Math 6 Notes: Ratios and Proportional Relationships

RATIO & PROPORTION

6.RP.A.1 Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities.

Beginning middle school students typically can reason with one variable (called univariate reasoning), but working with two quantities (bivariate reasoning) requires some attention/exposure/experience. For example, given a series of numbers or geometric shapes, students can examine the pattern and identify the next (number/figure in the series). Working with two quantities, as we do in ratios, creates a new challenge for students.

For example, students were shown a container of orange juice and were told it was made from orange concentrate and water. Two glasses – one large glass and one small glass – were filled with the orange juice from the container. The students were then asked if they thought the orange juice from the two glasses would taste equally orangey, or if they thought that the juice in one glass would taste more orangey than the juice in the other.

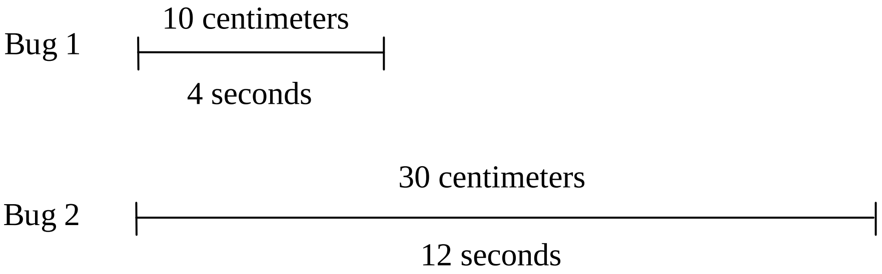
As adults we see this kind of situation so simply, we don't even recognize its importance until we hear the students thinking. Student responses were interesting – nearly half the class responded incorrectly. Approximately half of these student said that the juice in the large glass would taste more orangey, and the other half chose the smaller glass as more orangey. Their explanations suggest they focused on one quantity – the water or the orange concentrate – or they did not coordinate both quantities. Some students explained their thinking that "the larger glass is bigger, so it would hold more orange concentrate". Others explained that the juice in the small glass would taste more orangey because the smaller volume would allow less water to get in, which would leave more room for the orange concentrate.

The goal is to get students to understand that since the ratio of water to orange concentrate is the same within that container, the two glasses would taste equally orangey.

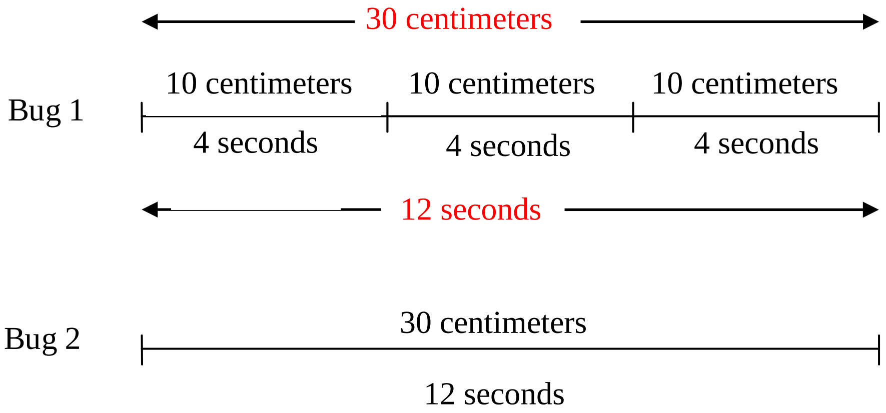
What happens when we give a situation such as: Bug 1 walks at the rate of 10 centimeters in 4 seconds. Bug 2 walks 30 centimeters in 12 seconds. Which bug is faster?

As adults and as mathematics teachers we jump either right into setting up ratios and then a proportion and we solve it or we mentally reason our way through the problem. With student learners we need to scaffold this thought process so our students truly understand how to work with ratios, proportions and rates.

It would help students to begin with a visual representation something like this:

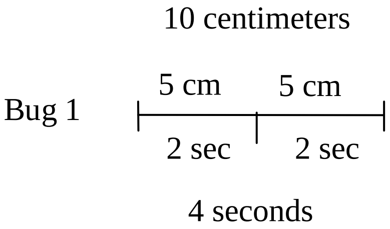


Some students will repeat (iterate) the composed unit until they find a match (or not). Below they will see that it is like Bug 1 walking the distance three times.

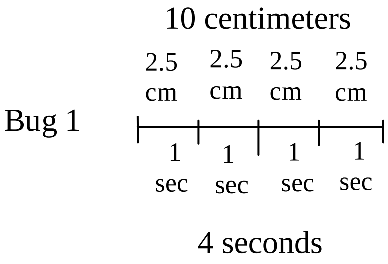


Once students see this visually they will realize that both bugs can walk 30 centimeters in 12 seconds so they are traveling at the same rate. Ask students to create other "same speed" values.

Hopefully they will see in the graph above that 20 centimeters in 8 seconds is the same. They may continue repeating this "joining" or they may begin to "partition" (break apart into equal sized sections).



Here we see another same rate value of 5 cm in 2 seconds.

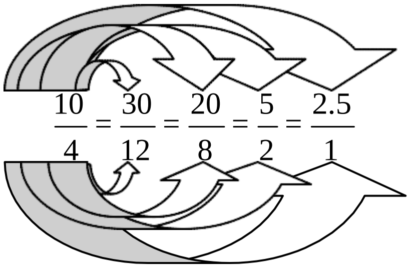


Once again, we see another same rate value of 2.5 cm in 1 second.
These ***equivalent ratios*** arise by multiplying each measurement in a ratio pair by the same positive number. Such pairs are said to be in the same ratio.
This can be described as:
2.5 cm *for each 1* second
2.5 cm *for each* second
2.5 cm *per* second
2.5 cm *for every* second

For this reason we must get students to attend to and coordinate <u>two</u> quantities.

Referring back to Bugs 1 and 2, we can put this in numerical form.

10:4 ⇨ 30:12 ⇨ 20:8 ⇨ 5:2 ⇨ 2.5:1 or

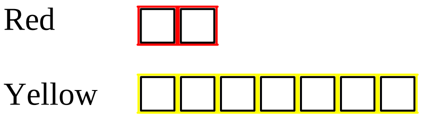


$$\frac{10}{4}=\frac{30}{12}=\frac{20}{8}=\frac{5}{2}=\frac{2.5}{1}$$

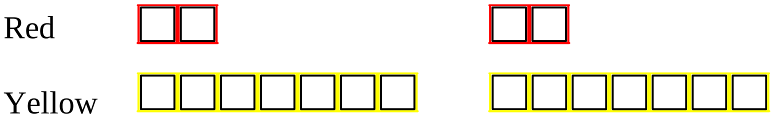
Point out to students that we began by tripling the original distance and tripling the time. Then we doubled the original distance and doubled the original time. Next we cut the original distance in half and the original time in half. Finally, we cut the original distance in fourths and the original time in fourths.

Remind students they must attend to <u>both</u> quantities equally.

Example: Suppose that you have a batch of orange paint by mixing 2 cans of red paint with 7 cans of yellow paint. What are some other combinations of numbers of cans of red paint and yellow paint that you can mix to make the same shade of orange? Solve the problem in two different ways – first by using a multiplicative comparison and then by using a composed unit.

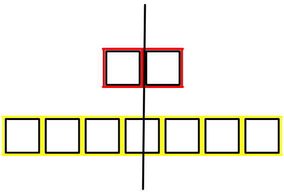


<u>Sample solution</u>
Doubling would yield:



$$\frac{2\,red}{7\ yellow}=\frac{4\,red}{14\ yellow}$$

Partitioning would yield:



$$\frac{2\,red}{7\,yellow} = \frac{1\,red}{3\frac{1}{2}\,yellow}$$

A **RATIO** in our textbooks is commonly defined as a comparison between two quantities. We use ratios every day; one Pepsi which costs 50 cents describes a ratio. On a map, the legend might tell us one inch is equivalent to 50 miles or we might notice one hand has five fingers. In our classrooms we are concerned with the student/teacher ratio and the ratio of boys to girls in a particular class. Other examples could include, the number of red M & M's to green M & M's in a bag of M&M candies, ratios found in recipes, etc. Those are all examples of comparisons – ratios. Students should have some background knowledge here, so this is a good place to begin.

3 Ways to Write a Ratio

A ratio can be written three different ways. For example, to show the comparison of one inch representing 50 miles on a map, the following notations can be used:

Method 1: 1 to 50 (Use "to" to separate the numbers being compared)

Method 2: 1:50 (Use a ":" to separate the numbers being compared)

Method 3: $\frac{1}{50}$ (Use a fraction bar to separate the numbers being compared)

(Most maps use Method 2 for the legend.)

Because ratios will be used to solve problems, it is easier to write the ratios using fractional notation (Method 3). Looking at the ratio of one inch representing 50 miles ($\frac{1}{50}$), 2 inches represents 100 miles ($\frac{2}{100}$), and 3 inches represents 150 miles ($\frac{3}{150}$) by repeating (iterating). These ratios can be related to equivalent fractions. These **equivalent ratios** arise by multiplying each measurement in a ratio pair by the same positive number. Such pairs are said to be in the same ratio.

With NVACS we need to dig deeper into the understanding of ratio, proportion and proportional reasoning so a clearer definition would be – **a ratio** *is a multiplicative comparision of two quantities, or it is a joining or composing two quantities in a way that preserves a multiplicative relationship.*

It is imperative to have students label the quantities, keep track of them and describe what the ratio means in context of that situation. For example:

The ratio of wings to beaks is 2:1, because for every 2 wings there was 1 beak.
For every vote Candidate A received, candidate C received 3 votes.
The ratio of 3:2 in this salad dressing means 3 parts vinegar to 2 parts oil.

Example: *The TransAlaska pipeline system is 800 miles long and cost $8 billion to build. Divide one number by the other. What is the meaning of the result?*

Solutions may include: $\frac{800\,miles}{\$8\,billion} = 100\,miles\ per\ 1\,billion\,dollars$ **OR**

$$\frac{\$8,000,000,000}{800\,miles} = \$10,000,000\ for\ each\ mile$$ **OR**

$$\frac{800\,miles}{\$8,000,000,000} = \frac{1}{10,000,000}\,of\ a\,mile\ per\,dollar$$

Example: *At times Neptune is about 2,700,000,000 miles from the Earth and Mercury is about 135,000,000 miles from Earth. Write a ratio to represent the distances and explain the meaning.*

Solutions may include: $\frac{\text{Neptune}}{Mercury} = \frac{2,700,000,000\,miles}{135,000,000\,miles} = \frac{20}{1}$

Neptune is 20 times farther from Earth than Mercury(is from Earth)

$$\frac{Mercury}{\text{Neptune}} = \frac{135,000,000\,miles}{2,700,000,000\,miles} = \frac{1}{20}$$

Mercury is 1/20 the distance from Earth than Neptune (is from Earth)

Example: *According to treasurydirect.gov, the total National Debt on October 1, 2011 was $14,707,406,820,591.87. The population in the US was 311, 800,000. Approximate the numbers and write a ratio. Explain the meaning of your ratio.*

Solutions may include: $\frac{\$14,707,406,820,591.87}{311,800,000\ people} \approx \frac{\$15,000,000,000,000}{300,000,000\,people} = \frac{\$50,000}{1}$

The national debt is approximately $50,000 per person

Example: *You are making a fruit punch for an afternoon meeting. The punch recipe makes 5 cups of punch by mixing 3 cups of cranberry juice with 2 cups of orange juice. Will the punch taste more like cranberry juice or orange juice? Explain.*

Example: *If the ratio of orchestra seats to balcony seats in a theater is 3:5, does this theater have more orchestra seats or balcony seats? Explain.*

6.RP.A.2 Understand the concept of a unit rate a/b associated with a ratio a:b with $b \neq 0$ ***and use rate language in the context of a ratio relationship. (Expectations for unit rates in this grade are limited to non-complex fractions.)***

Rate in the NVACS refers to a ratio that compares two quantities measured in the same units or different units. For example, cups to cups or meters to seconds.

Example: *Mike travels 300 miles in 5 hours, find Mike's rate.*

$$\text{Mike's } rate = \frac{300 \text{ miles}}{5 \text{ hours}}$$

Unit rate is a rate whose denominator is 1. To convert a rate to a unit rate, divide both the numerator and denominator by the denominator. (Remember to demonstrate and allow models for students who need them.) The problems below **show typical examples** of what we have been doing.

Example: *Find the unit rate of Mike's travel above.*

$$\frac{300 \text{ miles} \div 5}{5 \text{ hours} \div 5} = \frac{60 \text{ miles}}{1 \text{ hour}}; \quad \text{read 60 miles for each hour}$$

Example: *Bill's heart beats 520 times every four minutes, find Bill's heartbeat per minute.*

$$\frac{520 \text{ beats} \div 4}{4 \text{ minute} \div 4} = \frac{130 \text{ beats}}{1 \text{ minute}}; \quad \text{read 130 beats per 1 minute}$$

Example: *Find the rate of pay if you earn \$52 for 8 hours of work.*

$$\frac{\$52.00 \div 8}{8\,hours \div 8} = \frac{\$6.50}{1\,hour}; \quad \text{read \$6.50 per hour or \$6.50 for each hour}$$

Example: *An area of 3 acres measure 14,520 square yards. . How many square yards are there in one acre?*

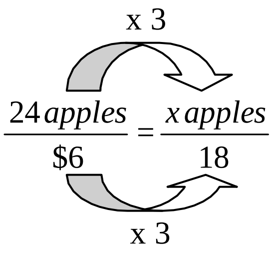
$$\frac{14,520\,sq.\,yd.}{3\,acres} = \frac{4,840}{1}; \qquad \text{4,840 sq yd for every acre}$$

Again, NVACS takes this skill (of finding unit rates) to a new level as demonstrated below.

Expectations for this grade level <u>exclude</u> complex fractions. Students should be able to demonstrate a variety of "modeling" techniques such as the use of the tape diagrams and double number line diagrams shown here, <u>in addition to procedural techniques</u>.

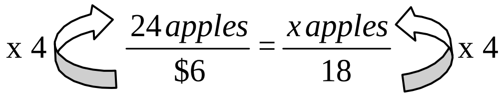
Another method that should be shown and developed is the factor of change method or scalar method.

<u>Between</u>



$$\frac{24\,apples}{\$6} = \frac{x\,apples}{18}$$

<u>Within</u>

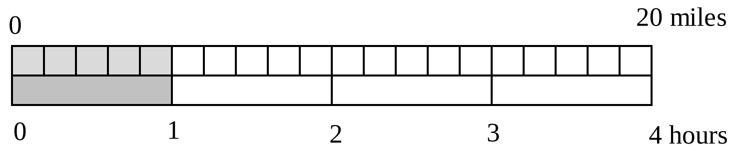


$$\frac{24\,apples}{\$6} = \frac{x\,apples}{18}$$

Example: *On a bicycle you can travel 20 miles in 4 hours. What are the <u>unit rates</u> in this situation (the distance you can travel in 1 hour <u>and</u> the amount of time required to travel 1 mile)?*

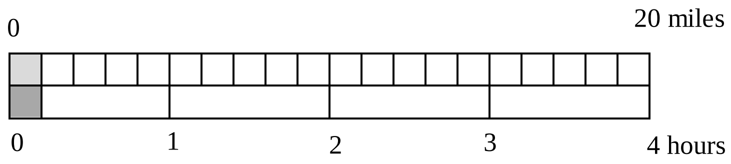
Using a model we could show….

Solution 1: *(the distance you can travel in 1 hour)*



5 miles in 1 hour or 5 mph

Solution 2: (*the amount of time required to travel 1 mile)*



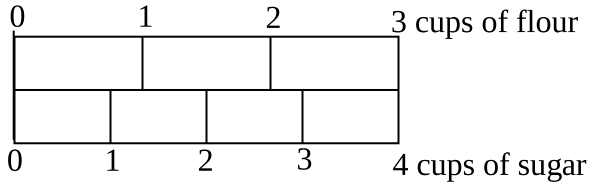
1/5 hour per mile

The first example above using a **tape diagram**, is a relatively simple one. The graphic is easily read. Below the use of the tape diagram graphic becomes more difficult to read so it was solved both graphically and procedurally.

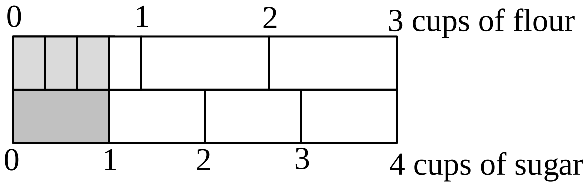
Note: When you are asked for both unit rates within a problem the unit rates will always be reciprocal of each other.

***Example**: A recipe has a ratio of 3 cups of flour to 4 cups of sugar. Find the per unit rate in terms of <u>each</u> ingredient.*

Begin with



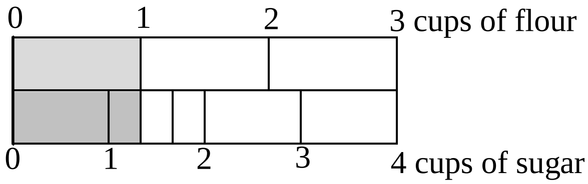
<u>Solution 1</u>:



$$\frac{3\,cups\,of\;\;flour}{4\,cups\,of\;\;sugar} = \frac{3 \div 4}{4 \div 4} = \frac{\frac{3}{4}}{1}$$

¾ cup of flour to each cup of sugar

<u>Solution 2</u>:

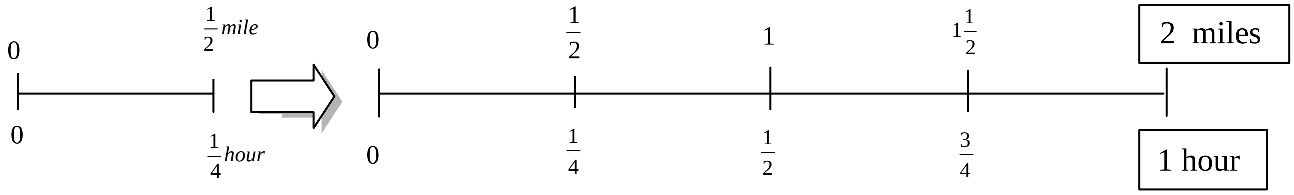


$$\frac{4\,cups\,of\;\;sugar}{3\,cups\,of\;\;flour} = \frac{4 \div 3}{3 \div 3} = \frac{\frac{4}{3}}{1}$$

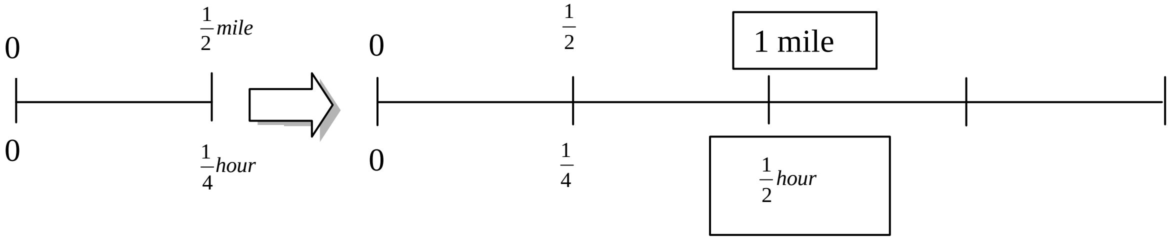
4/3 cups of sugar for each cup of flour
or
$1\frac{1}{3}$ **cups of sugar for each cup of flour**

Example: *If a person walks ½ mile in each ¼ hour, compute the unit rates.*

Students should use a ***double number line diagram*** *since they are* ***NOT*** *expected to work with complex fractions.*



Solution 1: **2 miles per hour**



Solution 2: **$\frac{1}{2}$ hour per mile**

Notice also in the double number line diagram that the reciprocal unit rate is shown. For each ½ hour the distance walked is 1 mile or ½ hour per mile.

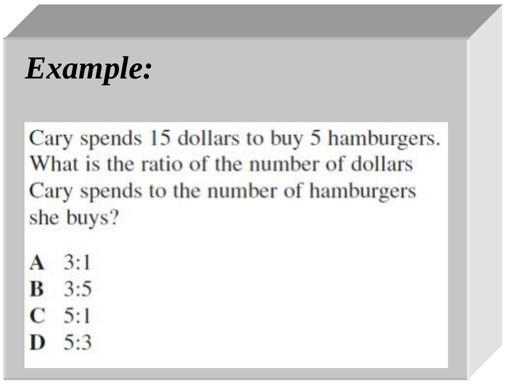
Another option for solving this problem could be the use of a table as shown below.

Distance (mi)	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$
Time (h)	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$

***Example**: Sandra tiles $\frac{5}{4}$ square yards in $\frac{1}{3}$ hour. Find the per unit rate.*

***Example**: If a person walks $\frac{1}{3}$ mile in each $\frac{1}{6}$ hour, compute both the unit rates.*

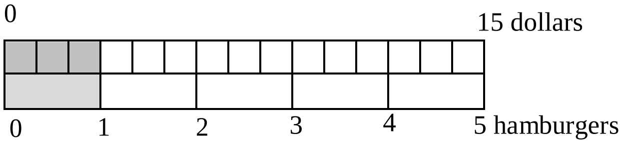
Example: *Peter mows* $\frac{1}{6}$ *acre in* $\frac{1}{4}$ *hour. How many acres does he mow in one hour?*



Example:

Cary spends 15 dollars to buy 5 hamburgers. What is the ratio of the number of dollars Cary spends to the number of hamburgers she buys?

A 3:1
B 3:5
C 5:1
D 5:3



$3.00 per hamburger 3:1

Simplifying Ratios

Because fractional notations are commonly used to describe ratios, it makes it easy to simplify ratios. To simplify a ratio, write it in fractional notation and reduce as you would a normal fraction.

Example: 3 inches represents 150 miles. Simplify this ratio.

Using method 3, the ratio can be written as a fraction: $\frac{3}{150}$. Reduce this fraction to $\frac{1}{50}$. The simplified ratios show that 1 inch represents 50 miles.

6.RP.A.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number lines or equations.

6.RP.A.3a Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

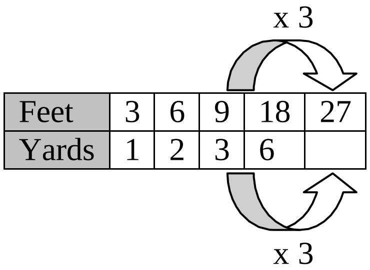
Example: *A lemon-lime punch calls for mixing 3 cups lemon with 2 cups lime. Complete the table below with equivalent ratios.*

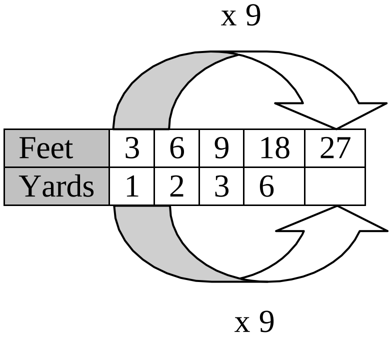
Lemon juice	*3*				
Lime juice	*2*				

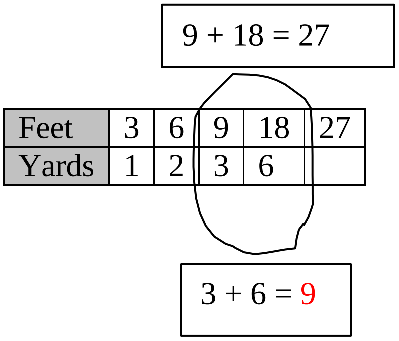
Example: *Using the information in the table, find the number of yards in 27 feet.*

Feet	3	6	9	18	27
Yards	1	2	3	6	

Solution: A variety of methods may be used, shown here are just a few.







9 yards in 27 feet

Example: *Carrie is packing apples for an orchard's mail order business. It takes 3 boxes to pack 2 bushels of apples. How many boxes will she need to pack 8 bushels of apples?*

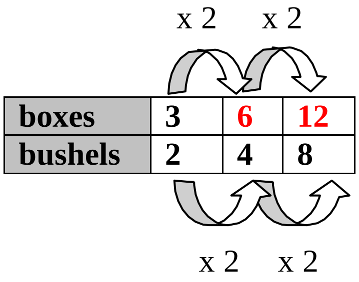
Solution methods may vary:

3 boxes for 2 bushels
x 4 x 4

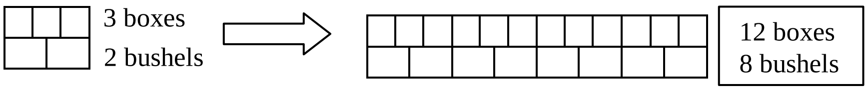
12 boxes for 8 bushels of apples

3 boxes = 2 bushels
6 boxes = 4 bushels

9 boxes = 6 bushels
12 boxes = 8 bushels



boxes	**3**	**6**	**12**
bushels	**2**	**4**	**8**

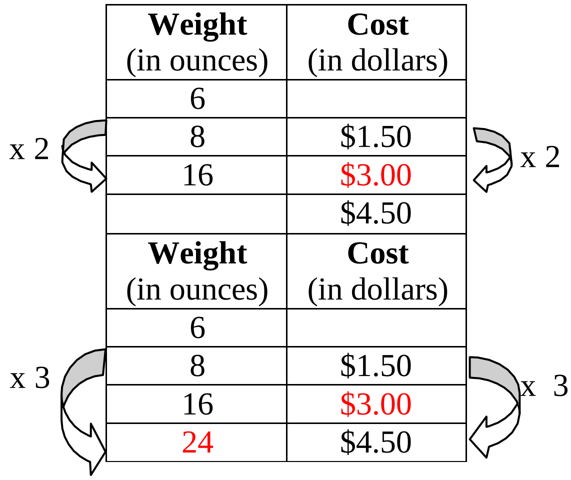


12 boxes for 8 bushels of apples

Example: *The Health Food Store sells granola by the ounce. The cost depends on the weight of the granola. Granola that weighs 8 ounces costs $1.50. Fill in the table with the appropriate cost or weight. Explain your reasoning.*

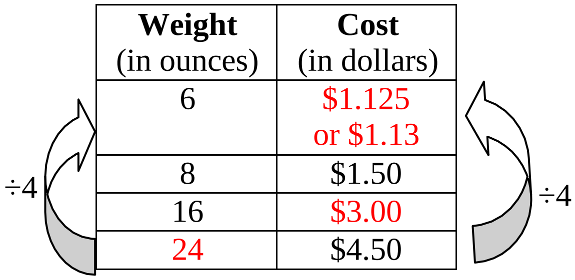
Weight (in ounces)	**Cost** (in dollars)
6	
8	$1.50
16	
	$4.50

Solution method may be:



Weight (in ounces)	**Cost** (in dollars)
6	
8	$1.50
16	$3.00
	$4.50

Weight (in ounces)	**Cost** (in dollars)
6	
8	$1.50
16	$3.00
24	$4.50



Weight (in ounces)	**Cost** (in dollars)
6	$1.125 or $1.13
8	$1.50
16	$3.00
24	$4.50

Example: *Paul's dog eats 20 pounds of food in 30 days. How long will it take Paul's dog to eat a 45 pound bag of dog food? Explain your thinking.*

Solution method may be:

20 pounds = 30 days

20 lbs. = 30 days
40 lbs. = 60 days
need 5 more pounds so 20 lbs.=30 days
10 lbs.=15 days
5 lbs.=7.5days

40 lbs 60 days
+ 5 lbs + 7.5 days
45 lbs 67.5 days

It will be
67.5 days

Example: *If 3 loaves of bread make 20 sandwiches, how many loaves of bread are needed to make 300 sandwiches?*

Solutions may vary:

x 2 x 2 x 2 x 2 Subtract initial ratio

Loaves of bread	*3*	*6*	*12*	*24*	*48*	*-3*	*45*
Sandwiches	*20*	*40*	*80*	*160*	*320*	*-20*	*300*

x 2 x 2 x 2 x 2 Subtract initial ratio

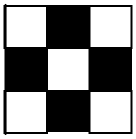
x 10 ÷2 x 3

Loaves of bread	**3**	**30**	**15**	**45**	
Sandwiches	**20**	**200**	**100**	**300**	

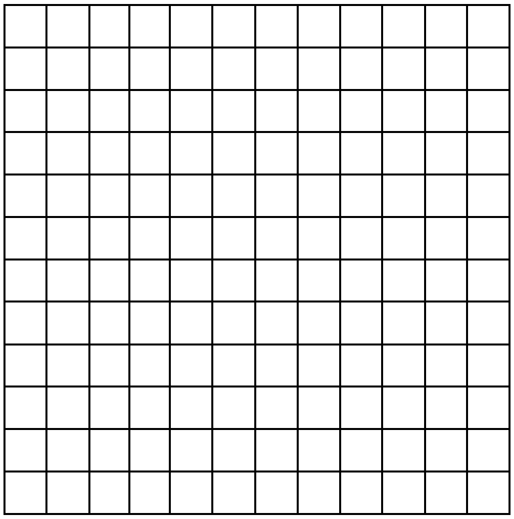
x 10 ÷2 x 3

45 loaves are needed to make 300 sandwiches

Example: *This grid has a black-to-white ratio of 4 to 5.*



Using the same ratio and pattern, complete the larger grid by coloring in the appropriate tiles. Then complete the table of equivalent ratios.



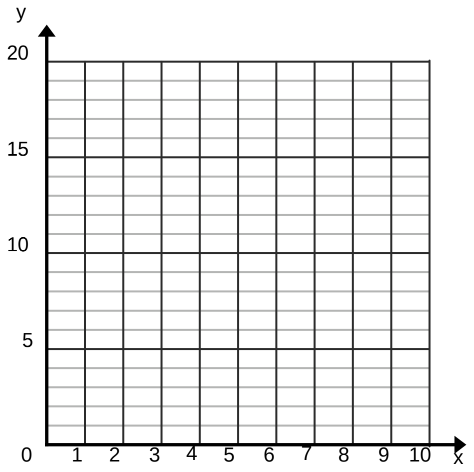
Black	*4*	*8*	*12*	*16*	*20*	*24*	*28*	*32*	*36*	*40*	*44*	*48*
White												

Additional Example: *Big Horn Ranch has 150 acres of pasture and raises 100 horses. Jefferson Ranch has 125 acres of pasture and raises 75 horses. Which ranch has more acres of pasture per horse? Explain your answer using words, pictures or diagrams.*

6.RP.A.3a *Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in tables, and* ***plot the pairs of values on the coordinate plane.*** *Use tables to compare ratios.*

Example: *Newly manufactured showerheads use 2 gallons of water per minute. Complete the table below and graph the values on the graph provided.*

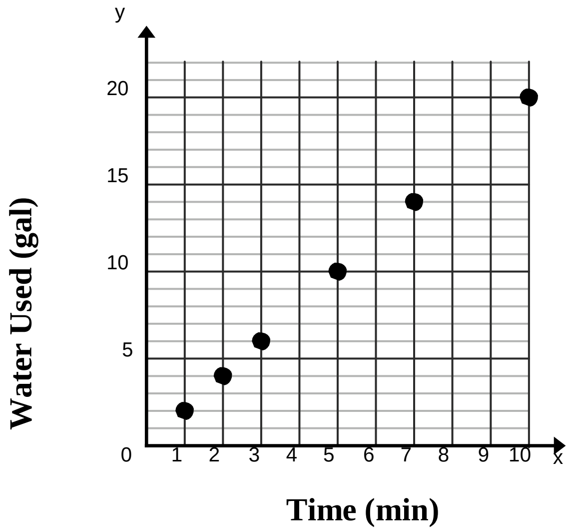
Time (min)	***Water Used (gal)***
1	
2	
3	
5	
7	
10	



Solution:

Time (min)	***Water Used (gal)***
1	***2***
2	***4***
3	***6***
5	***10***
7	***14***
10	***20***

Showerhead Water Usage



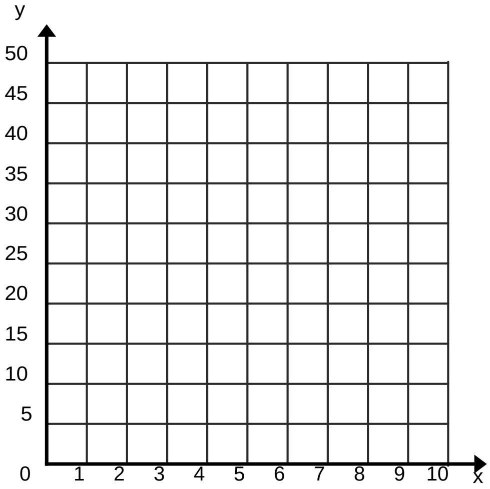
Follow Up Questions:

- If you connected the points on the graph, would it form a straight line?

- If a line was drawn through the points on the graph above, would you expect that it would go through the origin?
- Using the graph, could you find how much water was used in 8 minutes?
- Using the graph, how much water do you think would be used in 12 minutes? $1\frac{1}{2}$ minutes?
- If you used 12 gallons of water, how long was your shower?

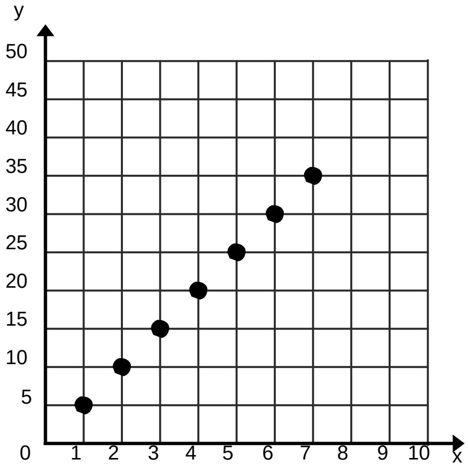
Example: *Sherrie bought 2* pounds *of almonds and paid $10. She is curious to see different poundage and the cost represented in a table. Create a table to show the different pounds that could be purchased and the cost for each. Then graph the values on the graph provided.*

# of pounds	Total Cost (in dollars)
1	
2	10
3	
4	
5	
6	
7	



Solution:

# of pounds	Total Cost (in dollars)
1	5
2	10
3	15
4	20
5	25
6	30
7	35



Be sure to have students compare **numerous** graphs of proportional relationships such as this. Conversations should include the following facts:

- the points create a straight line
- the origin would be included in that line
- the line is going uphill steadily
- additional points could be identified by extending the line
- the values of those additional points would also be solutions to the problem

6.RP.A.3a ***Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in tables, and plot the pairs of values on the coordinate plane.*** ***Use tables to compare ratios.***

Example: *Karl is at the grocery store comparing two brands of shampoo. Lots of Suds costs $4.50 for a 25-ounce bottle. Clean and Shine costs $3.80 for a 20-ounce bottle. Use the tables below to determine which is a better buy. Why?*

Clean and Shine

Ounces	Price ($)
20	**$3.80**
10	
5	
1	

Lots of Suds

Ounces	Price ($)
25	**$4.50**
5	
1	

Example: *The sizes and prices of three brands of laundry detergent are shown in the table.*
Find the unit price for each detergent and determine which is the best buy.

Brand	Size (oz)	Price ($)
A	32	$4.80
B	48	$5.76
C	128	$17.92

Example: *Carol's recipe for a punch requires 2 cups of pineapple juice and 3 cups of lemon-lime soda. Cheryl's recipe calls for 3 cups of pineapple juice and 5 cups of lemon-lime soda. Whose recipe makes stronger pineapple punch? How do you know? Show your work.*

Solutions may include:

Carol's Recipe

Pineapple Juice	*2*	*4*	*6*	*8*	*10*
Lemon-lime soda	*3*	*6*	*9*	*12*	*15*

Cheryl's Recipe

Pineapple Juice	*3*	*6*	*9*
Lemon-lime soda	*5*	*10*	*15*

Since $\frac{10}{15} > \frac{9}{15}$, *Carol's recipe has more parts of pineapple juice and is a stronger pineapple punch.*

6.RP.A.3b Solve unit rate problems including those involving unit pricing and constant speed.

Example: Recently 1 U. S. dollar was worth 0.7396 euros. If you exchanged $25 at that rate, how many euros would you get?

Solution: $\frac{\text{US dollar}}{\text{euro}} \quad \frac{1}{0.7396} = \frac{25}{x}$ 18.49 *euros*

Example: Recently 1 U.S. dollar was worth 13.70 Mexican pesos. If you had 109.60 pesos, how many U.S. dollars would you get?

Solution: $\frac{\text{pesos}}{\text{dollar}} \quad \frac{13.70}{1} = \frac{109.60}{x}$ $8.00*US*

Example: *If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

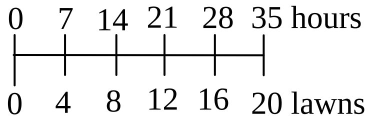
Solution methods may vary:

hours	**7**	**14**	**21**	**28**	**35**
lawns	**4**	**8**	**12**	**16**	**20**

x 5

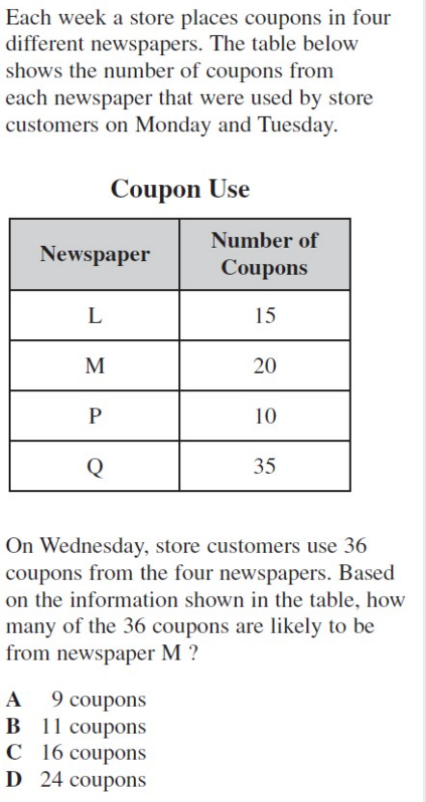
hours	**7**	**35**
lawns	**4**	**20**

x 5



$$\frac{7\,hours}{4\,lawns} \overset{\times 5}{\underset{\times 5}{=}} \frac{35\,hours}{x}$$

Example:



Each week a store places coupons in four different newspapers. The table below shows the number of coupons from each newspaper that were used by store customers on Monday and Tuesday.

Coupon Use

Newspaper	Number of Coupons
L	15
M	20
P	10
Q	35

On Wednesday, store customers use 36 coupons from the four newspapers. Based on the information shown in the table, how many of the 36 coupons are likely to be from newspaper M ?

A 9 coupons
B 11 coupons
C 16 coupons
D 24 coupons

Sample OnCore Examples:

Standard 6.RP.1 *DOK: 2* *Answer: D*

1. Use the table to write the ratio of soccer balls to the total number of balls in the store.

Type of Ball	Number of Balls
Baseballs	61
Softballs	10
Footballs	80
Soccer balls	33

a. 184:33
b. 184:80
c. 33:80
d. 33:184

Standard 6.RP.2 and 6.RP.3b *DOK: 2* *Answer: C*

2. Bars of soap come in packages of 6 and packages of 20. The 6-bar pack costs $7.86, and the 20-bar pack costs $25.00. Which is the better deal? What is the price per bar of the better deal?

a. 6-bar pack at $1.25 per bar
b. 6-bar pack at $0.39 per bar
c. 20-bar pack at $1.25 per bar
d. 6-bar pack at $1.31 per bar

Standard 6.RP.3 *DOK: 2* *Answer: B*

3. The fuel for a chain saw is a mix of oil and gasoline. The label says to mix 6 ounces of oil with 16 gallons of gasoline. How much oil would you use if you had 32 gallons of gasoline?

a. 3 ounces
b. 12 ounces
c. 18 ounces
d. 85.3 ounces

Standard 6.RP.3 *DOK: 2* *Answer: B*

4. Seven out of 12 people say they will vote for Mrs. Meekus. If 5,592 people vote, how many are likely to vote for Mrs. Meekus?

a. 2,121
b. 3,262
c. 4,868
d. 9,586

Standard 6.RP.3 *DOK: 2* *Answer: C*

5. Which proportion has a solution of $x = 8$?

a. $\frac{2}{48} = \frac{x}{12}$

b. $\frac{2}{x} = \frac{48}{12}$

c. $\frac{2}{x} = \frac{12}{48}$

d. $\frac{2}{12} = \frac{48}{x}$

Standard 6.RP.3 *DOK: 2* *Answer: B*

6. Which ratio is NOT proportional to $\frac{48}{30}$?

a. $\frac{24}{15}$

b. $\frac{12}{8}$

c. $\frac{16}{10}$

d. $\frac{8}{5}$

Standard 6.RP.3 *DOK: 2* *Answer: Each cupcake should cost $0.40* *Short Answer*

7.Audrey is using a recipe that makes 24 cupcakes. The recipe is shown in the table.

	24 cupcakes	192 cupcakes
Flour	2 cups	
Sugar	2 cups	
Water	$\frac{3}{4}$ cup	
Eggs	2 eggs	
Milk	$\frac{3}{4}$ cup	
Shortening	$\frac{1}{2}$ cup	
Salt	1 tsp	
Baking soda	1 tsp	
Baking powder	$\frac{1}{2}$ tsp	
Vanilla	1 tsp	
Baking chocolate	4 ounces	

Part A: Audrey plans to make 192 cupcakes for a bake sale. Find the amount of each ingredient Audrey needs to make 192 cupcakes.

Part B: The ingredients to make 192 cupcakes cost $25.85. Determine the amount that Audrey should charge for each cupcake in order to make a profit of $50. Show your work.

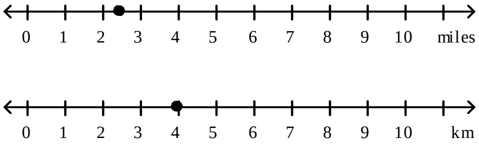
Standard 6.RP.3 *DOK: 2* *Answer: Hamburger = $3.50* *Short Answer*
Garden Salad = $4.00
French Fries = $3.25

8. The table shows the total costs for three restaurant orders. Find the cost of a hamburger, french fries, and garden salad. Then use these costs to complete the table.

Order	**Hamburger**	**French fries**	**Garden salad**	**Cost**
1	2	0	0	$7.00
2	1	0	2	$11.50
3	1	2	1	$14.00
4	3	2	1	
5	4	4	0	
6	2	4	2	
7	5	5	5	
8	4	6	3	

Standard 6.RP.3 *DOK: 3* *Answer: 4,480 km*

9. The points graphed on each number line show equivalent distances measured in kilometers and in miles.



The distance from New York City, NY, to San Diego, CA, is approximately 2,800 miles. Explain how you could use the number lines to estimate the distance between the two cities in kilometers.

Standard 6.RP.3a *DOK: 1* *Answer: A*

10. Juana is making fruit salad. The table shows the numbers of apples and bananas needed for various amounts of fruit salad. How many apples does Juana need to make 5 bowls of fruit salad?

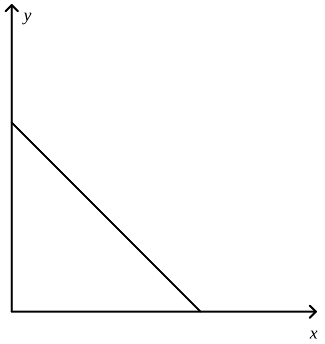
Bowls of fruit salad	2	4	6
Apples needed	12	24	36
Bananas needed	14	28	42

a. 30
b. 35
c. 6
d. 7

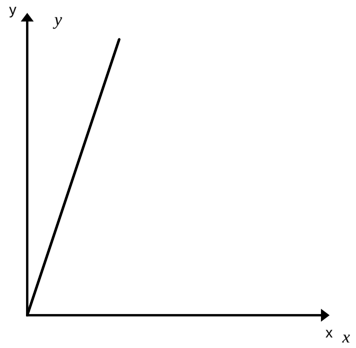
Standard 6.RP.3a *DOK: 2* *Answer: C*

11. Which graph could have been constructed from a table of equivalent ratios?

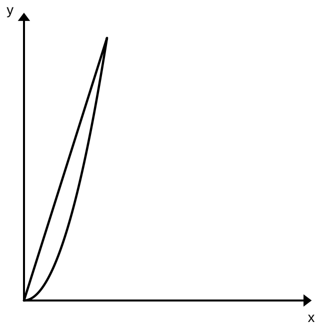
a.



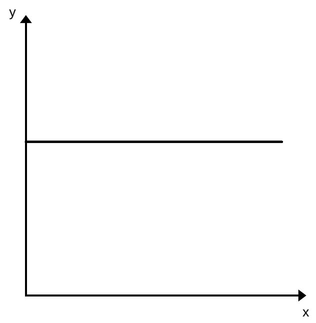
c.



b.



d.

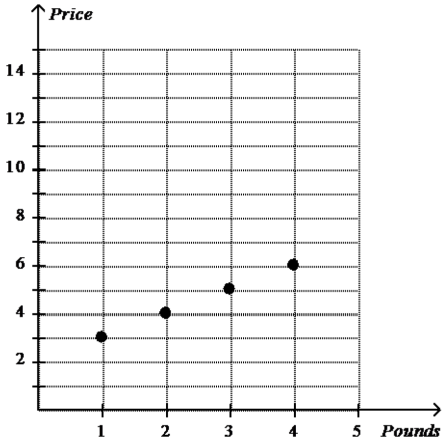


Standard 6.RP.3a *DOK: 1* *Answer: D*

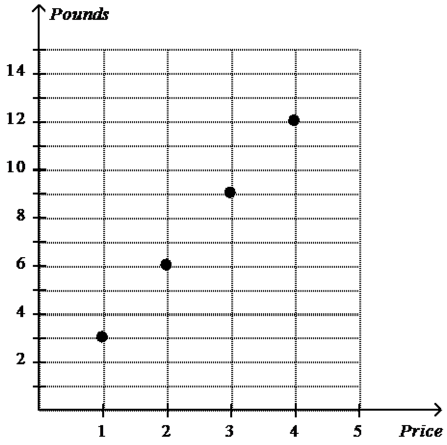
12. The table shows prices per pound. Which graph shows the same data?

Pounds	1	2	3	4
Price ($)	3	6	9	12

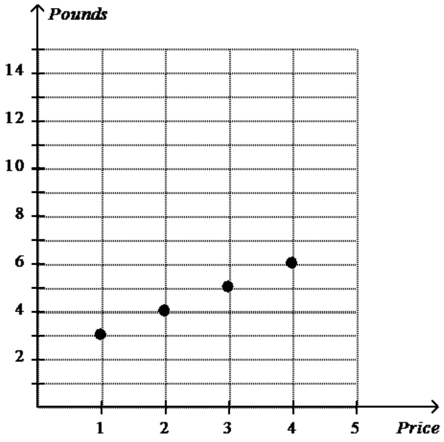
a.



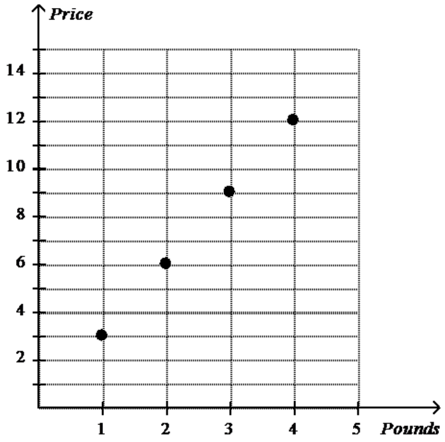
c.



b.



d.



Standard 6.RP.3a *DOK: 1* *Answer: C*

13. Examine the table. Do the quantities have a proportional relationship?

x	2	3	4	5
y	1	1.5	2	2.5

a. Yes; The *y*:*x* ratios are not constant.

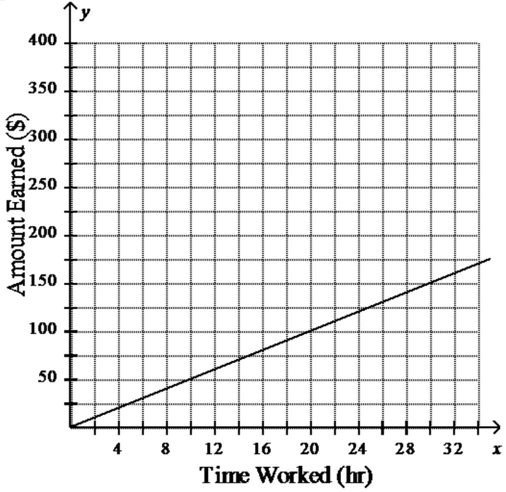
b. No; The *y*:*x* ratios are not constant.

c. Yes; The *y*:*x* ratios are constant.

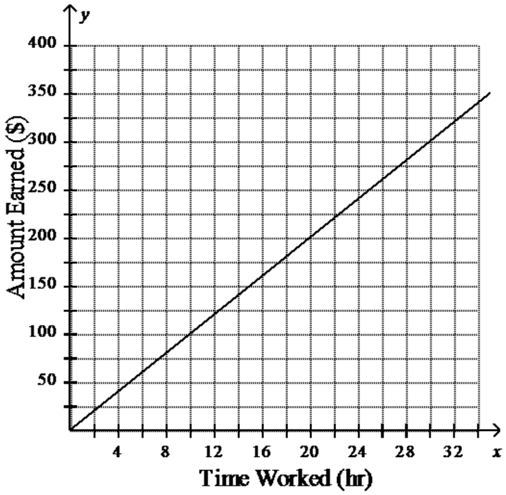
d. No; The *y*:*x* ratios are constant.

Standard 6.RP.3a *DOK: 2* *Answer: B*

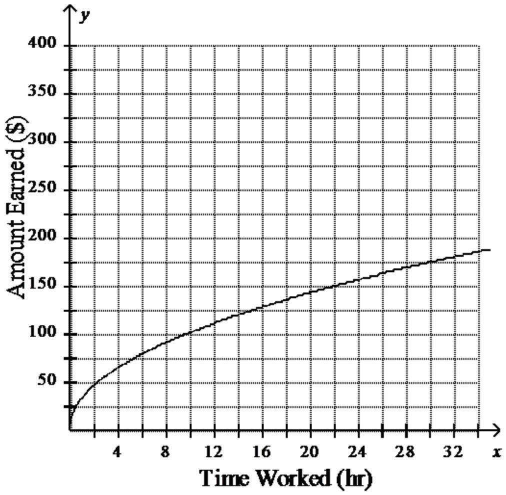
14. Dorell earns $10 per hour. Graph this relationship on the coordinate plane, and describe the pattern.

a.


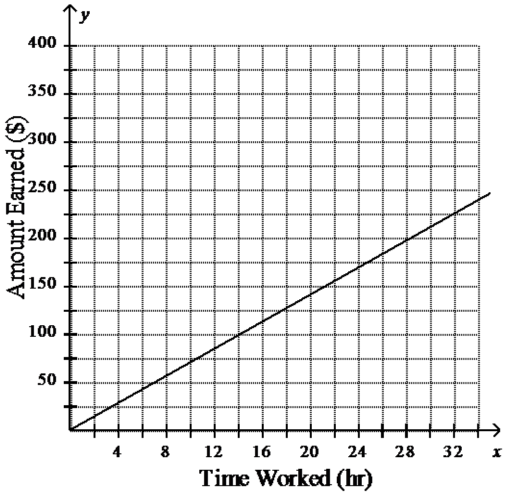
The points lie on a straight line.

b.


The points lie on a straight line.

c.


The points lie on a curve.

d.


The points lie on a straight line.

***Standard 6.RP.**3a* *DOK: 2* *Answer: B*

15. Determine whether the rates of change are constant or variable.

x	*y*
–5	13
–3	5
2	–15
4	–23

a. variable

b. constant

Standard 6.RP.3a *DOK: 3* *Answer: see below question*

16. Jace usually makes 17 out of every 20 free throws he attempts. Michael usually makes 21 out of every 25 free throws he attempts.

Part A: Explain how to make graphs showing the number of free throws Jace and Michael will usually make for various numbers of free throw attempts.

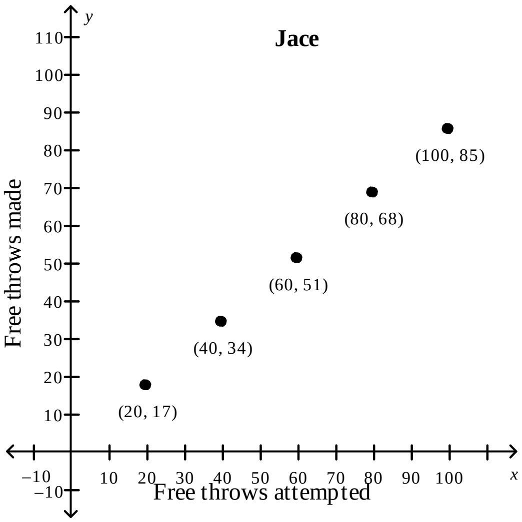
Part B: Make and use the graphs to find who will make more free throws if Jace and Michael each attempt 100 free throws. Explain your answer.

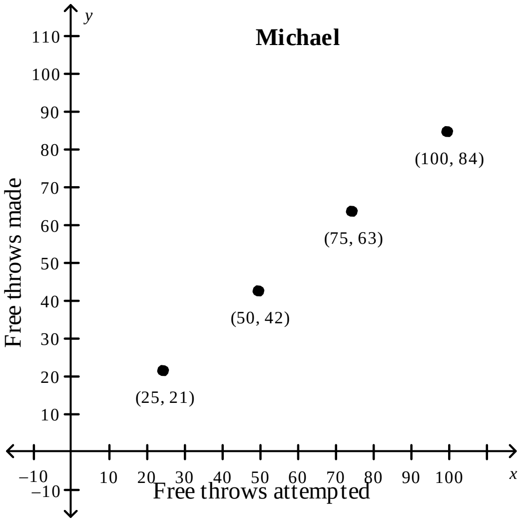
ANSWER: ***Part A:*** Make a graph for Jace and a graph for Michael. Each graph is a coordinate plane with the *x*-axis labeled "Free throws attempted" and the *y*-axis labeled "Free throws made."

Graph (20, 17) on Jace's graph. Then graph more points by multiplying the coordinates by 2, 3, 4, etc.

Graph (25, 21) on Michael's graph. Then graph more points by multiplying the coordinates by 2, 3, 4, etc.

Part B:





To find who will make more free throws in 100 attempts, compare the *y*-coordinates for the points whose *x*-coordinates are 100. Jace will make more free throws because 85 > 84.

2 The table below shows the numbers of tablespoons in various numbers of cups. One number is missing in the table.

Tablespoons and Cups

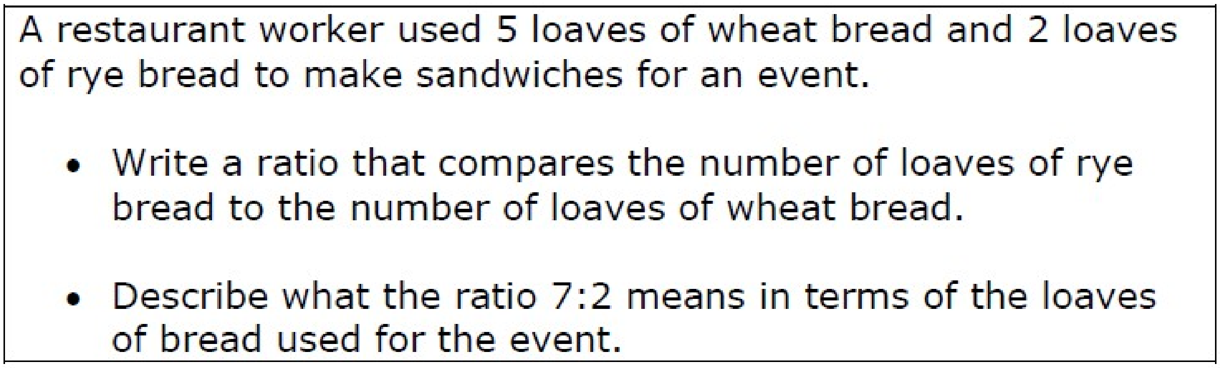
Number of Tablespoons	Number of Cups
48	3
112	
144	9
224	14

What is the missing number in the table?

A 4
B 6
C 7
D 8

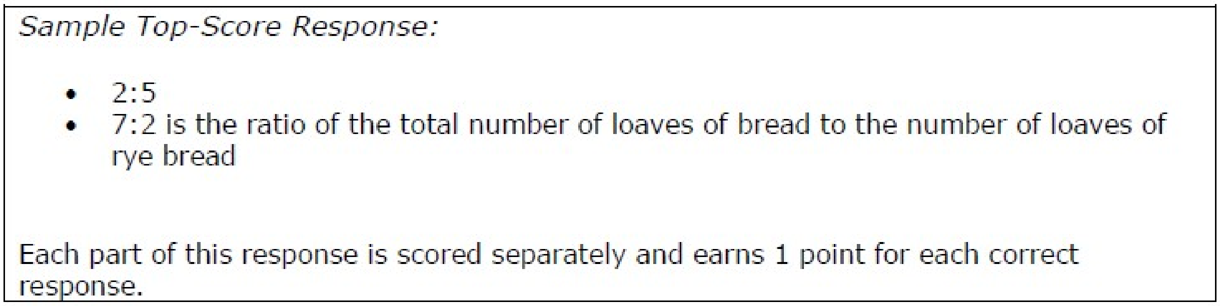
SBAC Example Assessment Questions:

Standard 6.RP.1 *DOK: 2* *Difficulty: Low* *Question Type: CR(constructed response)*



A restaurant worker used 5 loaves of wheat bread and 2 loaves of rye bread to make sandwiches for an event.

- Write a ratio that compares the number of loaves of rye bread to the number of loaves of wheat bread.
- Describe what the ratio 7:2 means in terms of the loaves of bread used for the event.

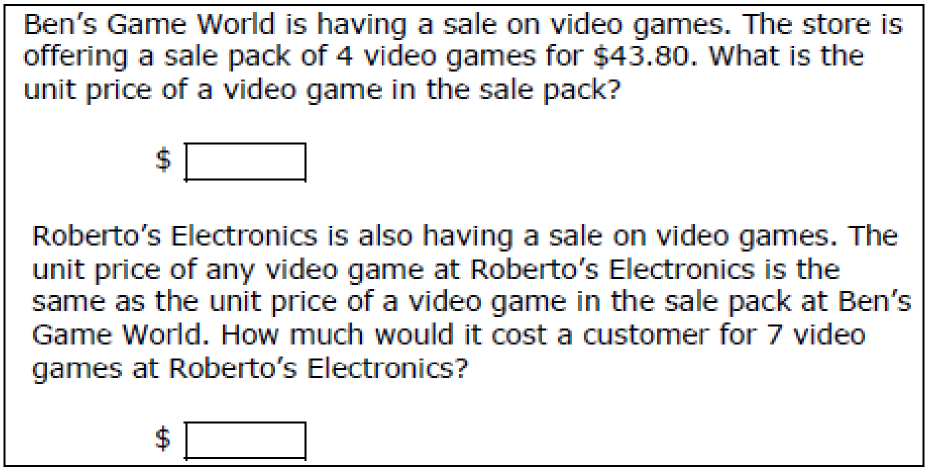


Sample Top-Score Response:

- 2:5
- 7:2 is the ratio of the total number of loaves of bread to the number of loaves of rye bread

Each part of this response is scored separately and earns 1 point for each correct response.

Standard 6.RP.3 *DOK: 2* *Difficulty: Medium* *Question Type: CR(constructed response)*



Ben's Game World is having a sale on video games. The store is offering a sale pack of 4 video games for $43.80. What is the unit price of a video game in the sale pack?

$ ☐

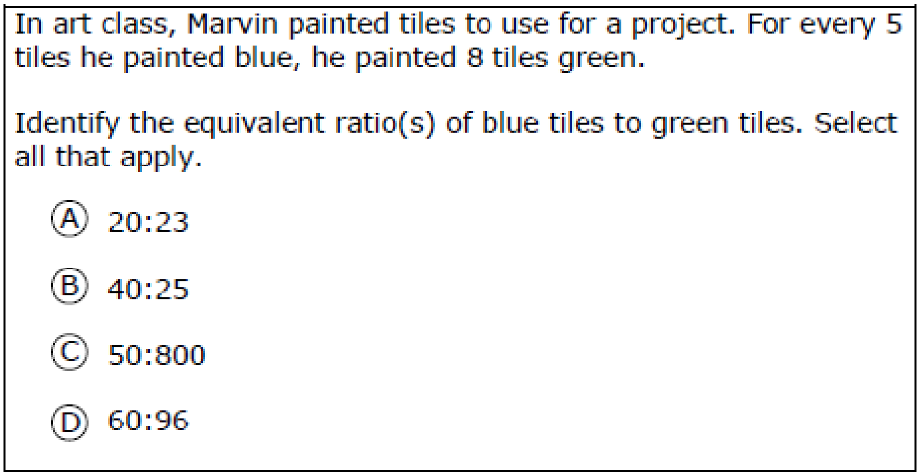
Roberto's Electronics is also having a sale on video games. The unit price of any video game at Roberto's Electronics is the same as the unit price of a video game in the sale pack at Ben's Game World. How much would it cost a customer for 7 video games at Roberto's Electronics?

$ ☐

Key:

$10.95
$76.65

Standard 6.RP.3 *DOK: 2* *Difficulty: Medium* *Question Type: SR (student-response)*



In art class, Marvin painted tiles to use for a project. For every 5 tiles he painted blue, he painted 8 tiles green.

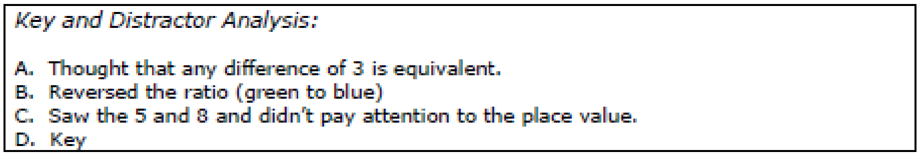
Identify the equivalent ratio(s) of blue tiles to green tiles. Select all that apply.

Ⓐ 20:23

Ⓑ 40:25

Ⓒ 50:800

Ⓓ 60:96



Key and Distractor Analysis:

A. Thought that any difference of 3 is equivalent.
B. Reversed the ratio (green to blue)
C. Saw the 5 and 8 and didn't pay attention to the place value.
D. Key

Grades 6-8, Claim 2

Task Model 1

DOK Levels 2, 3

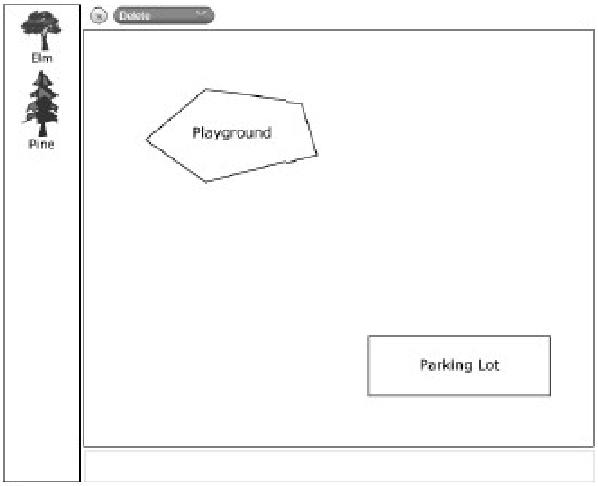
Target A: Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace.

Task Expectations:
- Mathematical information is presented in a table or graph or extracted from a context.
- Student is asked to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace.

Example Item 1 (Grade 6):
Primary Target 2A (Content Domain RP), Secondary Target 1A (CCSS 6.RP.3), Tertiary Target 2C)

A landscape designer is planning the layout of trees in a park.
- There are two types of trees: elm and pine.
- There should be at least 16 total trees but no more than 30.
- The ratio of elm trees to pine trees will be 3:2.

Drag trees anywhere to the model to show a possible number of each type of tree.

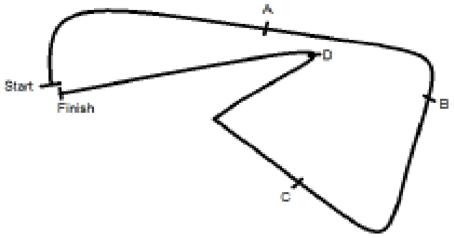


Rubric: (1 point) Student correctly places 16 to 30 trees in the response area with a 3 elm to 2 pine ratio (e.g., 12 elm, 8 pine; 15 elm, 10 pine).

Response Type: Drag and Drop

Example Item 4 (Grade 6):
Primary Target 2A (Content Domain RP), Secondary Target 1A (CCSS 6.RP.3), Tertiary Target 2D

It takes Shaun 90 minutes to complete a 15 mile race. The route, with four checkpoints (labeled A, B, C, and D), is shown.



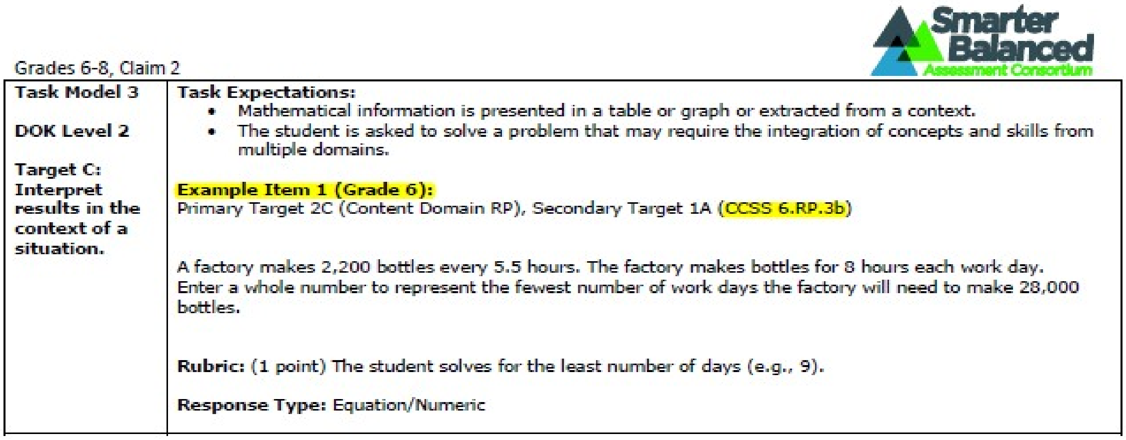
Assume Shaun runs at a constant rate during the race.

Complete the table to show Shaun's time, in minutes, and distance, in miles, at each checkpoint.

Checkpoint	A	B	C	D	Finish
Number of minutes	6			60	90
Number of miles		$2\frac{3}{4}$	7.5		15

Rubric: (2 points) The student correctly determines all four missing values.
(1 point) The student correctly determines both minutes (e.g., 16.5, 45) or both miles (e.g., 1, 10) or three out of four values correct.

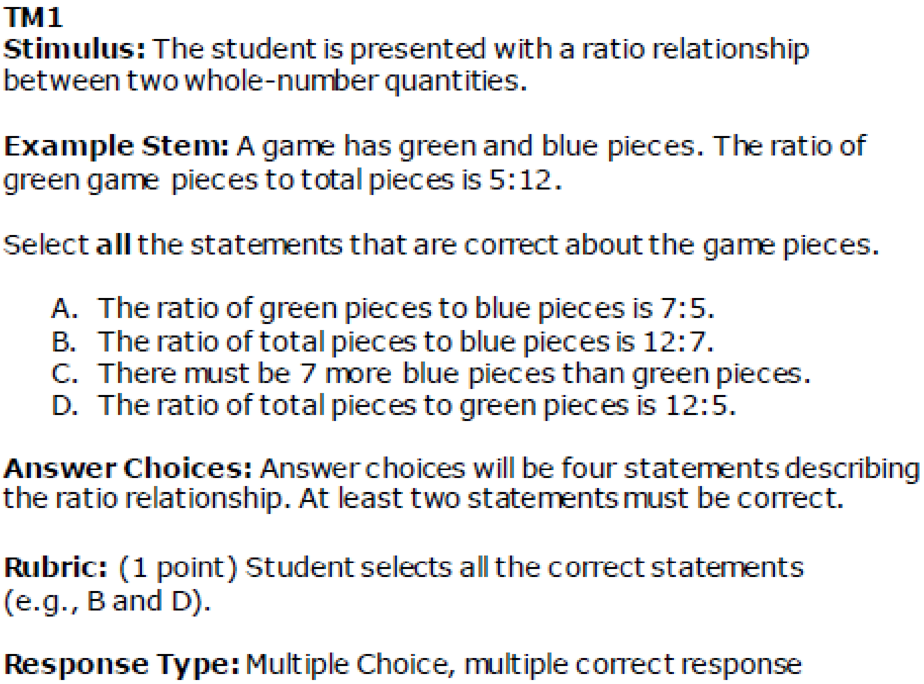
Response Type: Fill-in Table



Grades 6-8, Claim 2

Task Model 3 **DOK Level 2** **Target C: Interpret results in the context of a situation.**	**Task Expectations:** • Mathematical information is presented in a table or graph or extracted from a context. • The student is asked to solve a problem that may require the integration of concepts and skills from multiple domains. **Example Item 1 (Grade 6):** Primary Target 2C (Content Domain RP), Secondary Target 1A (CCSS 6.RP.3b) A factory makes 2,200 bottles every 5.5 hours. The factory makes bottles for 8 hours each work day. Enter a whole number to represent the fewest number of work days the factory will need to make 28,000 bottles. **Rubric:** (1 point) The student solves for the least number of days (e.g., 9). **Response Type:** Equation/Numeric

Standard: 6.RP.A.1 ***DOK:1*** ***Question Type: Multiple Choice Multiple Correct Response***



TM1

Stimulus: The student is presented with a ratio relationship between two whole-number quantities.

Example Stem: A game has green and blue pieces. The ratio of green game pieces to total pieces is 5:12.

Select **all** the statements that are correct about the game pieces.

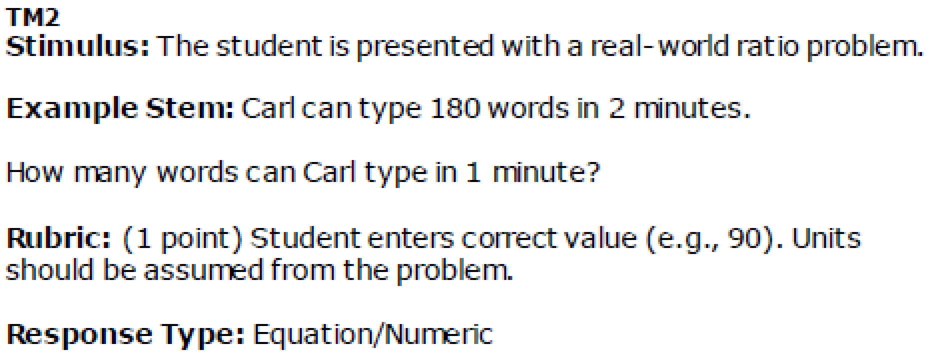
A. The ratio of green pieces to blue pieces is 7:5.
B. The ratio of total pieces to blue pieces is 12:7.
C. There must be 7 more blue pieces than green pieces.
D. The ratio of total pieces to green pieces is 12:5.

Answer Choices: Answer choices will be four statements describing the ratio relationship. At least two statements must be correct.

Rubric: (1 point) Student selects all the correct statements (e.g., B and D).

Response Type: Multiple Choice, multiple correct response

Standard: 6.RP.A.2 ***DOK:2*** ***Question Type: Equation/Numeric***



TM2
Stimulus: The student is presented with a real-world ratio problem.

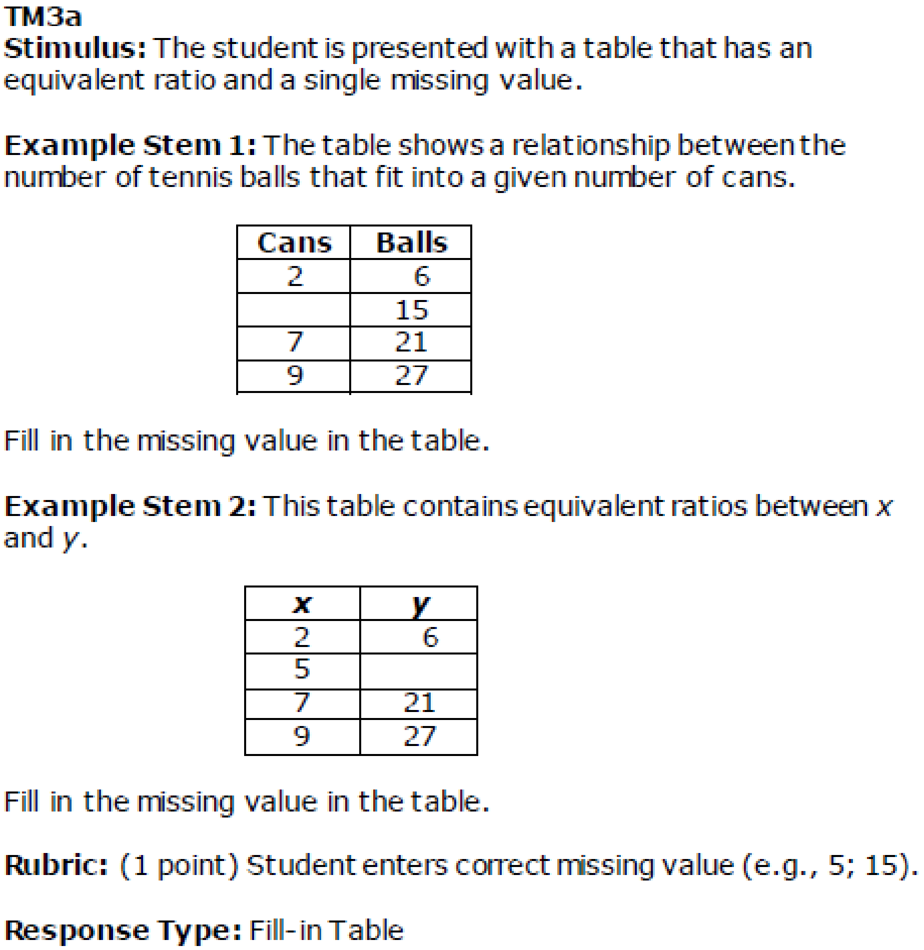
Example Stem: Carl can type 180 words in 2 minutes.

How many words can Carl type in 1 minute?

Rubric: (1 point) Student enters correct value (e.g., 90). Units should be assumed from the problem.

Response Type: Equation/Numeric

Standard: 6.RP.A.3a ***DOK:1*** ***Question Type: Fill-in Table***



TM3a
Stimulus: The student is presented with a table that has an equivalent ratio and a single missing value.

Example Stem 1: The table shows a relationship between the number of tennis balls that fit into a given number of cans.

Cans	**Balls**
2	6
	15
7	21
9	27

Fill in the missing value in the table.

Example Stem 2: This table contains equivalent ratios between *x* and *y*.

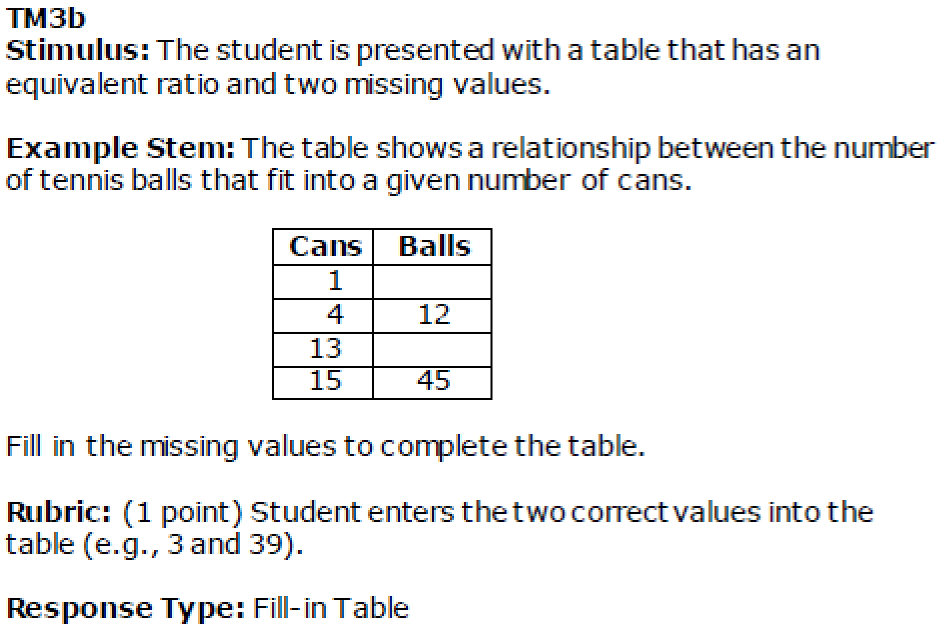
x	***y***
2	6
5	
7	21
9	27

Fill in the missing value in the table.

Rubric: (1 point) Student enters correct missing value (e.g., 5; 15).

Response Type: Fill-in Table

Standard: 6.RP.A.3a ***DOK:1*** ***Question Type: Fill-in***



TM3b

Stimulus: The student is presented with a table that has an equivalent ratio and two missing values.

Example Stem: The table shows a relationship between the number of tennis balls that fit into a given number of cans.

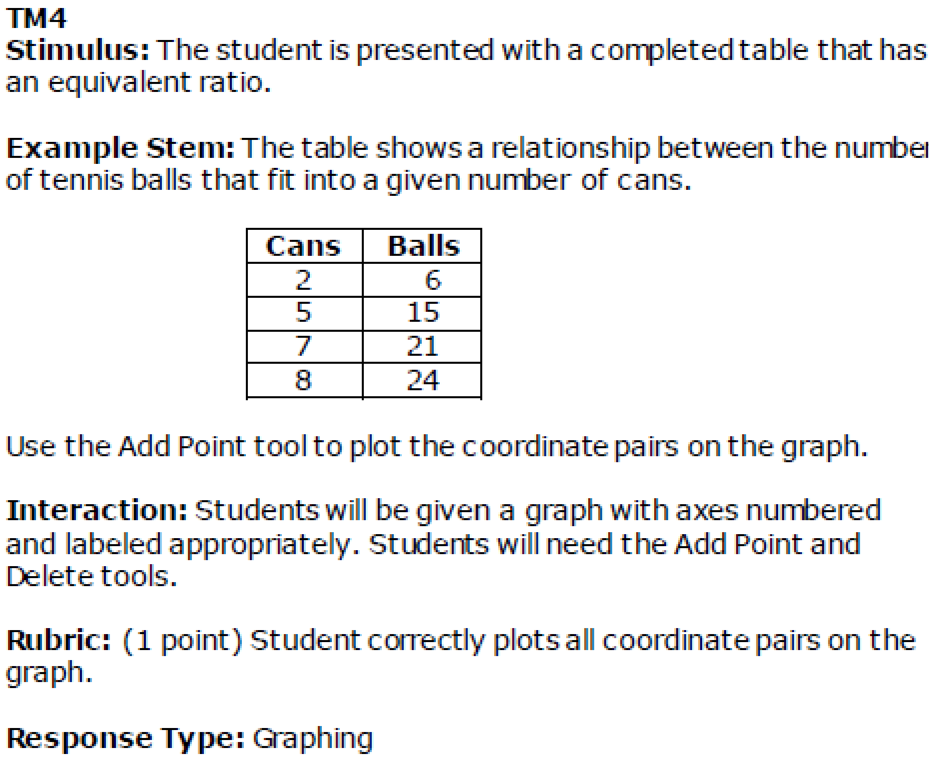
Cans	Balls
1	
4	12
13	
15	45

Fill in the missing values to complete the table.

Rubric: (1 point) Student enters the two correct values into the table (e.g., 3 and 39).

Response Type: Fill-in Table

Standard: 6.RP.A.3a ***DOK:1*** ***Question Type: Graphing***



TM4

Stimulus: The student is presented with a completed table that has an equivalent ratio.

Example Stem: The table shows a relationship between the numbe of tennis balls that fit into a given number of cans.

Cans	Balls
2	6
5	15
7	21
8	24

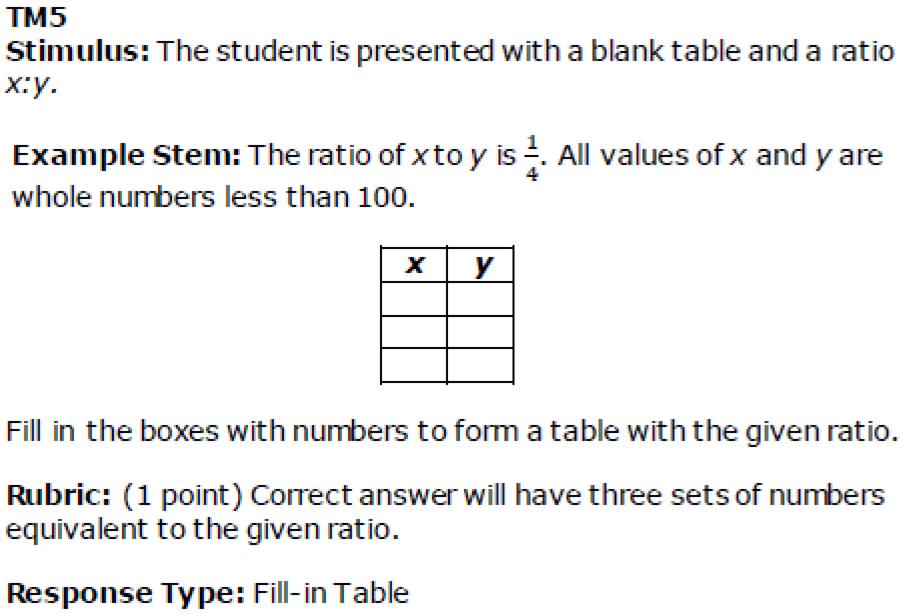
Use the Add Point tool to plot the coordinate pairs on the graph.

Interaction: Students will be given a graph with axes numbered and labeled appropriately. Students will need the Add Point and Delete tools.

Rubric: (1 point) Student correctly plots all coordinate pairs on the graph.

Response Type: Graphing

Standard: 6.RP.A.3a ***DOK:2*** ***Question Type: Fill-in Table***



TM5
Stimulus: The student is presented with a blank table and a ratio *x*:*y*.

Example Stem: The ratio of *x* to *y* is $\frac{1}{4}$. All values of *x* and *y* are whole numbers less than 100.

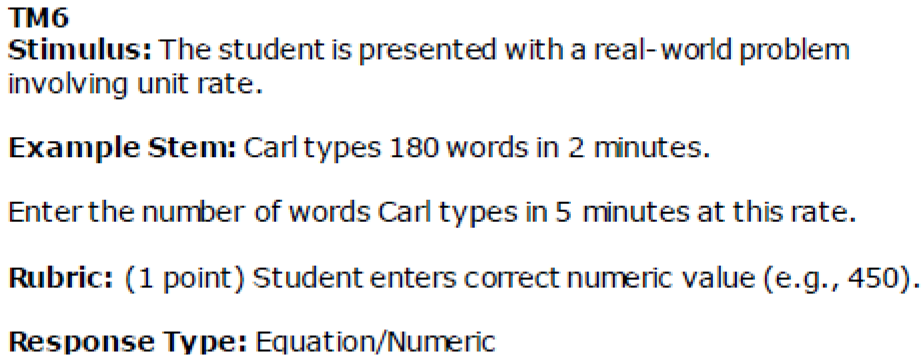
x	***y***

Fill in the boxes with numbers to form a table with the given ratio.

Rubric: (1 point) Correct answer will have three sets of numbers equivalent to the given ratio.

Response Type: Fill-in Table

Standard: 6.RP.A.3b ***DOK:2*** ***Question Type: Equation/Numeric***



TM6
Stimulus: The student is presented with a real-world problem involving unit rate.

Example Stem: Carl types 180 words in 2 minutes.

Enter the number of words Carl types in 5 minutes at this rate.

Rubric: (1 point) Student enters correct numeric value (e.g., 450).

Response Type: Equation/Numeric