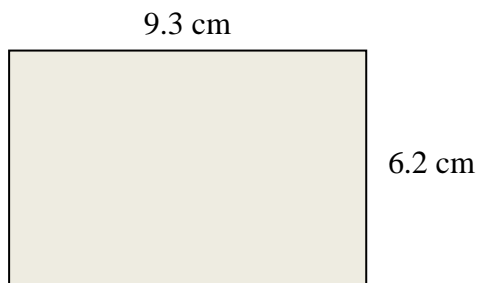
(a) Find the area of a parallelogram with height of 12.1 inches and base of 6 inches.
Area = (12.1)(6), which is 72.6. However, the lowest number of significant digits is one, so we would round the answer to 70 square inches.

(b) $14.2 \div 0.05$
 $14.2 \div 0.05 = 284$. *However, the lowest number of significant digits is one, so we would round the answer to 300.*



SAMPLE EXAM QUESTIONS

1. Use the diagram below.



A rectangle's sides are measured to be 6.2 cm and 9.3 cm. What is the rectangle's area rounded to the correct number of significant digits?

- (A) 57.66 cm^2
(B) 57.7 cm^2
(C) 58 cm^2
(D) 60 cm^2

ANS: C

2. How many significant figures are there in the number 0.02040?

- (A) 2
(B) 3
(C) 4
(D) 5

ANS: B

3. The answer to the following addition problem should be expressed as:

$$10.11111 + 1.02 + 100.00001 + 0.04000 = 111.17112$$

- (A) 111.1711
(B) 111.171
(C) 111.2
(D) 111.17

ANS: D

4. The density of a metal is calculated as follows. The weight of 57.4 grams is divided by the volume 6.2 cm^3 . Using the rule for significant figures in this calculation, the value of the density, in g/cm^3 , should be reported as:

- (A) 9
(B) 9.3
(C) 9.26
(D) 9.258

ANS: B



UNIT ANALYSIS

Learning Targets: Unit 2.3 To convert from one unit to another within the context of solving problems.

BIG IDEA: Writing the units of each variable in a real-life problem helps you determine the units for the answer. This is called unit analysis and it is often used in problem solving in science. When the same units of measure occur in the numerator and the denominator of an expression, you can cancel the units.

Note: It is helpful to have conversion tables available for units of time, capacity, mass, linear measure; etc. Abbreviations of units should also be included.

In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly “stands out” as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

N.Q.A. Reason quantitatively and use units to solve problems.

N.Q.A.1: Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

N.Q.A.2: Define appropriate quantities for the purpose of descriptive modeling.

An important component of number and quantity is the concept of ratio and proportion. Students should be able to simplify common ratios to find unit rates and to set up and solve proportions.

NOTE: These standards should be taught and reinforced throughout the year when applicable.

PREREQUISITE KNOWLEDGE/SKILLS:

- students must have a clear understanding of rates and unit rate and proportions
- students must understand reciprocals
- students must know unit conversions in measurement

**VOCABULARY:**

- Conversion factor: a factor used to convert a measured quantity to a different unit of measure without changing the relative amount
- Unit analysis or Dimensional analysis: a process that uses rates to convert measurements from one unit to another

SKILLS:

- write expressions, equations and inequalities including the units in order to correctly label real-world problems
- convert given rates to equivalent forms using different units

REVIEW AND EXAMPLES:

Ex 6: Convert $4\frac{1}{2}$ feet to inches. Which ratio should be used?

$$\frac{12 \text{ in}}{1 \text{ ft}} \text{ or } \frac{1 \text{ ft}}{12 \text{ in}}$$

$$\frac{4.5 \cancel{\text{ ft}}}{1} \cdot \frac{12 \text{ in}}{1 \cancel{\text{ ft}}} = \frac{54 \text{ in}}{1} = 54 \text{ in}$$

Ex 7: A **pipe** is leaking at a rate of 8 fluid ounces per minute. Use unit (dimensional) analysis to find out how many gallons the pipe is leaking per hour.

- (A) 3,840 gal/h (C) 3.75gal/h
 (B) 0.02 gal/h (D) 17.07 gal/h

We start by listing the given leaking rate: 8 ounces/minute then multiply by conversion factors for liquid volume so we can cancel units using unit analysis.

$$\frac{8 \cancel{\text{ fl oz}}}{1 \text{ min}} \cdot \frac{1 \cancel{\text{ c}}}{8 \cancel{\text{ fl oz}}} \cdot \frac{1 \cancel{\text{ pt}}}{2 \cancel{\text{ c}}} \cdot \frac{1 \cancel{\text{ qt}}}{2 \cancel{\text{ pt}}} \cdot \frac{1 \text{ gal}}{4 \cancel{\text{ qt}}} = \frac{1 \text{ gal}}{16 \text{ min}}$$

The conversion factor for minutes to hours is 60 minutes = 1 hour

$$\frac{1 \text{ gal}}{16 \cancel{\text{ min}}} \cdot \frac{60 \cancel{\text{ min}}}{1 \text{ hr}} = 3.75 \frac{\text{gal}}{\text{hr}}$$



Ex 8: A 25 ft tree casts a $6\frac{1}{4}$ ft shadow. At the same time, a fence casts a 2 ft shadow. How tall is the fence?

- (A) $\frac{1}{2}$ ft (B) 8 ft
(C) 4 ft (D) $12\frac{1}{2}$ ft

Set up a proportion including units.

$$\frac{25 \text{ ft}}{6.25 \text{ ft}} = \frac{x}{2 \text{ ft}} \qquad \frac{\cancel{2 \text{ ft}}}{1} \cdot \frac{25 \text{ ft}}{\cancel{6.25 \text{ ft}}} = \frac{x}{\cancel{2 \text{ ft}}} \cdot \frac{\cancel{2 \text{ ft}}}{1} \qquad x = 8 \text{ ft}$$

SAMPLE EXAM QUESTIONS

- In the formula $F = I + at$, F and I are measured in meters per second and t is measured in seconds. In what units is a measured?

(A) meters
(B) seconds
(C) meters per second
(D) meters per second squared

ANS: D

- Sam drives his car 150 miles in 3 hours. Find the unit rate.

(A) Sam drives 1 mile per 50 hours.
(B) Sam drives 50 miles per hour.
(C) Sam drives 30 miles per hour.
(D) Sam drives 150 miles per 3 hours.

ANS: B

- A model of a boat is 13.2 inches long. The scale of the model to the original is 2 inches : 5 feet. How long is the boat?

(A) 5 ft (C) 33 ft
(B) 6.6 ft (D) 132 ft

ANS: C



4. Kevin ran at a rate of 13 km/h. Convert his speed to meters per minute.

- (A) ≈ 0.2 m/min
(B) ≈ 217 m/min
(C) ≈ 3.6 m/min
(D) $\approx 12,000$ m/min

ANS: C

5. Molly jogged at a rate of 6 miles per hour. What was her speed in feet per second?

- (A) 8.8 ft/s
(B) 14.7 ft/s
(C) 176 ft/s
(D) 528 ft/s

ANS: A

6. You want to model the speed of a motorcycle. Which units would be appropriate for measuring this quantity?

- (A) kilometers per mile
(B) miles per hour
(C) minutes per hour
(D) hours per mile

ANS: B

7. You want to model the pressure of water on a given area of a submarine. Which units would be appropriate for measuring this quantity?

- (A) pounds per inch
(B) grams per cubic centimeter
(C) pounds per square inch
(D) square meters per liter

ANS: C

8. Anna and Joe are measuring the speed of a snail. Which units would be *most* appropriate for measuring this quantity?

- (A) miles per hour
(B) minutes per millimeter
(C) meters per second
(D) millimeters per minute

ANS: D

9. A printer prints 24 pages in 3 minutes. Which unit rate is correct?

- (A) 0.125 pages per minute
(B) 8 pages per minute
(C) 72 pages per hour
(D) 1440 pages per hour

ANS: B



10. A proofreader read 175 pages in 7 hours. To the nearest hundredth, what is her reading rate in pages per minute?

ANS: 0.46 pages/min

11. In baseball, the pitcher's mound is 60.5 feet from home plate. If a pitcher throws a baseball 90 mph, how many seconds will it take the baseball to reach home plate?
 (A) 0.46 sec (B) 0.67 sec
 (C) 1.49 sec (D) 90.75 sec

ANS: A

12. The "strike zone" or distance across home plate is 17 inches. Determine how long a baseball pitched at 90 mph is in the strike zone.
 (A) 0.27 sec (B) 0.19 sec
 (C) 0.13 sec (D) 0.01 sec

ANS: D

SHORT ANSWER

1. A train travels 120 miles in one hour. What is its speed in miles per minute?

ANS: 2 mi/min

2. The ratio of the sale price of a textbook to the original price is 1:3. The original price is \$81. What is the sale price?

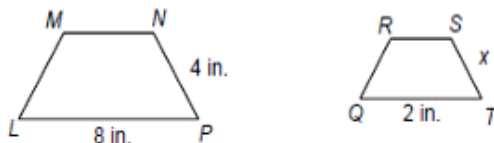
ANS: \$27

3. Shayla is 5 feet tall and casts a shadow 2 feet long. At the same time, a nearby lamppost casts a shadow 8 feet long. Write and solve a proportion to find the height of the lamppost.

ANS: 20 ft

4. Find the value of x in the diagram.

$$LMNP \sim \triangle QNST$$



ANS: $x = 1$ in

5. A boat travels 36 miles in one hour. What is its speed in miles per minute?

ANS: 0.6 mi/min