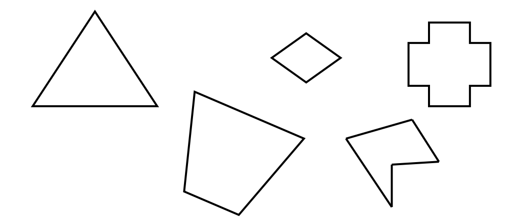
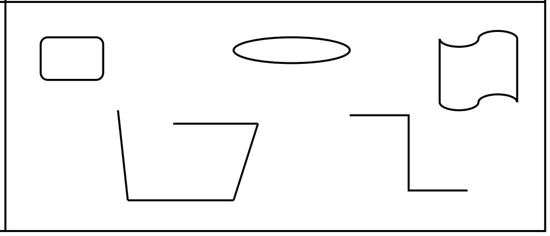
Math 6 NOTES: Geometry
2-Dimensional Figures

Prep for 6.G.A.1

Classifying Polygons

A **polygon** is defined as a closed geometric figure formed by connecting line segments endpoint to endpoint.

Polygons	Not Polygons
	

Polygons are named by the number of sides. We know a triangle has 3 sides.

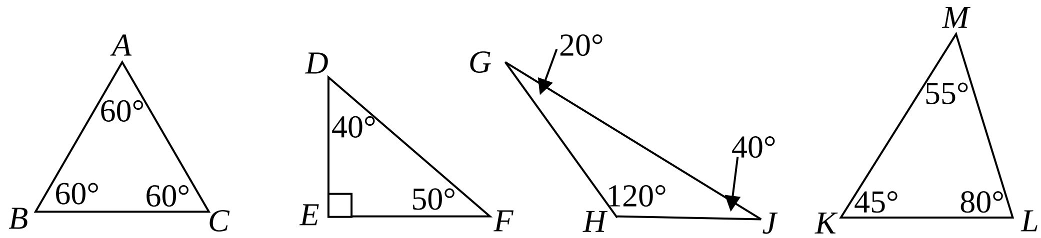
Prep for 6.G.A.1

Classifying Triangles

Triangles can be classified by the measures of their angles:

- ***acute*** triangle—3 acute angles
- ***right*** triangle—1 right angle
- ***obtuse*** triangle—1 obtuse angle

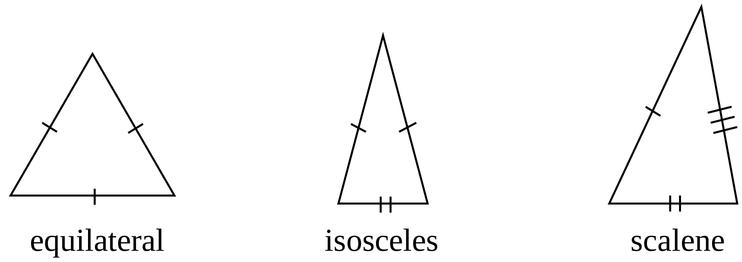
Example: Classify each triangle by their angle measure:



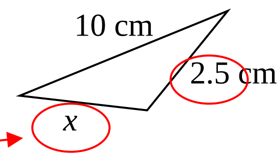
Acute (Equiangular) **Right** **Obtuse** **Acute**

Triangles can also be classified by the lengths of their sides. You can show tick marks to show congruent sides.

- ***equilateral*** triangle—3 congruent sides
- ***isosceles*** triangle—at least 2 congruent sides
- ***scalene*** triangle—no congruent sides



Example: *Classify the triangle. The perimeter of the triangle is 15 cm.*



Using the information given regarding the perimeter:

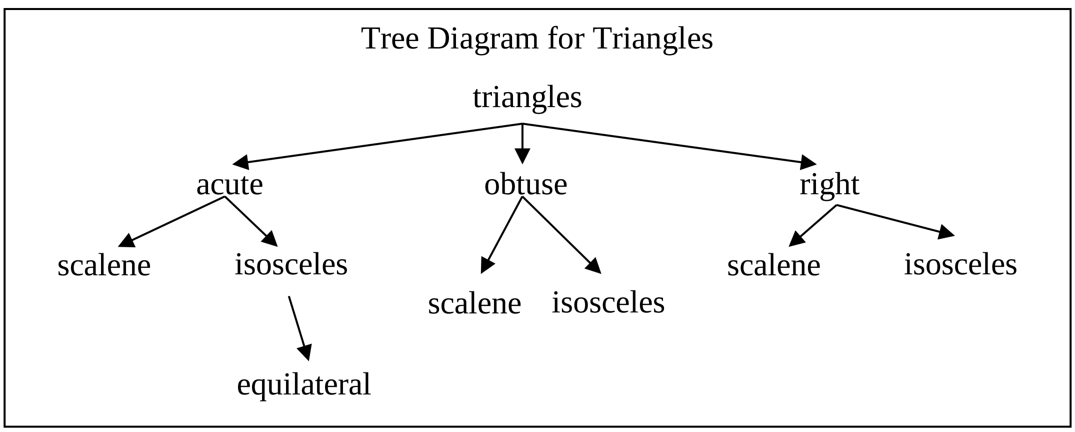
$x + 2.5 + 10 = 15$

$x + 12.5 = 15$

$x = 2.5$

Since 2 sides are congruent, the triangle is isosceles.

A tree diagram could also be used to show the triangle relationships.

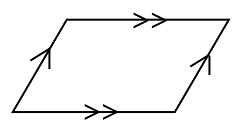


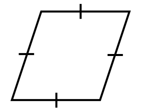
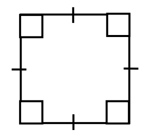
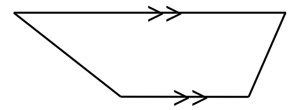
Regular polygons have all sides and all angles congruent. Also in regular polygons, there is a point (its center) which is equidistant from all of its vertices. Which of the triangles we have identified so far, would be a regular triangle? Equilateral Triangles

Prep for 6.G.A.1

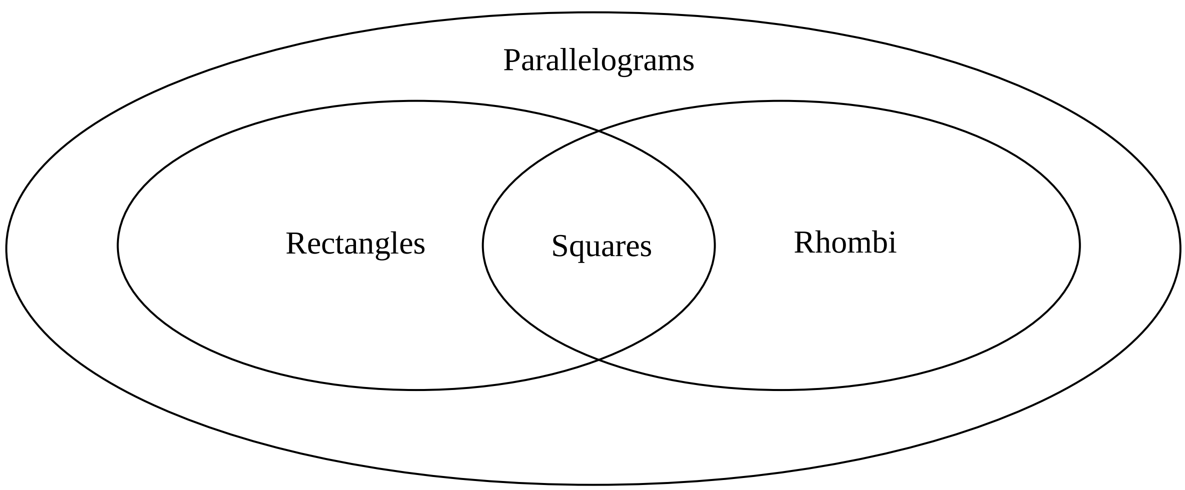
Classifying Quadrilaterals

A ***quadrilateral*** is a plane figure with four sides and four angles. They are classified based on congruent sides, parallel sides and right angles.

Quadrilateral Type	Definition	Example
Parallelogram	Quadrilateral with both pairs of opposite sides parallel.	

Rhombus	Parallelogram with four congruent sides.	 *Note: This polygon is a parallelogram.*
Rectangle	Parallelogram with four right angles.	 *Note: This polygon is a parallelogram.*
Square	Parallelogram with four right angles and four congruent sides.	 *Note: This polygon is a parallelogram.*
Trapezoid	Quadrilateral with exactly one pair of parallel sides.	

Another way to show the relationship of the parallelograms is to complete a Venn diagram as shown below.



Which of the polygons we have identified so far, would be a regular polygons? Equilateral triangles and squares

Example: *A quadrilateral has both pairs of opposite sides parallel. One set of opposite angles are congruent and acute. The other set of angles is congruent and obtuse. All four sides are NOT congruent. Which name below best classifies this figure?*

A. parallelogram
B. rectangle
C. rhombus
D. trapezoid

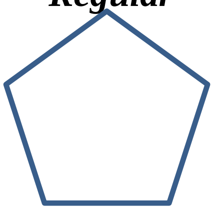
Vocabulary becomes very important when trying to solve word problems about quadrilaterals.

We have both pairs of opposite sides parallel, so it cannot be the trapezoid. Since the angles are not 90° in measure, we can rule out the rectangle. We are told that the 4 sides are not congruent, so it cannot be the rhombus. ***Therefore, we have a parallelogram. (A)***

Classifying Other Polygons

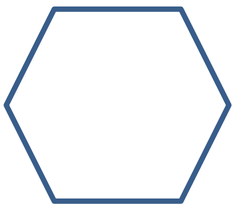
Polygons	Pentagon	Hexagon	Heptagon	Octagon	Nonagon	Decagon
# of sides	5	6	7	8	9	10

Regular ***Irregular***



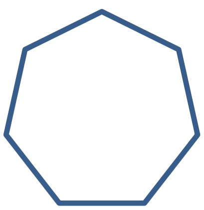
Regular pentagon

An example of an irregular pentagon

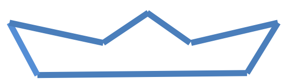


Regular hexagon

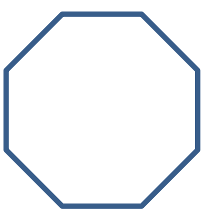
An example of an irregular hexagon



Regular heptagon

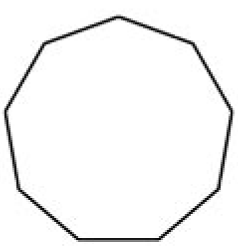


An example of an irregular heptagon

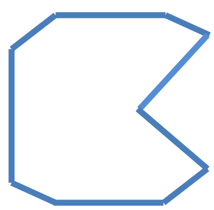


Regular octagon

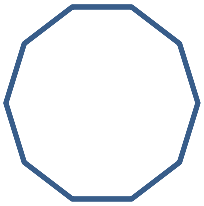
An example of a irregular octagon



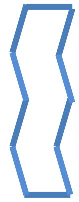
Regular nonagon



An example of a irregular nonagon



Regular decagon



An example of a irregular decagon

6.G.A.1 ***Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.***

Area of Triangles and Quadrilaterals

One way to describe the size of a room is by naming its dimensions. A room that measures 12 ft. by 10 ft. would be described by saying it's a 12 by 10 foot room. That's easy enough.

There is nothing wrong with that description. In geometry, rather than talking about a room, we might talk about the size of a rectangular region.

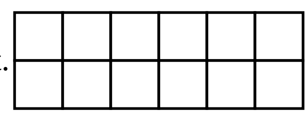
For instance, let's say I have a closet with dimensions 2 feet by 6 feet (sometimes given as 2×6). That's the size of the closet.

2 ft.

6 ft.

Someone else might choose to describe the closet by determining how many one foot by one foot tiles it would take to cover the floor. To demonstrate, let me divide that closet into one foot squares.

2 ft.



6 ft.

By simply counting the number of squares that fit inside that region, we find there are **12 squares.**

If I continue making rectangles of different dimensions, I would be able to describe their size by those dimensions, or I could mark off units and determine how many equally sized squares can be made.

Rather than describing the rectangle by its dimensions or counting the number of squares to determine its size, we could multiply its dimensions together.

Putting this into perspective, we see the number of squares that fits inside a rectangular region is referred to as the ***area***. A shortcut to determine that number of squares is to multiply the base by the height.

The ***area of a rectangle*** is equal to the product of the length of the base and the length of a height to that base.

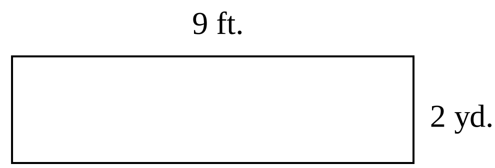
That is $A = bh$. Most books refer to the longer side of a rectangle as the length (***l***), the shorter side as the width (***w***). That results in the formula $A = lw$. The answer in an area problem is always given in square units because we are determining how many squares fit inside the region.

Example: *Find the area of a rectangle with the dimensions 3 m by 2 m.*

$$A = lw$$
$$A = 3 \cdot 2$$
$$A = 6$$

The area of the rectangle is 6 m²^{2}.

Example: *Find the area of the rectangle.*



Be careful! Area of a rectangle is easy to find, and students may quickly multiply to get an answer of 18. This is wrong because the measurements are in different units. We must first convert feet into yards, or yards into feet.

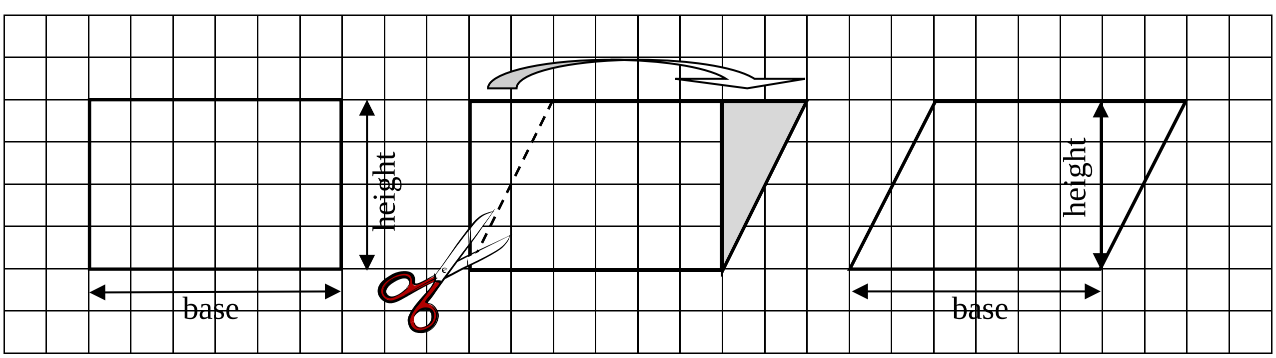
$$\frac{yards}{feet} \rightarrow \frac{1}{3} = \frac{x}{9}$$
$$9 = 3x$$
$$3 = x$$

We now have a rectangle with dimensions 3 yd. by 2 yd.

$$A = lw$$
$$A = (3)(2)$$
$$A = 6$$

The area of our rectangle is 6 square yards.

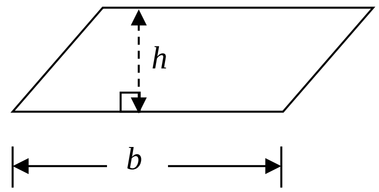
If I were to cut one corner of a rectangle and place it on the other side, I would have the following:



A parallelogram! Notice, to form a parallelogram, we cut a piece of a rectangle from one side and placed it on the other side. Do you think we changed the area? The answer is **no**. All we did was rearrange it; the area of the new figure, the parallelogram, is the same as the original rectangle.

This allows us to find a formula for the area of a parallelogram.

Since the bottom length of the rectangle was not changed by cutting, it will be used as the base length (***b***), the height of the rectangle was not changed either, we'll call that ***h***.



Now we arrive at the formula for the **area of a parallelogram.**
$A = bh$.

Example: *The height of a parallelogram is twice the base. If the base of the parallelogram is 3 meters, what is its area?*

First, find the height. Since the base is 3 meters, the height would be twice that or 2(3) or 6 m.
To find the area,

$$A = bh$$
$$A = 3 \cdot 6$$
$$A = 18$$

The area of the parallelogram is 18 m² — ***The area of the parallelogram is 18 m^2.***

We have established that the area of a parallelogram is $A = bh$. Let's see how that helps us to understand the area formula for a triangle and trapezoid.

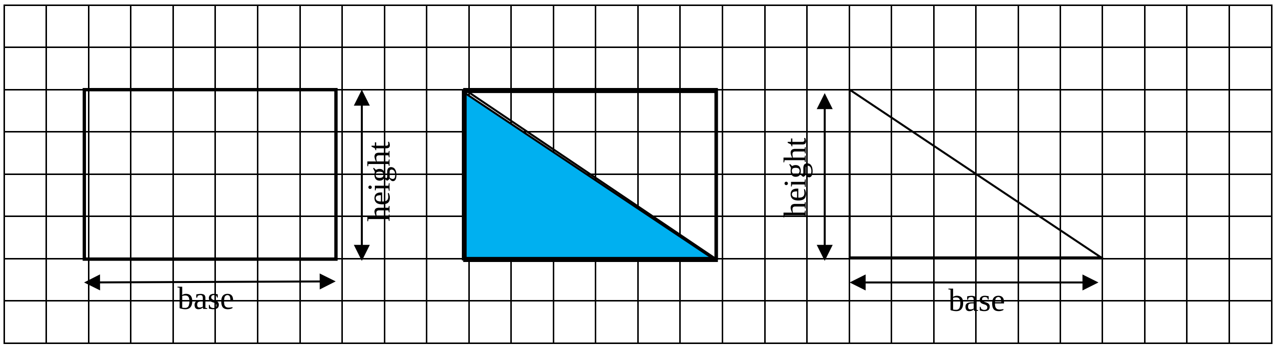
Remember: Once a formula for a figure has been developed, it can be used for any figure that meets its criteria.

For example: The parallelogram formula can be used for rectangles, rhombi, and squares.
The rectangle formula can be used for squares.
The rhombus formula (derived in HS Geometry) can be used for squares.

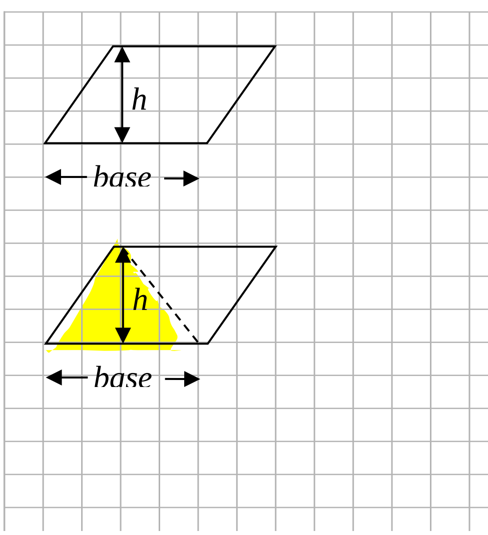
This is based on the Venn Diagram given previously (pg. 3 of these notes). The inner sets have all the same attributes and properties of the sets they are contained within. Therefore, what must be true about any element of the outer set must be true of **all** elements of that set.

Let’s see how that helps us to understand the area formula for a triangle.

If I were to cut the rectangle using its diagonal, I would have the following:



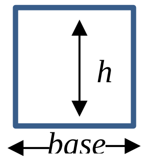
Cutting the rectangle as shown above to create the triangle allows me to see that the triangle is half the rectangle. Therefore since the area of the rectangle is $A = bh$ and the triangle is half of that rectangle, the area of the triangle should be $A = \frac{1}{2}bh$. We can easily see/count the rectangle contains 24 square units and the triangle contains 12 square units. Mathematically, we could show the area of the rectangle is $A = 6 \bullet 4 = 24 units^2$. The **Area of the triangle =** $\frac{1}{2}\boldsymbol{bh}$ or the **Area of the triangle =** $\frac{bh}{2}$**.** To compute the area we get $A = \frac{1}{2} \bullet 6 \bullet 4 = \frac{1}{2} \bullet 24 = 12 units^2$ or $A = \frac{bh}{2} = \frac{6 \bullet 4}{2} = \frac{24}{2} = 12 \text{ } square \ units$.



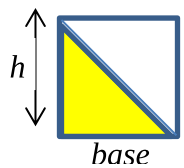
For this parallelogram, its base is 4 units and its height is 3 units. Therefore, the area is $4 \cdot 3 = 12 \text{ units}^2$.

If we draw a diagonal, it cuts the parallelogram into 2 triangles. That means one triangle would have one-half of the area or 6 units^2. Note the base and height stay the same. So for a triangle,

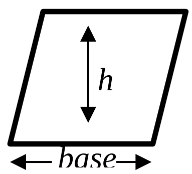
$A = \frac{1}{2}bh, \text{ or } \frac{1}{2}(4)(3) = 6 \text{ units}^2$



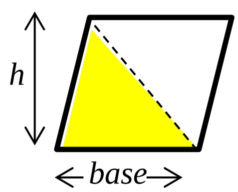
For this square, let's say its base is 4 units. Then, the area is $4 \cdot 4 = 16 \text{ units}^2$.



If we draw a diagonal, it cuts the square into 2 triangles. That means one triangle would have one-half of the area or 6 units^2. Note the base and height stay the same. So for a triangle, $A = \frac{1}{2}bh$, or $\frac{1}{2}(4)(4) = 8 \text{ units}^2$



For this rhombus, let's say its base is 4 units and its height is 3 units. Then, the area is $4 \cdot 3 = 12 \text{ units}^2$.



If we draw a diagonal, it cuts the rhombus into 2 triangles. That means one triangle would have one-half of the area or 6 units^2. Note the base and height stay the same. So for a triangle,

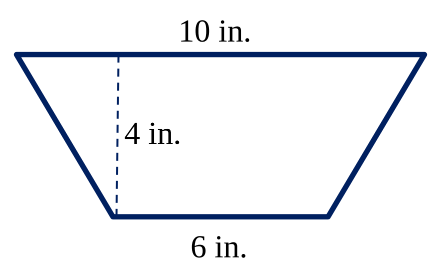
$A = \frac{1}{2}bh$, or $\frac{1}{2}(4)(3) = 6 \text{ units}^2$

We have shown that half the rectangle, half the square, half the parallelogram and half the rhombus (when cut by a diagonal) form triangles whose area is half the original polygon. To us this may seem redundant, but students need to see and experience this for themselves.

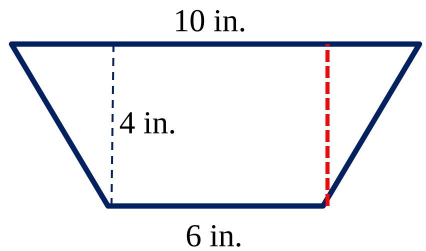
NVACS calls for students to compose and decompose figures using this knowledge. At this point we do NOT teach the formula for trapezoids but instead have them decompose (divide the figure) into parts they know.

Composite Figures are figures made up of multiple shapes. (linkage - Composite numbers have multiple factors) In order to find the area of these oddly-shaped figures they must be decomposed into figures we are familiar with.

Let's start with this isosceles trapezoid…



We can decompose this trapezoid into a rectangle and two triangles. The area of this trapezoid would be the area of the rectangle added to the areas of the two triangles. In this case, **because the trapezoid is isosceles**, the two triangles will be congruent.



Notice the b_1 length 10 is equal to the b_2 length 6, plus 4 more inches. Those 4 inches can be split into 2 and 2 and would indicate the lengths of the bases of the two triangles.

Now we can use the rectangle formula and triangle formula (twice) to find the total area.

<u>Rectangle</u>

$A = lw$

$A = (6)(4)$

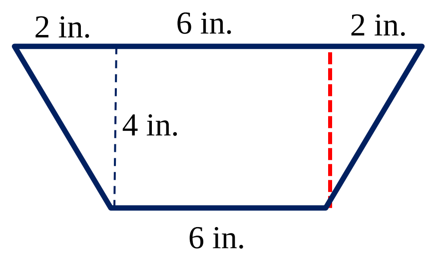
$A = 24$

<u>Each Triangle</u>

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(2)(4)$

$A = 4$



Since the two triangles are congruent,

$A_{total} = A_{rectangle} + (2)A_{triangle}$

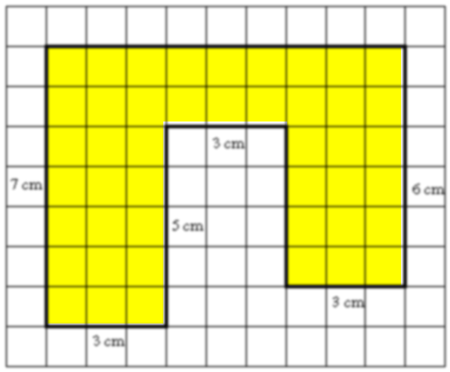
$A = 24 + (2)(4)$

$A = 32$

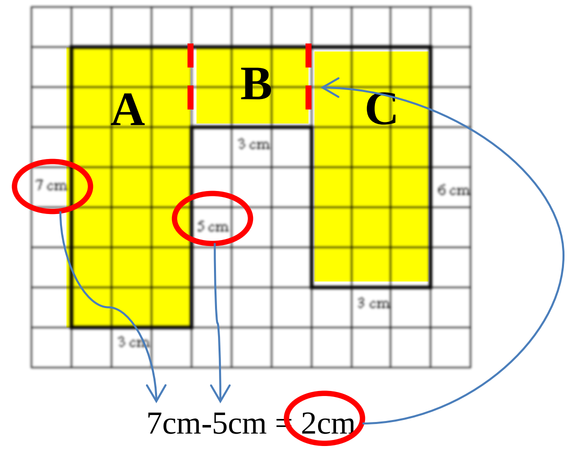
The area of this trapezoid is 32 square inches.

Now that we see that it can be done, we can explore the areas of other, less common, composite shapes. These shapes must be decomposed and the area formulas of the decomposed parts can be used to find each individual area. The total area of the figure is the sum of the areas of its decomposed parts.

Example*: Find the area of the given shape.*



Start by decomposing…



One way to decompose is given above. Students may find multiple ways to decompose the given figures. If the needed measurements are not available for them to complete the problem, they may want to consider trying a different combination. (Re-decompose?)

From the diagram we can see that the total area of this figure will be the sum of the areas of the three rectangles that composed it. Notice that the measures of the sides of the figure can be found by counting the blocks that run along the side of that portion. (Be careful of the scale of the diagram, sometimes a block represents more than one unit.)

$$A_{TOTAL} = A_{rectangle\ A} + A_{rectangle\ B} + A_{rectangle\ C}$$

$$A = (7)(3) + (2)(3) + (6)(3)$$

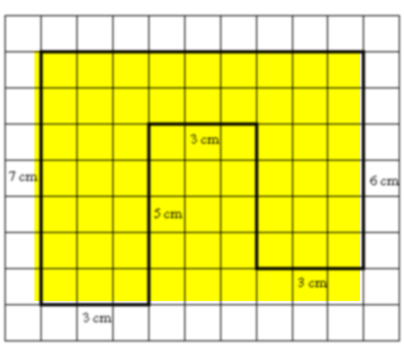
$$A = 21 + 6 + 18$$

$$A = 45$$

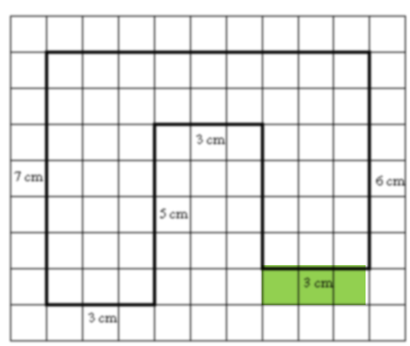
The area of this figure is 45 square cm.

Students can verify this answer by counting the squares inside the figure.

Another way to find the area of the above shape is to complete a rectangle. Then subtract off the section(s) not needed.

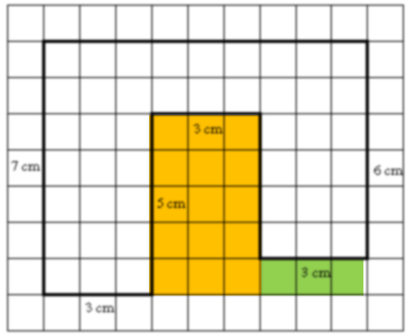


$$A_{TOTAL} = 7(3+3+3) = 7(9) = 63$$



Minus the area shown in green (and below) in orange

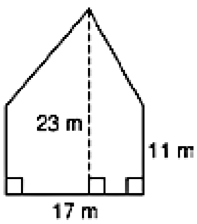
$$A_{small\ rectangle} = 1(3) = 3$$



$A_{\text{large rectangle}} = 5(3) = 15$

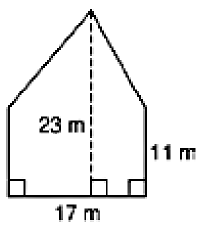
Area of the composite figure **=** $A = 63 - 3 - 15 = 45$ ***45square cm***

Example*: Find the area of the given polygon.* *(DOK 2)*



A. 102 m^2
B. 187 m^2
C. 289 m^2
D. 391 m^2

First, the figure must be decomposed…
Students should be able to find a rectangle and a triangle.



$A_{TOTAL} = A_{rectangle} + A_{triangle}$

remember... $A_{rectangle} = lw$ and $A_{triangle} = \frac{1}{2}bh$

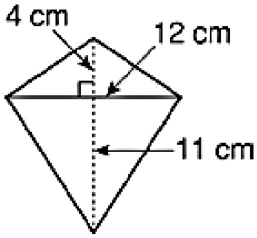
$A = (11)(17) + \frac{1}{2}(17)(12)$

$A = 187 + 102$

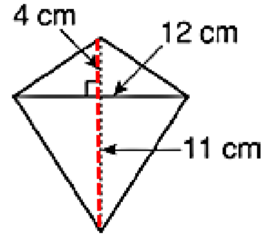
$A = 289$

So, the area of this figure is 289 m^2. (C)

Example*: Find the area of the given polygon.* *(DOK 2)*



A. 45 cm^2
B. 90 cm^2
C. 110 cm^2
D. 180 cm^2



This figure can be decomposed a few ways:
Vertically down the middle, forming two congruent triangles:
$b = 11 + 4 = 15$ **cm,** $h = 12 \div 2 = 6$ **cm**
Therefore, the total area is **twice** the area of one triangle.

$$A_{triangle} = \frac{1}{2}bh$$

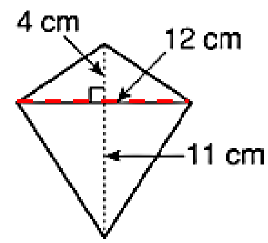
$$A_{TOTAL} = (2)A_{triangle} = (2)\frac{1}{2}bh$$

The total area is 90 cm^2. (b)

$$A_{TOTAL} = (2)\frac{1}{2}(15)(6)$$

$$A = 90$$

<u>Another way would be:</u>
Horizontally, along the diagonal, forming two non-congruent triangles:
b = 12 cm (on both), h_1 = 4 cm (top Δ) & h_2 = 11 cm (bottom Δ)
In this case we must find the sum of both areas to find the total sum.



$$A_{triangle} = \frac{1}{2}bh$$

$$A_{TOTAL} = A_{top\,\Delta} + A_{bottom\,\Delta}$$

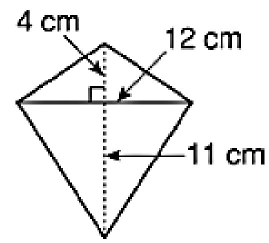
$$A_{TOTAL} = \frac{1}{2}(12)(4) + \frac{1}{2}(12)(11)$$

The total area is 90 cm^2. (b)

$$A_{TOTAL} = 24 + 66$$

$$A = 90$$

<u>A final alternative would be:</u>
Horizontally and vertically, forming four right triangles, with two sets congruent: (these measurements are interchangeable based on your perspective when viewing the triangles)
b_1 = 4 cm (both smaller Δs) & b_2 = 11 cm (both larger Δs),
h = 12 ÷ 2 = 6 cm
This time we must add the areas of all four triangles together, but recall that there were two sets of congruent triangles formed when we decomposed the original figure in this manner. So…



We can find the area of one small triangle and double it, then find the area of one larger triangle and double it, and finally add those two doubled areas together.

$$A_{triangle} = \frac{1}{2}bh$$

$$A_{TOTAL} = (2)A_{smaller\,\Delta} + (2)A_{larger\,\Delta}$$

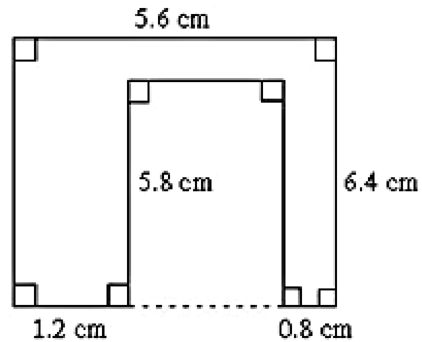
$$A_{TOTAL} = (2)\left(\frac{1}{2}\right)(4)(6) + (2)\left(\frac{1}{2}\right)(11)(6)$$

Taa daa…The total area is 90 cm^2. (b) AGAIN!!

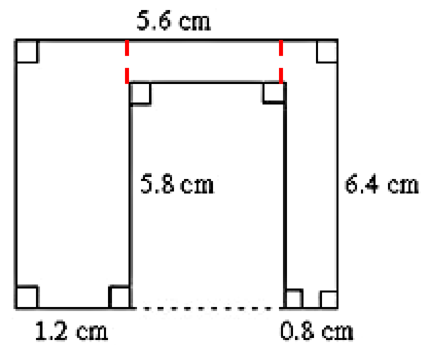
$$A_{TOTAL} = 24 + 66$$

$$A = 90$$

Example: *Find the area of the given polygon.*



The area of this figure can be found multiple ways. It could be decomposed into rectangles this way…



$A_{rectangle} = lw$

$A_{TOTAL} = A_{rectangle\ A} + A_{rectangle\ B} + A_{rectangle\ C}$

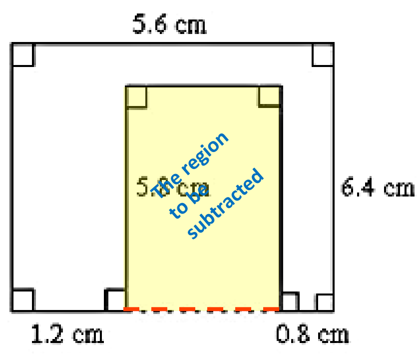
$A = (1.2)(6.4) + (3.6)(0.6) + (0.8)(6.4)$

$A = 7.68 + 2.16 + 5.12$

$A = 14.96$

The total area is 14.96 cm².

An alternative to this method and extension on this topic is to decompose and subtract. In this case, students can picture an imaginary rectangle surrounding the entire figure then subtract the region that is not a part of the original figure.



$A_{rectangle} = lw$

$A_{TOTAL} = A_{outer\ rectangle} - A_{inner\ rectangle}$

$A = (5.6)(6.4) - (3.6)(5.8)$

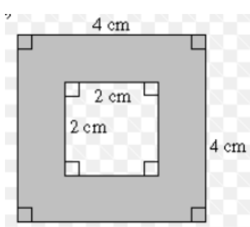
$A = 35.84 - 20.88$

$A = 14.96$

Again, the total area is 14.96 cm².

Extensions Examples: *Find the areas of the shaded regions in the figures below.*

a)



$A_{rectangle} = lw \rightarrow\rightarrow$ All squares are rectangles...

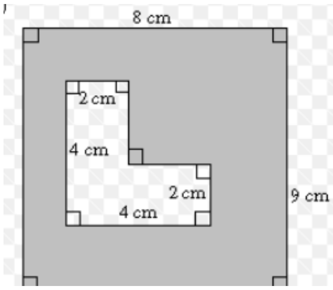
$A_{TOTAL} = A_{outer\ square} - A_{inner\ square}$

$A = (4)(4) - (2)(2)$

$A = 16 - 4$

$A = 12$

The area of the shaded region is 12 square cm.

b) 

$A_{shaded\ region} = A_{outer\ region} - A_{inner\ region}$

To find the area of the inner region, we must decompose it.

$A_{inner\ region} = A_{rectangle\ A} + A_{rectangle\ B}$

$A_{inner\ region} = (4)(2) + (2)(2)$

$A = 8 + 4$

$A = 12$

Now the shaded region can be found.

$A_{shaded\ region} = A_{outer\ region} - A_{inner\ region}$

$A = (8)(9) - 12$

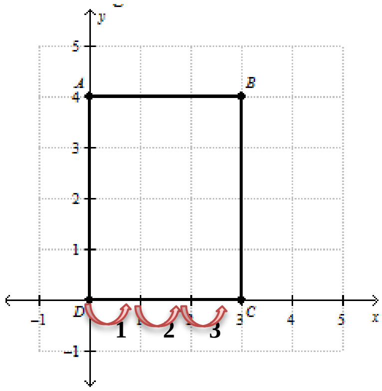
$A = 72 - 12$

$A = 60$

The area of the shaded region is 60 square cm.

6.G.A.3 ***Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.***

***Example**: Find the length of segment CD.* (DOK 1)

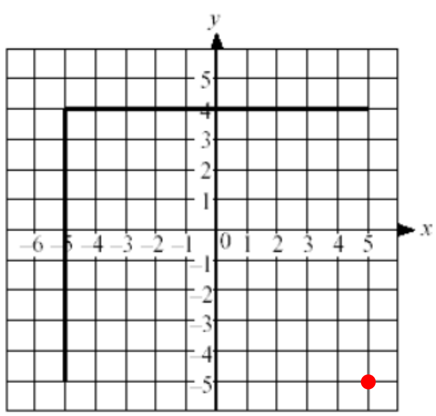


Simply count the units along the indicated side.
Common error…
Remind students to start at "0" and count each "swoop" as 1 unit.

CD = 3 units

***Example**: In each part below two sides of a rectangle are shown. Write the coordinates of the fourth corner of each rectangle. Then answer the questions.*

a)



The fourth corner is at (5, -5).

Can the perimeter and area of this rectangle be found?
If so, what are they?
If not, why not?

Yes, they are…

$P = 2l + 2w$

$P = 2(10) + 2(9)$

$P = 20 + 18$

$P = 38$

and

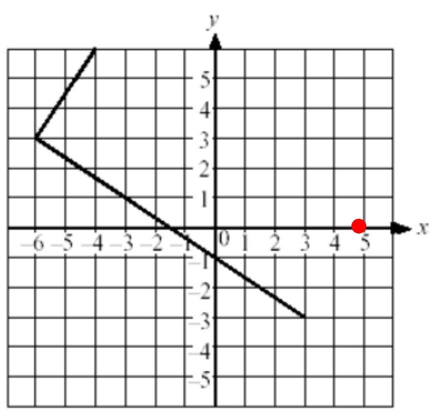
$A = bh$

$A = (10)(9)$

$A = 90$

The perimeter is 38 units.
The area is 90 units^2.

b)

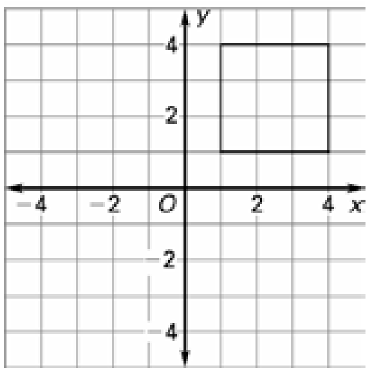


The fourth corner is at (5, 0).

Can the perimeter and area of this rectangle be found?
If so, what are they?
If not, why not?

NO, because the sides are not horizontal or vertical so the measures cannot be determined.

Example: *Which gives the perimeter of the square?* (DOK 2)



a) 4
b) 8
c) 10
d) 12

By counting the units we can see that *l = 3* & *w = 3*, therefore…

$P = 2l + 2w$

$P = 2(3) + 2(3)$

$P = 6 + 6$

$P = 12$

OR

Since the side lengths of a square are always equal, it may be faster for students to find one side length and multiply by 4.

$P_{square} = 4s$

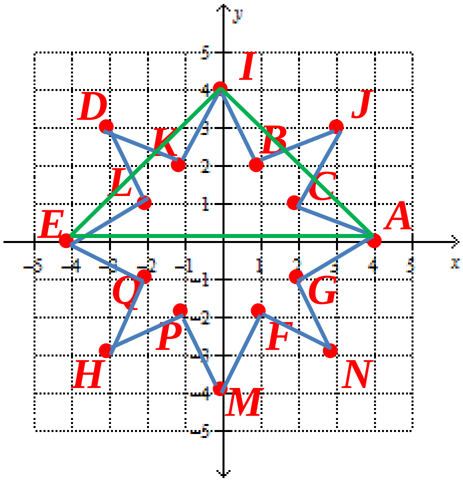
$P = 4(3)$

$P = 12$

Both methods yield the answer 12 units. (d)

Example: *Plot the sixteen points in the table below on this graph. After graphing the points, connect them to make a 16-pointed star.*

POINTS	***POINTS***	***POINTS***	***POINTS***
A(4, 0)	***E(-4, 0)***	***I(0, 4)***	***M(0, -4)***
B(1, 2)	***F(1, -2)***	***J(3, 3)***	***N(3, -3)***
C(2, 1)	***G(2, -1)***	***K(-1, 2)***	***P(-1, -2)***
D(-3, 3)	***H(-3, -3)***	***L(-2, 1)***	***Q(-2, -1)***



a) *Find all horizontal lengths:*

DJ = __**6**__ *units* — *KB* = __**2**__ *units* — *LC* = __**4**__ *units*
EA = __**8**__ *units* — *QG* = __**4**__ *units* — *PF* = __**2**__ *units*
HN = __**6**__ *units*

b) *Find all vertical lengths:*

DH = __**6**__ *units* — *LQ* = __**2**__ *units* — *KP* = __**4**__ *units*
IM = __**8**__ *units* — *BF* = __**4**__ *units* — *CG* = __**2**__ *units*
JN = __**6**__ *units*

c) *Find the area of the triangle found by connecting the vowels (ΔAEI)*

$$A_{triangle} = \frac{1}{2}bh$$

$$A = \left(\frac{1}{2}\right)(8)(4)$$

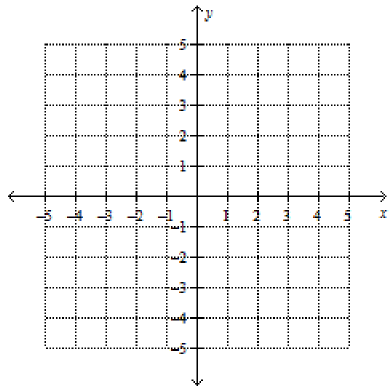
The area of ΔAEI is 16 square units or 16 units^2.

$$A = 16$$

d) ***Discussion:*** *I purposely skipped labeling any point with an "O", what point is usually labeled with an "O"? Give its name and coordinates.* **The origin, (0, 0)**

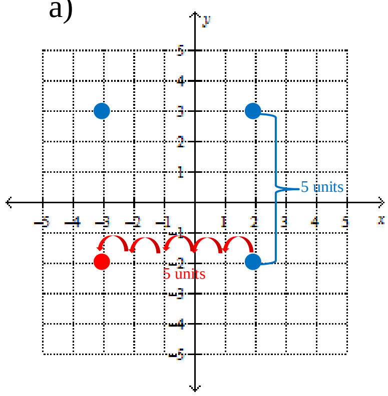
Example*: In each question below the coordinates of three corners of a square are given. Find the coordinates of the other corner in each case. You may find it helpful to draw a sketch.*

a) *(2, -2), (2, 3) and (-3, 3). The other corner of the square is at (___, ___).* **(-3, -2)**
b)
c) *(2, 3), (3, 4) and (1, 4). The other corner of this square is at (___, ___).* **(2, 5)**
d)
e) *(2, 2), (4, 4) and (4, 0). The other corner of this square is at (___, ___).* **(6, 2)**

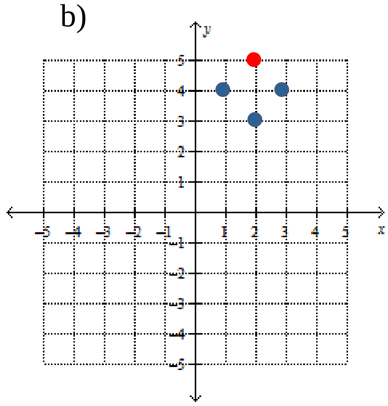


Answers:

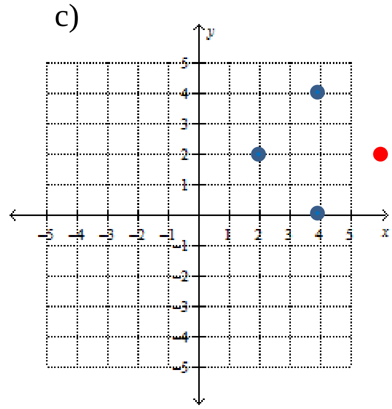
a)



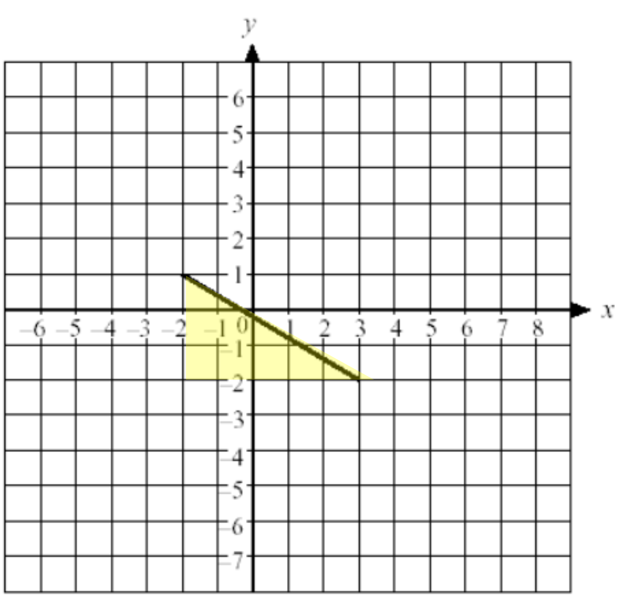
b)



c)



Example*: The line marked on the coordinate grid below is one side of a square:*
What are the possible coordinates of the corners of the square?



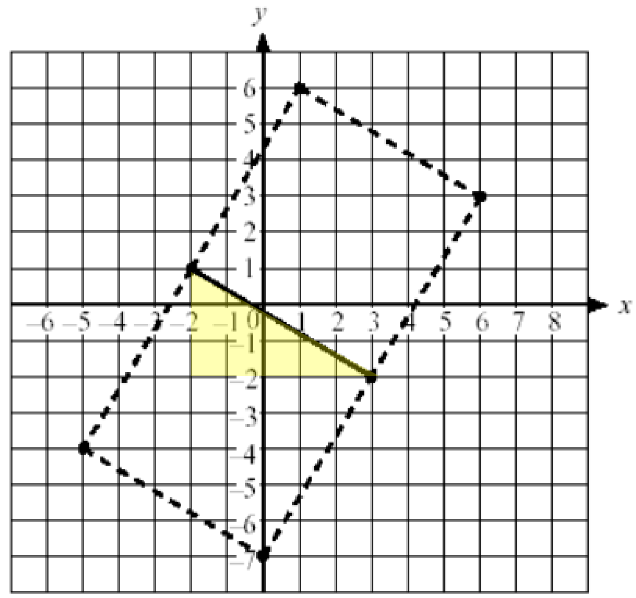
There are two possible places the square could be placed.

Students can visualize (or sketch) the imaginary right triangle with the given line as the hypotenuse.

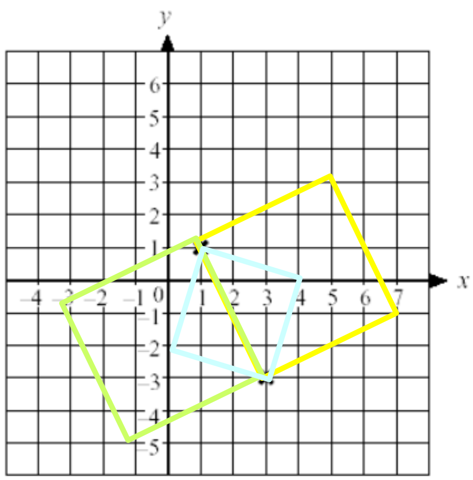
By using it as a reference, students can then find the location of the missing vertices of the square.

LINKAGE: slope, rate of change

The missing coordinates could be at (6, 3) and (1, 6) OR (-5, -4) and (0, -7).



Example*: Two corners of a square are shown in the coordinate plane below:*



a) If the third corner is at (7, -1), where is the fourth corner?

(5, 3)

b) If the third corner is at (-3, -1) where is the fourth corner?

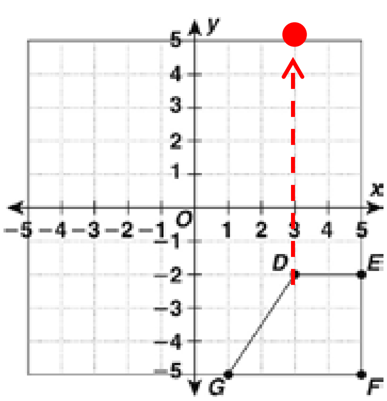
(-1, -5)

It is possible to make a different square to those above by placing the third and fourth points in two new positions.

c) What are the coordinates which need to be plotted?

By looking at the given points as opposite vertices, we can find the other corners at (4, 0) and (0, -2).

Example*: Identify the coordinates of vertex D after quadrilateral DEFG is translated 7 units up:*
(DOK 2)

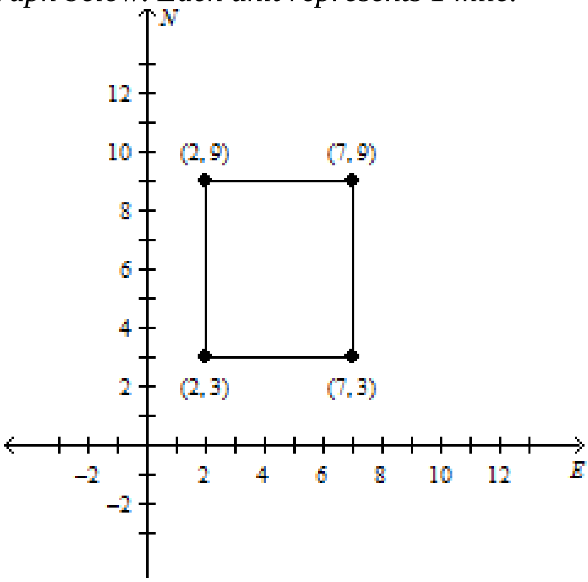


A translation up will change the y-coordinate only and move the figure up seven units. Therefore, *D(3, -2)* would move to

- *a)* *(3, -2)*
- *b)* *(3, 5)*
- *c)* *(5, 3)*
- *d)* *(-4, -2)*

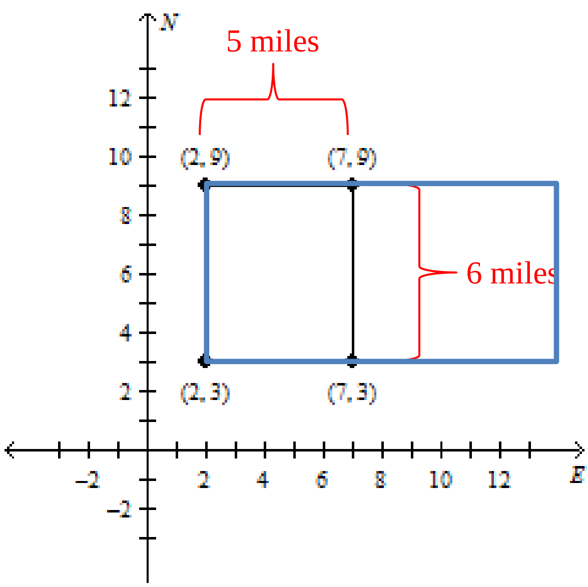
(3, 5). (b)

Example: *John rode his bike on Monday and again on Friday. His path for Monday is shown on the graph below. Each unit represents 1 mile.* *(DOK 3)*



On Friday, John rode 14 miles farther than he did on Monday. How could his path have changed while remaining rectangular?

a) John could have ridden 14 miles farther north.
b) John could have ridden 3½ miles farther north and 7 miles farther east.
c) John could have ridden 7 miles farther east.
d) John could have ridden 1¾ miles farther north and 3½ miles farther east.



Students will need to determine the perimeter of the rectangle graphed to answer this question.

$P = 2l + 2w$

$P = (2)(5) + (2)(6)$

$P = 10 + 12$

$P = 22$

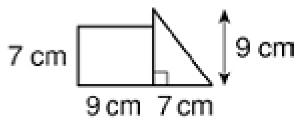
Then… adding 14 would result in a larger perimeter of 26.

Students must remember that each change in a dimension will result in twice as much change in the perimeter. Since the perimeter must increase by 14, the only choice that will result in that amount of change would be when John rides an additional 7 miles east. ***(d)***

Sample Questions from OnCore

Example:

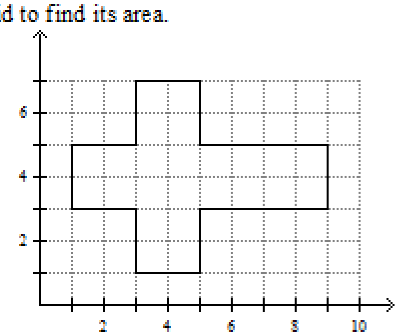
Find the area of the figure.



a. 112 cm^2
b. 94.5 cm^2
c. 126 cm^2
d. 144 cm^2

Example:

Donny needs to put carpet in the hallway of his house, and drew the following diagram. Use the coordinate grid to find its area.



a. 28 ft^2
b. 24 ft^2
c. 20 ft^2
d. 30 ft^2

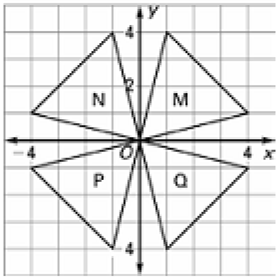
Example:

Tina wants to install a hot tub in her backyard. The backyard is 15 ft by 40 ft, and the hot tub will be 7 ft by 7 ft. What is the area of the backyard that will not be covered by the hot tub?

a. 82 ft^2
b. 551 ft^2
c. 41 ft^2
d. 438 ft^2

Example:

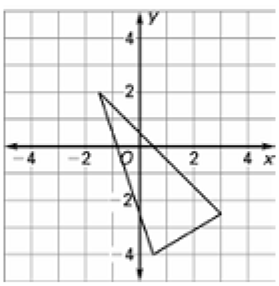
Which triangle has vertices $(0, 0)$, $(-1, 4)$, and $(-4, 1)$?

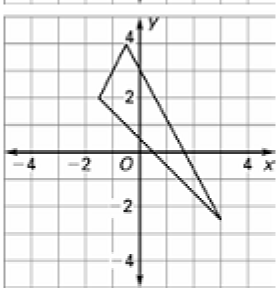


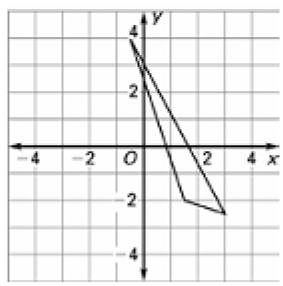
a. Triangle M
b. Triangle N
c. Triangle P
d. Triangle Q

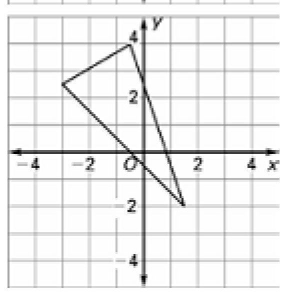
Example:

Which triangle has vertices $\left(-\frac{1}{2}, 4\right)$, $\left(3, -2\frac{1}{2}\right)$, and $\left(-1\frac{1}{2}, 2\right)$?

a.


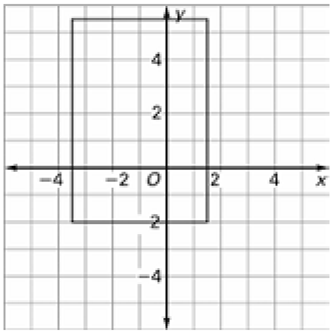
b.


c.


d.


Example:

Find the perimeter of the rectangle.



a. $17\frac{1}{2}$

b. 18

c. 21

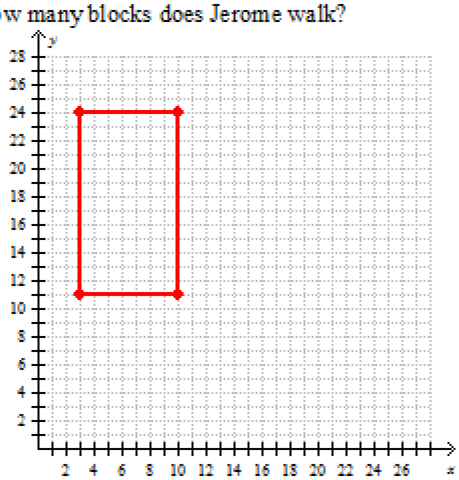
d. 25

Example:

Suppose that all blocks in a city are the same length. Streets run north and south. They are numbered consecutively, starting with First Street. Avenues run east and west. They are also numbered consecutively, starting with First Avenue.

Jerome lives at the corner of Third Street and Twenty-Fourth Avenue. He walks to the bank at Third Street and Eleventh Avenue, to the post office at Tenth Street and Eleventh Avenue, and then to the barbershop on Tenth Street and Twenty-Fourth Avenue. Then Jerome walks home.

How many blocks does Jerome walk?



a. 20 blocks

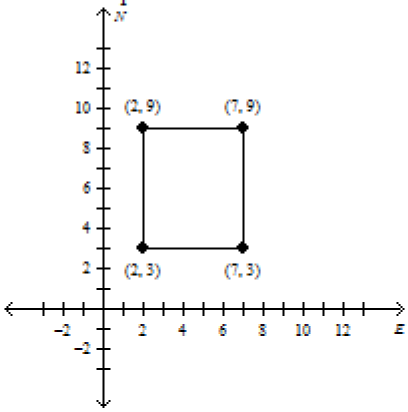
b. 40 blocks

c. 91 blocks

d. 96 blocks

Example:

Miguel rode his bike on Monday and again on Friday. His path for Monday is shown on the graph below. Each unit represents 1 mile.



On Friday, Miguel rode 14 miles farther than he did on Monday. How could his path have changed while remaining rectangular?

a. Miguel could have ridden 14 miles farther north.

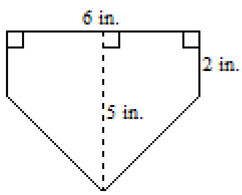
b. Miguel could have ridden $3\frac{1}{2}$ miles farther north and 7 miles farther east.

c. Miguel could have ridden 7 miles farther east.

d. Miguel could have ridden $1\frac{3}{4}$ miles farther north and $3\frac{1}{2}$ miles farther east.

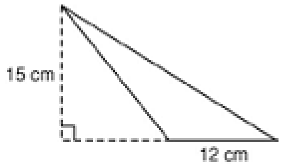
Example:

Gloria's grandmother gave her a quilt made up of pieces that have the following dimensions. Find the area of the quilt piece. Show your work.



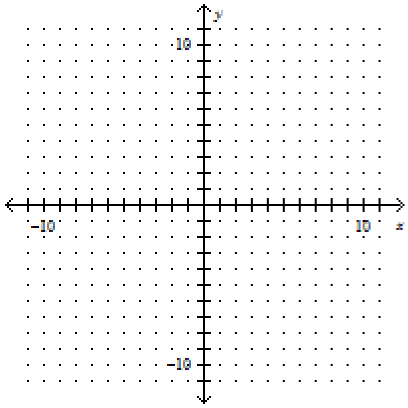
Example:

What is the area of the triangle?



Example:

Plot and connect the points to form a rectangle. Then find the length, width, and area of the rectangle.
$A(8, -3)$, $B(8, 2)$, $C(4, 2)$, $D(4, -3)$



Example:

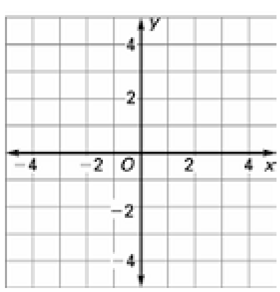
Jarek is having a pool installed in his backyard. The contractors have made plans for the installation by graphing his backyard on a coordinate plane. The pool, which will be rectangular, will have three of its corners at the points $K(-2, -2)$, $L(-2, 7)$, and $M(8, 7)$.

Part A: Explain how the contractor can find the coordinates of the fourth corner, N.
Part B: Draw a polygon representing the pool on the coordinate plane.

Example:

You are working as an assistant to a graphic artist. The artist has asked you to answer the following questions.

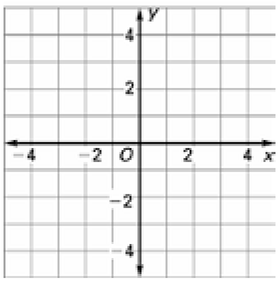
Graph a polygon with vertices $(-4, 1)$, $(-2, -1.5)$, $(2, -1.5)$, and $(4, 1)$. Then draw a rectangle with vertices $(-2, 1)$, $(-2, 2.5)$, $(2, 2.5)$, and $(2, 1)$ on the same coordinate grid. What does the following figure represent?



Example:

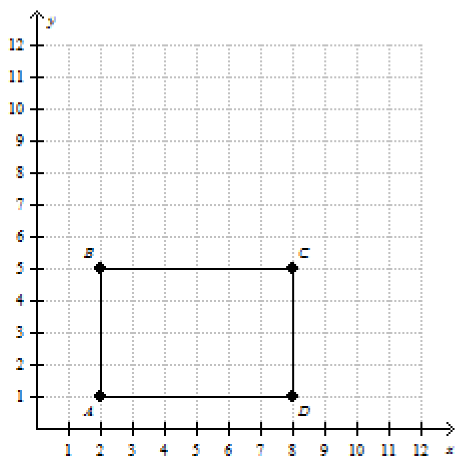
You are working as an assistant to a graphic artist. The artist has asked you to answer the following questions.

Graph a polygon with vertices $(-4, 1)$, $(-2, -1.5)$, $(2, -1.5)$, and $(4, 1)$. Then draw a rectangle with vertices $(-2, 1)$, $(-2, 2.5)$, $(2, 2.5)$, and $(2, 1)$ on the same coordinate grid. What does the following figure represent?



Example:

The figure below represents Hideki's rectangular garden. Each unit represents 1 foot. Hideki decides to increase the size of the garden by moving point B to point (2, 8.5) and point C to (8, 8.5).



Part A: Describe the change to the dimensions of the garden. Make a graph showing the new garden.
Part B: The garden is surrounded by a small picket fence. How much additional fencing must Hideke buy to accommodate the new garden? Show your work.

Example:

A square has an area of approximately 58.06 cm^2. Find the length of a side of the square in inches. Round your answer to the nearest tenth of an inch. (*Hint*: 2.54 cm ≈ 1 in.)

Example:

Graph the points $A(8,\ 2)$, $B(2,\ 2)$, $C(2,\ 6)$, $D(5,\ 9)$, $E(8,\ 6)$ on a coordinate grid and connect them to form a polygon. Then classify the polygon.

Example:

A quadrilateral has vertices $W(3, 7)$, $X(3, 1)$, $Y(9, 1)$ and $Z(9, 7)$.
Is the quadrilateral:

- a square,
- a rectangle that is not a square,
- a rhombus that is not a square,
- a parallelogram that is neither a rhombus nor a square, or,
- a quadrilateral that is not a square?

Justify your answer without using a graph..

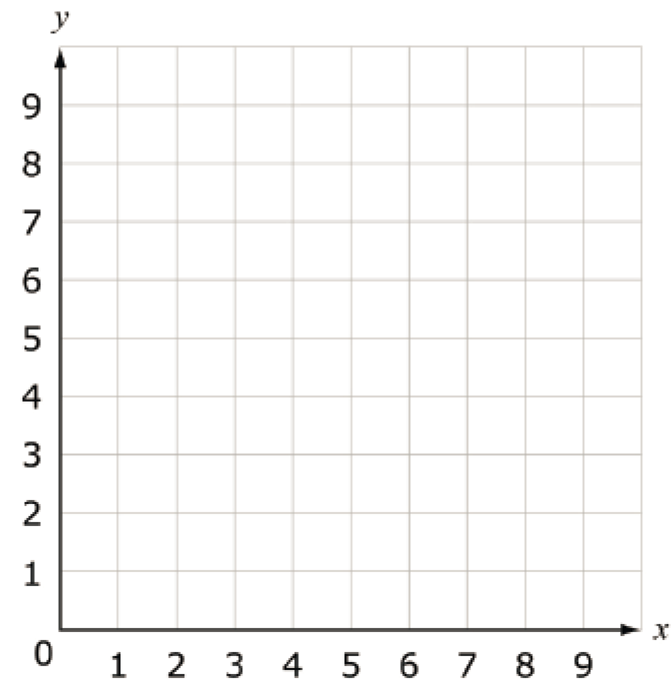
2013 SBAC Questions

Standard: 6.G.A.1, 6.G.A.3 ***DOK:2*** ***Difficulty: Medium*** ***Question Type: TE Technology Enhanced***

Part A

On the coordinate grid, plot the following points in order and connect each plotted point to the previous one in the order shown to form a figure.

1. Point A (2, 5)
2. Point B (2, 9)
3. Point C (5, 7)
4. Point D (8, 9)
5. Point E (8, 5)
6. Point A (2, 5)

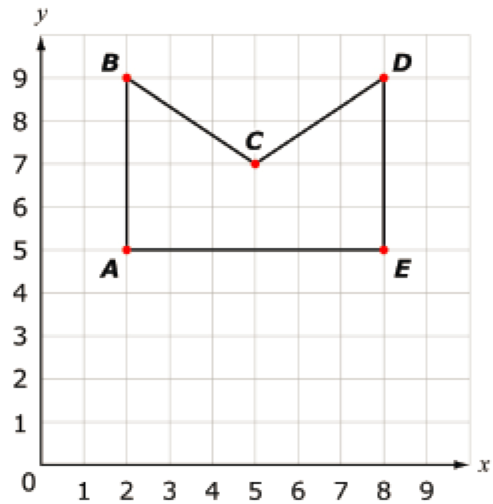


Part B

What is the area, in square units, of the enclosed figure?

[ ] square units

Sample Top-Score Response:



What is the area, in square units, of the enclosed figure?

18 square units

Suggested Scoring Rubric:

Each part is scored independently; worth 1 point each.

TE information:

Item Code: MAT.06.TE.1.0000G.H.071

Template: Vertex Based Polygons (does not exist at this time)

Interaction Space Parameters:

A. False

B. (0,0), (10,10),1, the axes are labeled *x* and *y*, no axis titles

C. True

D. False

E. N/A

F. Limit number of vertices to 5

Scoring Data (Specific to Each Item):

1. False
2. True
    a. (2, 5), tolerance = 0
    b. (2, 9), tolerance = 0
    c. (5, 7), tolerance = 0
    d. (8, 9), tolerance = 0
    e. (8, 5), tolerance = 0
3. False
4. False
5. False

2014 SBAC Questions

Grade 6 Mathematics Item Specification C1 TH

Task Model 1

Response Type: Equation/Numeric

DOK Level 1

6.G.1
Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Evidence Required:
1. The student determines the area of triangles, special quadrilaterals, and polygons using composition and decomposition in solving real-world and mathematical problems.

Tools: Calculator

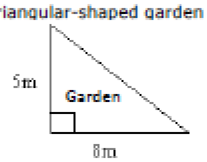
Prompt Features: The student is prompted to determine the areas of triangles in solving mathematical and real-world problems.

Stimulus Guidelines:

- If used, context should be familiar to students 11 to 13 years old.
- Rational numbers used should be appropriate for the situation.
- Item difficulty can be adjusted via these example methods:
    - Measurements of shapes can be whole numbers, fractions, or decimals.
    - Students find the area of right triangles.
    - Students find the area of non-right triangles such as isosceles triangle, equilateral triangle, or scalene triangle.

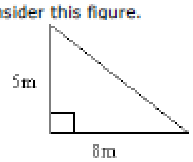
TM1a
Stimulus: The student is presented with a mathematical or real-world problem involving triangles.

Example Stem 1: A triangular-shaped garden is shown.



Enter the area of the garden, in square meters.

Example Stem 2: Consider this figure.



Enter the area of the right triangle in square meters.

Rubric: (1 point) Student enters the correct area of the figure (e.g., 20; 20). Correct answer should be a single numerical value and units should be assumed from the stem.

Response Type: Equation/Numeric

Grade 6 Mathematics Item Specification C1 TH

Task Model 1

Response Type: Equation/Numeric

DOK Level 2

6.G.1
Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Evidence Required:
1. The student determines the area of triangles, special quadrilaterals, and polygons using composition and decomposition in solving real-world and mathematical problems.

Tools: Calculator

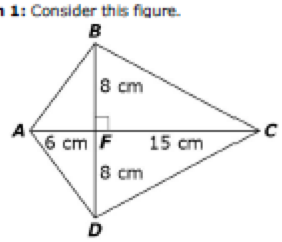
Prompt Features: The student is prompted to determine the areas of triangles, special quadrilaterals, and other polygons in solving mathematical and real-world problems.

Stimulus Guidelines:

- If used, context should be familiar to students 11 to 13 years old.
- Rational numbers used should be appropriate for the situation.
- Item difficulty can be adjusted via these example methods:
    - Students find area of non-right triangles/special quadrilaterals with whole-number measures.
    - Students find area of polygon that can be decomposed into quadrilaterals and triangles with whole number measures.
    - Students find area of triangles/special quadrilaterals with fraction/decimal measures.
    - Students find area of polygon that can be decomposed into quadrilaterals and triangles with fraction/decimal measures.

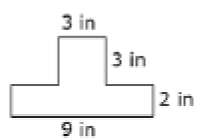
TM1b
Stimulus: The student is presented with a mathematical or real-world problem involving composition or decomposition of a triangle, special quadrilateral, or other polygon.

Example Stem 1: Consider this figure.



Enter the total area of kite *ABCD* in square centimeters.

Example Stem 2: A company is using this design for their shirts. The design is made by joining a square and a rectangle. This figure shows the design.



Enter the total area of the design in square inches.

Rubric: (1 point) Student enters the correct area of the figure (e.g., 168; 27). Correct answer should be a single numerical value and units should be assumed from the stem.

Response Type: Equation/Numeric

Grade 6 Mathematics Item Specification C1 TH

Task Model 3

Response Type:
Graphing

DOK Level 1

6.G.3
Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Evidence Required:
3. The student draws polygons in the coordinate plane, given coordinates for the vertices in the context of solving real-world and mathematical problems.

Tools: Calculator

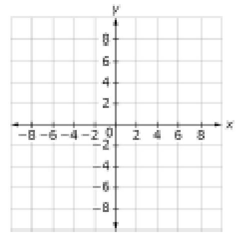
Prompt Features: The student is prompted to draw polygons in the coordinate plane given coordinates for the vertices.

Stimulus Guidelines:
- If used, context should be familiar to students 11 to 13 years old.
- Polygons should be limited to triangles, squares, rectangles, parallelograms, kites, rhombi, and trapezoids.
- Coordinates of the ordered pairs should be integers.
- Item difficulty can be adjusted via these example methods:
  - Students graph polygon in Quadrant I with one-unit increment axes.
  - Students graph polygon in all four quadrants with one-unit increment axes.
  - Students graph polygon in all four quadrants with varying integer increment axes.

TM3
Stimulus: The student is presented with the vertices of a polygon in the context of a real-world or mathematical problem.

Example Stem: Consider these ordered pairs.

Point A: (3, 2)
Point B: (−3, 2)
Point C: (3, −2)

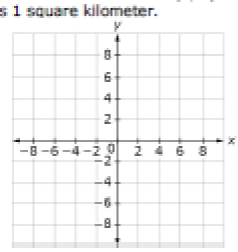


Use the Add Point and Connect Line tools to connect the three points to form triangle ABC.

Interaction: The student is given the Connect Line, Add Point, and Delete tools to draw the polygon in the coordinate plane.

Rubric: (1 point) Student plots all given points and connects the lines correctly.

Response Type: Graphing

Task Model 4
 	
Response Type: Equation/Numeric
 	
DOK Level 2
 	
6.G.3
Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
 	
Evidence Required:
4. The student determines the length of a side of a polygon in the coordinate plane, given coordinates for the vertices in the context of solving real-world and mathematical problems.
 	
Tools: Calculator
	Prompt Features: The student is prompted to determine the length of a side of a polygon in the coordinate plane given coordinates for the vertices that have the same first coordinate or the same second coordinate.
	
	Stimulus Guidelines:
	• If used, context should be familiar to students 11 to 13 years old.
	• Polygons should be limited to triangles, squares, rectangles, parallelograms, kites, rhombi, and trapezoids.
	• Coordinates of the ordered pairs should be integers.
	• Item difficulty can be adjusted via these example methods:
	  ○ Coordinates of the side used are in the same quadrant.
	  ○ Coordinates of the side used are in different quadrants.
	TM4
	Stimulus: The student is presented with coordinates for the side of a polygon in the coordinate plane with either the same first coordinate or the same second coordinate in the context of a mathematical or real-world problem.
	
	Example Stem 1: A triangle has these coordinates:
	
	Point A: (−5, 2)
	Point B: (−5, 6)
	Point C: (7, 2)
	
	Enter the length of side AC.
	
	Example Stem 2: Refer to the map as a coordinate grid. On the map, the library is located at (−5, 2), the bus station is located at (−5, 6), and the courthouse is located at (7, 2). Each square unit in the grid represents 1 square kilometer.
	
	
	Enter the distance from the courthouse to the library in kilometers.
	
	Rubric: (1 point) Student enters the correct length (e.g., 12; 12). Correct answer should be a single numerical value and units should be assumed from the stem.
	
	Response Type: Equation/Numeric