Math 6 Part B-NOTES – Fraction Operations

Review/Prep for 6.NS Adding and Subtracting Fractions With Unlike Denominators

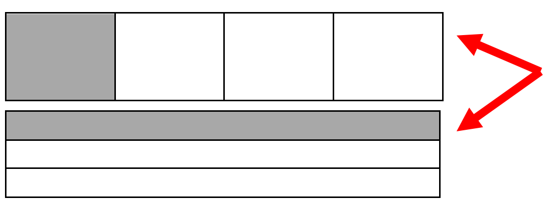
Note: **You can use *any* common denominator to add and subtract unlike fractions;** but the least common denominator (LCD), which is the least common multiple (LCM) of the denominators, is the simplest**.**

***Example**:* There is a giant laundry pile of dirty clothes in the laundry room. If $\frac{1}{4}$ of the dirty clothes are Ashton's dirty clothes and $\frac{1}{3}$ of the dirty clothes belong to Catherine, what fraction of the dirty clothes are Ashton and Catherine's?

To find the fraction of the dirty clothes that belong to Ashton and Catherine, you can add $\frac{1}{4}+\frac{1}{3}$, which are fractions with unlike denominators. Here is a visual representation of the two fractions:

$\frac{1}{4}$

$\frac{1}{3}$

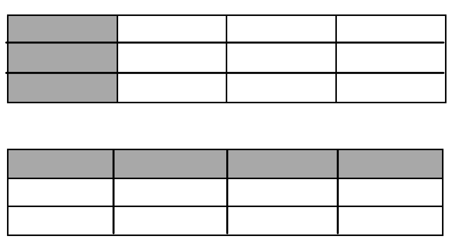


Be sure one diagram is cut vertically and the other horizontally to start.

Notice the pieces are not the same size. Making the same cuts in each fraction bar will result in equal size pieces:

$\frac{1}{4}$

$\frac{1}{3}$



$\frac{3}{12}$ Notice the horitontal cuts are added.

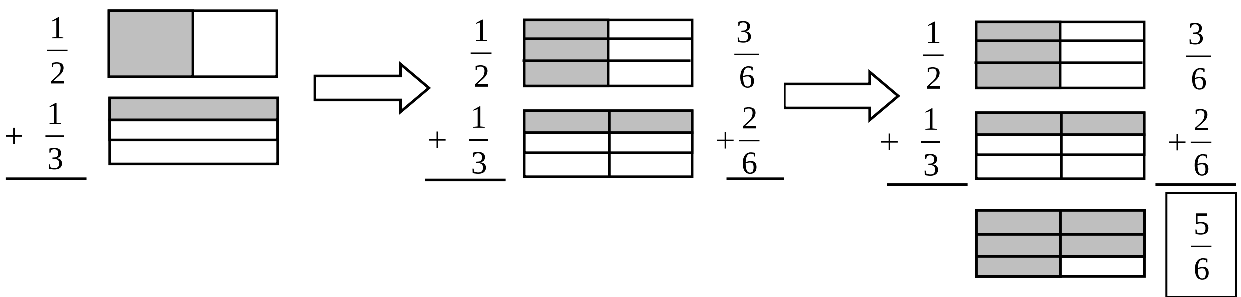
$\frac{4}{12}$ Notice the vertical cuts are added.

Now the two fraction bars are divided into 12 equal pieces:
$\frac{1}{4}$ can be re-written as $\frac{3}{12}$ and $\frac{1}{3}$ can be re-written as $\frac{4}{12}$. This problem is now much easier because the two fractions have the same denominator.

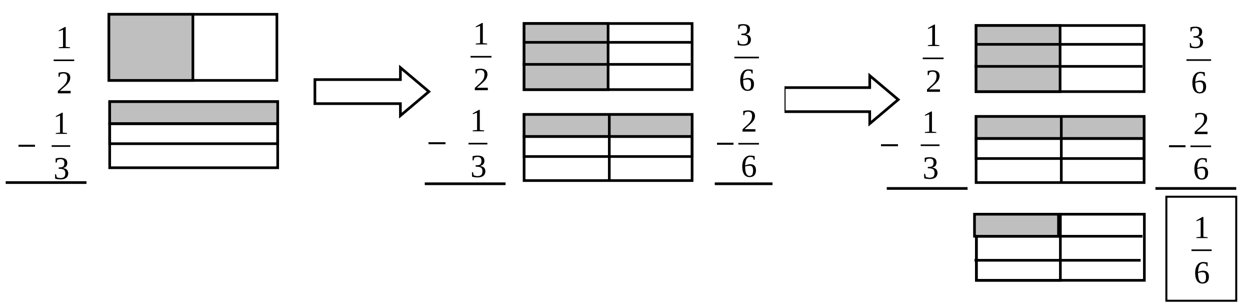
Math 6 Part B-NOTES – Fraction Operations

$$\begin{array}{rcl} \frac{1}{4} & = & \frac{3}{12} \\ +\frac{1}{3} & = & \frac{4}{12} \\ \hline & & \frac{7}{12} \end{array}$$

Example: *Draw a* ***model*** *to solve* $\frac{1}{2}+\frac{1}{3}=$



Example: *Draw a* ***model*** *to solve* $\frac{1}{2}-\frac{1}{3}=$



It is inconvenient to have to draw fraction bars for every addition or subtraction problem. The following is the algorithm for adding or subtraction fractions that do not have the same denominator:

Algorithm For Adding/Subtracting Fractions

Step 1. Find a common denominator.
Step 2. Make equivalent fractions.
Step 3. Add or subtract the numerators, keep the common denominator, then simplify if possible.

Math 6 Part B-NOTES – Fraction Operations

REVIEW: Remember to find a least common denominator (LCD), we can:

1. Multiply the denominators if they are prime or relatively prime.
2. List the common multiples for each denominator until you find one in common.
3. Work with factorization methods – factor trees, prime factorization, Venn Diagrams and/or ladder diagrams.
4. Use the Simplifying Method.

You will find explanations and examples of these methods on pages 36 - 39 of Unit 3 Part A notes under LCD's and more on pages 15 -18 of Unit 3 Part A notes under LCM.

Example: Add $\frac{1}{5}+\frac{2}{3}$.

Step 1 Find a common denominator: The common denominator of 5 and 3 is 15. (Best method is to multiply the denominators since they are prime numbers.)

Step 2 Make equivalent fractions:

$$\frac{1}{5}=\frac{3}{15} \quad \text{and} \quad \frac{2}{3}=\frac{10}{15}$$

Step 3 Add the numerators, keep the common denominator:

$$\frac{3}{15}+\frac{10}{15}=\frac{13}{15}$$

Example: Subtract $\frac{5}{7}-\frac{1}{3}$. This problem can also be written vertically. Using the algorithm, first find the common denominator (multiply the denominators since they are prime numbers) then make equal fractions. Once you complete that, you subtract the numerators and place that result over the common denominator and simplify.

$$\begin{array}{r} \frac{5}{7}=\frac{15}{21} \\ -\ \frac{1}{3}=\frac{7}{21} \\ \hline \frac{8}{21} \end{array}$$

Multiply denominators since they are relatively prime.

Since $\frac{6}{9}=\frac{2}{3}$ 18

LCD=18

Math 6 Part B-NOTES – Fraction Operations

Example:

$$\frac{4}{9}+\frac{5}{8}=$$
$$\frac{32}{72}+\frac{45}{72}=$$
$$\frac{77}{72}=1\frac{5}{72}$$

Example:

$$\frac{5}{6}-\frac{4}{9}=$$
$$\frac{15}{18}-\frac{8}{18}=$$
$$\frac{7}{18}$$

Example: Add $4\frac{3}{5}+3\frac{3}{20}$. This problem can also be written vertically. In fact, it may be easier to separate the whole numbers from the fractions if the problem is written vertically.

$$\begin{array}{r} 4\frac{3}{5} \\ +\,3\frac{3}{20} \\ \hline \end{array} \longrightarrow \begin{array}{r} 4\frac{12}{20} \\ +\,3\frac{3}{20} \\ \hline \end{array} \longrightarrow \begin{array}{r} 4\frac{12}{20} \\ +\,3\frac{3}{20} \\ \hline 7\frac{15}{20}=7\frac{3}{4} \end{array}$$

Step 1: Find a common denominator (Use 20 since 4 is a factor of 20.)

Step 2: Make equivalent fractions

Step 3: Add the numerators and keep the common denominator. Then reduce.

Notice that the whole numbers are just added together.

Example: Subtract $7\frac{3}{5}-2\frac{2}{7}$. Another approach to evaluating fractions that involve mixed numbers is to change all mixed numbers to improper fractions then follow the algorithm.

Change $7\frac{3}{5}-2\frac{2}{7}$ to improper fraction: $\frac{38}{5}-\frac{16}{7}$. Now follow the algorithm for subtracting fractions.

Step 1 Find the common denominator: The common denominator of 5 and 7 is 35 (multiply the denominators since they are prime numbers).

Step 2 Make equivalent fraction using the common denominator:

$$\frac{38}{5}\frac{\bullet 7}{\bullet 7}-\frac{16}{7}\frac{\bullet 5}{\bullet 5}=\frac{266}{35}-\frac{80}{35}$$

Step 3 Subtract the numerators and keep the common denominator, reduce, change back to a mixed number:

Math 6 Part B-NOTES – Fraction Operations

$$\_\_\_ \quad \_\_\_ \quad \frac{266}{35}-\frac{80}{35}=\frac{186}{35}=5\frac{11}{35}$$

As you can see many times the fractions get quite large which makes computation difficult for students. So again I will show the other format for solving for this problem.

$$\begin{array}{rcr} 7\frac{3}{5} & = & 7\frac{21}{35} \\ -2\frac{2}{7} & = & -2\frac{10}{35} \\ \hline & & 5\frac{11}{35} \end{array}$$

Review/Prep for 6.NS Estimation with Fractions

Benchmarks for Rounding Fractions		
Round to **0** if the numerator is much smaller than the denominator. Examples: $\frac{3}{20}, \frac{1}{8}, \frac{7}{100}$	Round to ½ if the numerator is about half the denominator. Examples: $\frac{11}{20}, \frac{5}{9}, \frac{47}{100}$	Round to **1** if the numerator is nearly equal to the denominator. Examples: $\frac{17}{20}, \frac{7}{8}, \frac{87}{100}$

1. $\frac{7}{9}\approx 1$ 2. $\frac{27}{50}\approx\frac{1}{2}$ 3. $\frac{2}{25}\approx 0$

Estimating Sums and Differences

Round each fraction or mixed number to the nearest half, and then simplify using the rules for signed numbers.

Example: Estimate $\frac{7}{8}+\frac{1}{9}$.

$$\begin{array}{rl} \frac{7}{8} & \approx 1 \\ +\frac{1}{9} & \approx 0 \\ \hline & 1 \end{array}$$

is our estimate

Math 6 Part B-NOTES – Fraction Operations

Example: Estimate $\frac{7}{8}-\frac{2}{5}$.

$$\begin{array}{r} \frac{7}{8} \approx \ \ 1 \\ -\frac{2}{5} \approx -\frac{1}{2} \\ \hline \frac{1}{2} \end{array}$$ is our estimate

Example: Estimate $8\frac{1}{7}-5\frac{3}{4}$

$$\begin{array}{r} 8\frac{1}{7} \approx \ \ 8 \\ -5\frac{3}{4} \approx -6 \\ \hline 2 \end{array}$$ is our estimate

Review/Prep for 6.NS Regrouping To Subtract Mixed Numbers

Review: Before teaching how to borrow with fractions, it is a good idea to revisit the following topic (from Unit 4): *Borrowing when subtracting mixed numbers.*

The concept of borrowing when subtracting with fractions has been typically a difficult area for kids to master. For example, when subtracting $12\,{}^{1}\!/_{6} - 4\,{}^{5}\!/_{6}$, students usually answer $8\,{}^{4}\!/_{6}$ if they subtract this problem incorrectly. In order to ease the borrowing concept for fraction, it would be a good idea to go back and review borrowing concepts kids are familiar with.

Subtracting fractions and borrowing is as easy as getting change for your money.

Example: You have 8 one dollar bills and you have to give your friend \$3.25. How much money would you have left?

Since you don't have any coins, you would have to change one of the dollars into 4 quarters. Why not ten dimes? Because you have to give your friend a quarter, so you get the change in terms of what you are working with, in this case – quarters.

8 dollars	→	7 dollars 4 quarters
– 3 dollars 1 quarter		– 3 dollars 1 quarter
	Subtract to get:	4 dollars 3 quarters

Example: For this problem you have 6 dollars and 1 quarter in your pocket, and you have to give your brother \$2.75.

Since we don't have enough quarters, we change 1 dollar for 4 quarters, adding that to the quarter we already had, gives you 5 quarters.

$$\begin{array}{r} \text{6 dollars 1 quarter} \\ \underline{-\text{ 2 dollars 3 quarters}} \end{array} \longrightarrow \begin{array}{r} \text{5 dollars 4 quarters+1 quarter} \\ \underline{-\text{ 2 dollars }\quad\text{ 3 quarters}} \end{array} \longrightarrow \begin{array}{r} \text{5 dollars 5 quarters} \\ \underline{-\text{ 2 dollars 3 quarters}} \\ \text{3 dollars 2 quarters} \end{array}$$

Example: Finals are in 5 weeks. You will be on vacation for 2 weeks and 3 days. How much free time do you have to spend just studying for finals?

$$\begin{array}{l} \text{ 5 weeks} \\ -\underline{\text{2 weeks 3 days}} \end{array} \longrightarrow \begin{array}{l} \text{ 4 weeks 7 days} \\ -\underline{\text{2 weeks 3 days}} \\ \text{ 2 weeks 4 days} \end{array}$$

Example: John is 6 feet 3 inches tall. Sally is 4 feet 11 inches tall. How much taller is John?

$$\begin{array}{l} \text{ 6 feet 3 inches} \\ -\underline{\text{4 feet 11 inches}} \end{array} \longrightarrow \begin{array}{l} \text{ 5 feet 12inches+3 inches} \\ -\underline{\text{4 feet }\quad\text{ 11 inches}} \end{array} \longrightarrow \begin{array}{l} \text{ 5 feet 15 inches} \\ -\underline{\text{4 feet 11 inches}} \\ \text{ 1 foot 4 inches} \end{array}$$

Example: Sam has 2 dollar bills and 3 nickels. Sara has 1 dollar bill and 15 nickels. How much more does Sam have than Sara?

$$\begin{array}{l} \text{ 2 dollars 3 nickels} \\ -\underline{\text{1 dollar 15 nickels}} \end{array} \longrightarrow \begin{array}{l} \text{ 1 dollar 20 nickels+3 nickels} \\ -\underline{\text{1 dollar }\quad\text{ 15 nickels}} \end{array} \longrightarrow \begin{array}{r} \text{1 dollar 23 nickels} \\ -\underline{\text{1 dollar 15 nickels}} \\ \text{8 nickels} \end{array}$$

Example: Take away 3 hours 47 minutes from 5 hours 16 minutes.

$$\begin{array}{r} \text{5 hrs 16 min} \\ \underline{-\text{ 3 hrs 47 min}} \\ \text{?????????} \end{array}$$

Subtracting the hours is not a problem but students will see that 47 minutes cannot be subtracted from 16 minutes. In this case, students will see that 1 hour must be borrowed from 5 hrs and added to 16 minutes:

$$\begin{array}{r} 4hrs\quad\quad\quad \\ \text{~~5 hrs~~ ~~16 min~~} \\ \underline{-\text{ 3 hrs 47 min}} \\ \text{?????????} \end{array} \quad 16\,\text{min} + 1hr = 16\,\text{min} + 60\,\text{min} = 76\,\text{min}$$

Now the subtraction problem can be rewritten as:

$$\begin{array}{r} \text{4 hrs 76 min} \\ \underline{-\text{ 3 hrs 47 min}} \\ \text{???????????} \end{array} \Longleftrightarrow \begin{array}{r} \text{4 hrs 76 min} \\ \underline{-\text{ 3 hrs 47 min}} \\ \text{1 hr 29 min} \end{array}$$

If students can understand the borrowing concept from the previous example, the same concept can be linked to borrowing with mixed numbers. Lets go back to the first

Math 6 Part B-NOTES – Fraction Operations

example: $12\frac{1}{6} - 4\frac{5}{6}$. It may be easier to link the borrowing concept if the problem is rewritten vertically:

$$\begin{array}{r} \overset{11}{\cancel{12}}\frac{1}{6} \\ -\ 4\frac{5}{6} \\ \hline ??????? \end{array} \quad \frac{1}{6}+1=\frac{1}{6}+\frac{6}{6}=\frac{7}{6} \quad \Longleftrightarrow \quad \begin{array}{r} 11\frac{7}{6} \\ -\ 4\frac{5}{6} \\ \hline 7\frac{2}{6}=7\frac{1}{3} \end{array}$$

Example:

$$\begin{array}{r} 13\frac{2}{5} \\ -7\frac{1}{2} \\ \hline \end{array} \qquad \begin{array}{r} 13\frac{4}{10} \\ -7\frac{5}{10} \\ \hline \end{array}$$

Step 1. Find a common denominator: The common denominator is 10.

Step 2. Make equivalent fractions using 10 as the denominator.

Step 3. It is not possible to subtract the numerators. You cannot take 5 from 4!! Use the concept of borrowing as described in the above examples to re-write this problem. Borrow from 1 from 13 and add 1 ($\frac{10}{10}$) to $\frac{4}{10}$.

$$\begin{array}{r} \overset{12}{\cancel{13}}\frac{4}{10}+\frac{10}{10} \\ -7\frac{5}{10} \\ \hline \end{array} \quad \longrightarrow \quad \begin{array}{r} 12\frac{14}{10} \\ -7\frac{5}{10} \\ \hline 5\frac{9}{10} \end{array}$$

Example: Catherine has a canister filled with $5\frac{1}{2}$ cups of flour. She used $1\frac{3}{4}$ cups of flour to bake a cake. How much flour is left in the canister?

Subtract $5\frac{1}{2} - 1\frac{3}{4}$.

Step 1. Find a common denominator: The common denominator for 2 and 4 is 4.

Step 2. Make equivalent fractions using 4 as the common denominator.

$$5\frac{2}{4}-1\frac{3}{4}$$

Step 3. When subtracting the numerators, it is not possible to take 3 from 2, therefore borrow. It may be easier to follow the borrowing if the problem is rewritten vertically .

Math 6 Part B-NOTES – Fraction Operations

$$\begin{array}{r} {\color{red}4}\ \not{5}\ \frac{2}{4} {\color{red}+\frac{4}{4}} \\ -1\ \frac{3}{4} \\ \hline \end{array} \longrightarrow \begin{array}{r} 4\ \frac{6}{4} \\ -1\ \frac{3}{4} \\ \hline 3\frac{3}{4} \end{array}$$

There are $3\frac{3}{4}$ cups of flour left in the canister.

Review/Prep for 6.NS — Multiplying Fractions

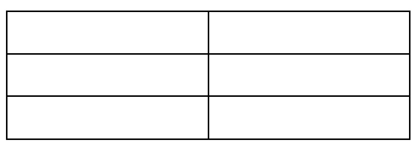
Before we learn how to multiply fractions, let's revisit the concept of multiplication using whole numbers. When I have an example like $3 \times 2 =$ we can model that in several ways.

Since it can be read "three groups of 2" we can use the **repeated addition** model.

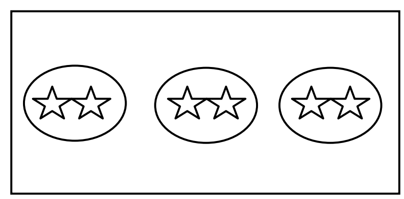
$$\begin{array}{r} 2 \\ 2 \\ \underline{+2} \\ 6 \end{array}$$

We can show a **rectangular array**.

2

3 

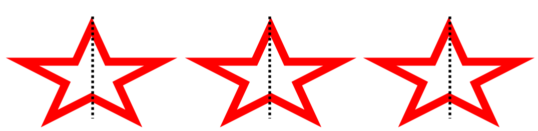
Show 3 **groups** of 2 items



Each representation shows a total of 6. Mathematically, we say $3 \times 2 = 6$.

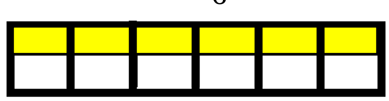
Multiplication is defined as repeated addition. That won't change because we are using a different number set. In other words, to multiply fractions, I could also do repeated addition.

Example: $6 \times \frac{1}{2} \rightarrow \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{6}{2} = 3$ repeated addition



Groups - six halves

6

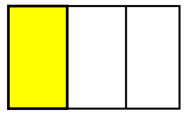
$\frac{1}{2}$ 

array

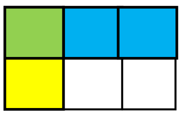
Math 6 Part B-NOTES – Fraction Operations

***Example**:* $\frac{1}{2}\times\frac{1}{3}=$

A visual representation of multiplication of fractions would look like the following. In this example, we want half of one third. So the visual begins with the one third we have, shaded yellow.



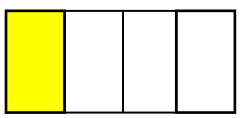
Since we want only half of it, we cut the visual in half and shade it blue. (Remember yellow mixed with blue makes green.)



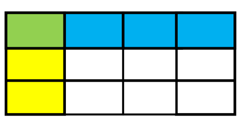
Now we see 1 part double shaded and 6 total parts or $\frac{1}{6}$.

Example: $\frac{1}{3}\times\frac{1}{4}=$

In this example, we want to take one third of $\frac{1}{4}$**.** So the visual begins with the one fourth shaded yellow.



We want one third of it. We cut it into thirds horizontally and shade the 1 part doubly shaded (green) is one part out of 12 total.



So $\frac{1}{3}\times\frac{1}{4}=\frac{1}{12}$.

Multiplying fractions is pretty straight-forward. Here is the algorithm for multiplying fractions:

Algorithm for Multiplying Fractions:

Step 1. Make sure you have proper or improper fractions
Step 2. Cancel if possible
Step 3. Multiply numerators
Step 4. Multiply denominators
Step 5. Reduce

Example: $\frac{3}{7}\bullet\frac{1}{2}=\frac{3}{14}$

Math 6 Part B-NOTES – Fraction Operations

It is essential to get students to find common factors for any pair of numerator and denominator they can BEFORE multiplying. Start slowly but build until all are convinced the numbers get too big to handle if they don't cancel .

> I solved this problem in steps so you could see my "thinking".
> Note: The steps could be done in a different order, with different pairs of numbers, or all in one step; BUT the product must be the same.

Example: $\frac{2}{5}\bullet\frac{3}{8}=\frac{\overset{1}{\cancel{2}}}{5}\bullet\frac{3}{\underset{4}{\cancel{8}}}=\frac{3}{20}$

Example: $\frac{2}{3}\bullet\frac{5}{6}\bullet\frac{9}{10}=\frac{\overset{1}{\cancel{2}}}{3}\bullet\frac{5}{\underset{3}{\cancel{6}}}\bullet\frac{9}{10}=\frac{\overset{1}{\cancel{2}}}{3}\bullet\frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}}\bullet\frac{9}{\underset{2}{\cancel{10}}}=\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}}\bullet\frac{\overset{1}{\cancel{5}}}{\underset{3}{\cancel{6}}}\bullet\frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{10}}}=\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}}\bullet\frac{\overset{1}{\cancel{5}}}{\underset{\underset{1}{\cancel{3}}}{\cancel{6}}}\bullet\frac{\overset{\overset{1}{\cancel{3}}}{\cancel{9}}}{\underset{2}{\cancel{10}}}=\frac{1}{2}$

Example: Multiply $\frac{32}{45}\bullet\frac{55}{96}=\frac{\overset{1}{\cancel{32}}}{\underset{9}{\cancel{45}}}\bullet\frac{\overset{11}{\cancel{55}}}{\underset{3}{\cancel{96}}}=\frac{11}{27}$

Example: Multiply $3\frac{1}{2}\times\frac{4}{5}$.

Step 1. Make sure that the numbers are a proper or improper fractions.

Change $3\frac{1}{2}$ to an improper fraction and rewrite as follows: $\frac{7}{2}\times\frac{4}{5}$

Step 2. Cancel if possible: $\frac{7}{\underset{1}{\cancel{2}}}\times\frac{\overset{2}{\cancel{4}}}{5}=\frac{7}{1}\times\frac{2}{5}$

Step 3 and Step 4. Multiply numerators and denominators: $\frac{7}{1}\times\frac{2}{5}=\frac{14}{5}$

Step 5. Reduce: $\frac{14}{5}=2\frac{4}{5}$

Example: $3\frac{3}{5}\bullet2\frac{2}{9}\longrightarrow\frac{18}{5}\bullet\frac{20}{9}$ Make sure you have proper or improper fractions.

$\frac{\overset{2}{\cancel{18}}}{\underset{1}{\cancel{5}}}\bullet\frac{\overset{4}{\cancel{20}}}{\underset{1}{\cancel{9}}}$ Cancel 18 and 9 by common factor of 9.

Cancel 20 and 5 by common factor of 5.

$\frac{2}{1}\bullet\frac{4}{1}=\frac{8}{1}=8$ Multiply numerators, multiply denominators, simplify.

Math 6 Part B-NOTES – Fraction Operations

Multiply the following fractions:

1) $\frac{3}{4} \times \frac{2}{5}$ 2) $\frac{1}{2} \times 4\frac{3}{4}$ 3) $2\frac{1}{2} \times 3\frac{1}{4}$

Example: There are 105 members of the high school band. Of these members, $\frac{1}{5}$ play percussion instruments. How many members play percussion?

Solution: 21 percussion players

Estimating Products

Round each number to the nearest integer, and then simplify.

Example: $\frac{3}{4} \bullet 2\frac{5}{6} \approx 1 \bullet 3 \approx 3$ is our estimate

Example: $7\frac{5}{6} \bullet 11\frac{1}{5} \approx 8 \bullet 11 \approx 88$ is our estimate

Dividing Fractions

NEW CCSS 6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

Before we learn how to divide fractions, let's revisit the concept of division using whole numbers. When I ask, how many 2's are there in 8, I can write that mathematically three ways.

$2\overline{)8}$ $\frac{8}{2}$ $8 \div 2$

To find out how many 2's there are in 8, we will use the subtraction model:

$$\begin{array}{r} 8 \\ \underline{-2} \\ 6 \\ \underline{-2} \\ 4 \\ \underline{-2} \\ 2 \\ \underline{-2} \\ 0 \end{array}$$

Now, how many times did we subtract 2? Count them: there are 4 subtractions. So there are 4 twos in eight.

Mathematically, we say 8 ÷ 2 = 4.

Division is defined as repeated subtraction. That won't change because we are using a different number set. In other words, to divide fractions, I could also do repeated subtraction.

Math 6 Part B-NOTES – Fraction Operations

Example: $1\frac{1}{2} \div \frac{1}{4}$

Another way to look at this problem is using your experiences with money. How many quarters are there in $1.50? Using repeated subtraction we have:

$$1\frac{1}{2} = 1\frac{2}{4} \quad - \frac{1}{4} \quad = 1\frac{1}{4} \quad - \frac{1}{4} \quad = 1$$

$$1 = \frac{4}{4} \quad - \frac{1}{4} \quad = \frac{3}{4} \quad - \frac{1}{4} \quad = \frac{2}{4}$$

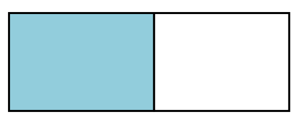
$$\frac{2}{4} \quad - \frac{1}{4} \quad = \frac{1}{4} \quad - \frac{1}{4} \quad = 0$$

How many times did we subtract $\frac{1}{4}$? Six. But this took a lot of time and space.

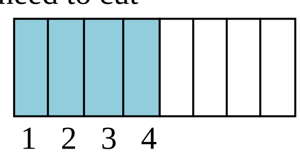
A visual representation of division of fractions would look like the following.

Example: $\frac{1}{2} \div \frac{1}{8} =$

We have $\frac{1}{2}$. Representing that would be

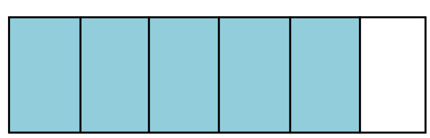


Since the question we need to answer is how many $\frac{1}{8}$'s are there in $\frac{1}{2}$, we need to cut this entire diagram into eighths. Then count each of the shaded one-eighths. As you can see there are four. So $\frac{1}{2} \div \frac{1}{8} = 4$.



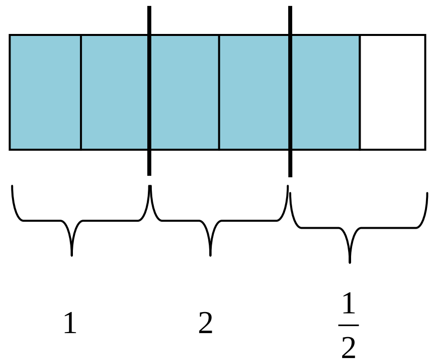
Example: $\frac{5}{6} \div \frac{1}{3} =$

We have $\frac{5}{6}$. Representing that would be



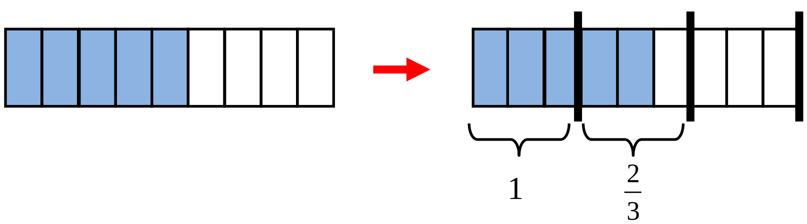
Math 6 Part B-NOTES – Fraction Operations

Since the question we need to answer is how many $\frac{1}{3}'s$ are there in $\frac{5}{6}$, we need to use the cuts for thirds only. Then count each of the one thirds.



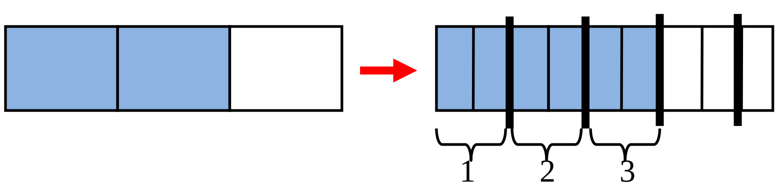
As you can see there are $2\frac{1}{2}$. So $\frac{5}{6} \div \frac{1}{3} = 2\frac{1}{2}$.

Example: $\frac{5}{9} \div \frac{1}{3} =$



So $\frac{5}{9} \div \frac{1}{3} = 1\frac{2}{3}$

Example: $\frac{2}{3} \div \frac{2}{9} =$



So $\frac{2}{3} \div \frac{2}{9} = 3$

Be careful to choose division examples that are easy to represent in visual form.

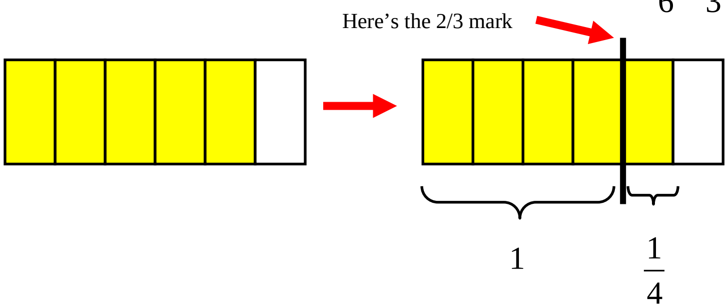
Examples:

$\frac{1}{2} \div \frac{1}{4} =$ $\frac{3}{4} \div \frac{1}{8} =$ $\frac{1}{2} \div \frac{3}{5} =$ $\frac{4}{9} \div \frac{1}{3} =$

$\frac{2}{3} \div \frac{1}{12} =$ $\frac{5}{8} \div \frac{1}{4} =$ $\frac{7}{8} \div \frac{1}{4} =$ $2\frac{1}{2} \div \frac{1}{4} =$

Math 6 Part B-NOTES – Fraction Operations

Example: *A very challenging example might be* $\frac{5}{6} \div \frac{2}{3} =$



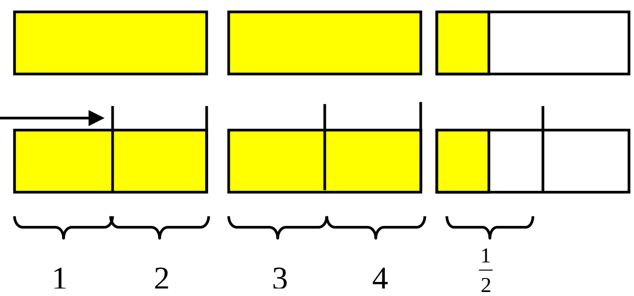
So you might wonder where did the answer $1\frac{1}{4}$ come from?
Visually you can see the one whole and the one piece left over. Remember the one whole was made up of 4 equal sized pieces so 1 piece shaded out of 4 for the whole gives you $1\frac{1}{4}$.

Example: *Draw a model to show* $2\frac{1}{4} \div \frac{1}{2} = 4\frac{1}{2}$.

Begin with a model of $2\frac{1}{4}$.

Show cuts for halves.

Count the number of # of halves shaded.



Because some enjoy playing with numbers, they found a quicker way of dividing fractions. They did this by looking at fractions that were to be divided and they noticed a pattern. And here is what they determined.

Algorithm for Dividing Fractions and Mixed Numbers

1. **Make sure you have proper or improper fractions.**
2. **Change** $\div$ to $\cdot$ **and invert the divisor (2nd number).** .
   (Keep-Change-Flip)
3. **Cancel, if possible.**
4. **Multiply numerators.**
5. **Multiply denominators.**
6. **Simplify if needed.**

The very simple reason we tip the divisor upside-down (use the reciprocal), then multiply (for division of fractions), is because it works. And it works faster than if we did repeated subtractions, not to mention it takes less time and less space.

Math 6 Part B-NOTES – Fraction Operations

Example: $\frac{3}{4} \div \frac{2}{5} \longrightarrow \frac{3}{4} \bullet \frac{5}{2} \longrightarrow \frac{15}{8} = 1\frac{7}{8}$

(Invert the divisor.)

Multiply numerators and denominators, and simplify.

Let's look at this same problem in the format of complex fractions. In this format we need to get the denominator to be equal to 1. To do that we multiply the top and bottom by $\frac{5}{2}$, remember $\left(\frac{\frac{5}{2}}{\frac{5}{2}}\right)$ this is another name for 1 whole. This is a much better explanation of why we invert the divisor and multiply.

Example: $\frac{3}{4} \div \frac{2}{5} = \frac{\frac{3}{4}}{\frac{2}{5}} = \frac{\frac{3}{4} \bullet \frac{5}{2}}{\frac{2}{5} \bullet \frac{5}{2}} = \frac{\frac{15}{8}}{1} = \frac{15}{8} \text{ or } 1\frac{7}{8}$

Example: $3\frac{1}{3} \div \frac{4}{9} \longrightarrow \frac{10}{3} \div \frac{4}{9}$ Make sure you have proper or improper fractions.

$\frac{10}{3} \bullet \frac{9}{4}$ Change the divide sign to multiply and invert the divisor.

$\frac{\overset{5}{\cancel{10}}}{\underset{1}{\cancel{3}}} \bullet \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{4}}}$ Cancel 10 and 4 by 2, and cancel 9 and 3 by 3.

$\frac{5}{1} \bullet \frac{3}{2} = \frac{15}{2}$ Multiply numerators; the multiply the denominators.

$\frac{15}{2} = 7\frac{1}{2}$ Simplify.

Another way to explain the above algorithm is to show students that division is simply multiplication by the reciprocal. Two numbers are **reciprocals** if their product is 1.

Let's demonstrate this concept by using simple whole numbers:

Math 6 Part B-NOTES – Fraction Operations

Example: Divide 12 by 4: In other words, $12 \div 4 = ?$

Students will know that $12 \div 4 = 3$. Show them that you get the same answer if you do $12 \times \frac{1}{4} = \frac{12}{4} = 3$.

Here are more examples:

$$10 \div 5 = 10 \times \frac{1}{5} = \frac{10}{5} = 2$$

$$24 \div 8 = 24 \times \frac{1}{8} = \frac{24}{8} = 3$$

$$6 \div 12 = 6 \times \frac{1}{12} = \frac{6}{12} = \frac{1}{2}$$

Lead students to notice that dividing is the same as multiplying by the reciprocal of the divisor!

Example: Divide $3\frac{2}{5} \div 2\frac{9}{10}$.

Step 1. Make sure you have fractions. Change both mixed numbers to improper fractions.

$$3\frac{2}{5} \div 2\frac{9}{10} = \frac{17}{5} \div \frac{29}{10}$$

Step 2. Keep Change Flip:

$$\frac{17}{5} \div \frac{29}{10} = \frac{17}{5} \times \frac{10}{29}$$

Keep Change Flip

Step 3. Multiply, reduce if possible:

$$\frac{17}{\cancel{5}_{1}} \times \frac{\cancel{10}^{2}}{29} = \frac{34}{29} = 1\frac{5}{29}$$

Estimating Quotients

Round each number to the nearest integer that is a compatible number, and then simplify. Remind students that we estimate problems so we can easily do the math mentally.

Example: $13\frac{1}{3} \div 2\frac{4}{5} \approx 12 \div 3 \approx 4$ is our estimate

Example: $17\frac{5}{6} \div (2\frac{4}{5}) \approx 18 \div 3 \approx 6$ is our estimate

CCSS 6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

Example: How much chocolate will each person get if 3 people share $\frac{1}{2}$ lb. of chocolate equally? Use a visual model to show your answer. *Solution:* $\frac{1}{6}$ *lb*

Example: How many $\frac{3}{4}$-cup servings are in $\frac{2}{3}$ of a cup of yogurt?

Solution: $\frac{8}{9}$ *of a cup*

Example: How wide is a rectangular strip of land with length $\frac{3}{4}$ mi and area $\frac{1}{2}$ square mi? *Solution:* $\frac{2}{3}$ *mi*

Example: Cedric made $\frac{7}{8}$ cup of soup. How many $\frac{1}{4}$ cups servings does he have? Draw and label a diagram to support your answer. *Solution:* $3\frac{1}{2}$ *servings*

Example: Manny has $\frac{1}{2}$ yard of fabric to make book covers. Each book is made from $\frac{1}{8}$ yard of fabric. How many book covers can Manny make? Draw and label a diagram to support your answer. *Solution: 4 book covers*

Example: Tyler has a piece of ribbon that is $\frac{3}{4}$ yard long. How many $\frac{1}{8}$ yard pieces can he cut? Draw and label a diagram to support your answer.
Solution: 6 pieces

Math 6 Part B-NOTES – Fraction Operations

Example: Josie has $24\frac{1}{2}$ pounds of birdseed. She puts $1\frac{3}{4}$ pounds of seed in her feeders each day. How many days will she be able to fill her feeders?

Solution: 14 days

Example: A concrete-mixing truck can mix $4\frac{1}{2}$ tons of concrete at a time. How many truckloads are needed for the foundations of a building that will use 150 tons?

Solution: $33\frac{1}{3}$ *truckloads*

Example: Pam bought 2 packages of ground beef at a local supermarket. One package was $2\frac{1}{2}$ pounds, and the other package was $3\frac{1}{8}$ pounds. Ann divided the total amount of beef into 5 equal packages for the freezer. How many pounds were in each package?

Solution: $1\frac{1}{8}$ pounds of beef

Example: A grocery chain released 300 balloons at an outdoor celebration of the opening of a new store. Of these balloons, $\frac{3}{4}$ were yellow. How many balloons were yellow?

Example: Ken Jackson worked $12\frac{1}{2}$ hours last week and earned $150. What was his hourly rate of pay?

Solution: $12 per hour

Example: A gasoline tank with a capacity of 15 gallons is $\frac{3}{4}$ full. How many gallons will it take to fill the tank?

Solution: $3\frac{3}{4}$ gallons of gasoline

Example: A town raised $\frac{5}{8}$ of the $32 million it needs to build a new library. How much more is it hoping to raise?

Solution: $12 million

ADD

Math 6 Part B-NOTES – Fraction Operations

Example Item 2 (Grade 6):
Primary Target 2D (Content Domain NS), Secondary Target 1C (CCSS 6.NS.1)

Day of Week	Laps Run
Monday	$1\frac{1}{4}$
Tuesday	$1\frac{3}{4}$
Wednesday	$1\frac{5}{8}$
Thursday	$2\frac{1}{2}$

Bill wants to run a total of 4000 meters in 5 days. The table shows how far he runs each day for 4 days.

- Each lap is 400 meters.

Enter the number of laps Bill should run on Friday so his total for the 5 days is exactly 4000 meters.

Rubric: (1 point) The student enters the correct number in the box (e.g., $2\frac{7}{8}$).

Response Type: Equation/Numeric