Math 6 Part A -NOTES – Fractions

Prep for 6.NS

Rules of Divisibility

It is beneficial for students to be familiar with rules of divisibility. When familiar with these rules, it allows students to be focused more on the concept and skill of the lesson rather than to be bogged with the arithmetic. Chances are students already know if a number is divisible by 2, 5 or 10. For instance, if asked to determine if a number is divisible by two, students can tell you that it has to be even. Divisibility rules for 10 and 5 are also familiar to most students.

Before starting to work with rational numbers it is imperative that students have some basic understanding of fractions. Everyone will benefit if you take a few minutes to review or learn the Rules of Divisibility. Employing these rules will make life a lot easier in the future; not to mention it will save you time and allow you to do problems very quickly when others are experiencing difficulty.

In general if you were asked if any given number was divisible by a second number you could divide them and if the remainder is 0, we would say yes the first number is divisible by the second number.

For example, if asked if 132 is divisible by 3 we could divide to see the remainder.

$$\begin{array}{r} 44 \\ 3\overline{)132} \\ \underline{12} \\ 12 \\ \underline{-12} \\ 0 \end{array}$$

Since the remainder is 0, we say that 132 is divisible by 3.

We could say 3 is a factor of 132 and 132 is a multiple of 3.

What if you were asked if 197 was divisible by 14? Again we could divide and see the remainder.

$$\begin{array}{r} 14 \\ 14\overline{)197} \\ \underline{-14} \\ 57 \\ \underline{-56} \\ 1 \end{array}$$

Since the remainder is not 0, we say that 197 is NOT divisible by 14.

We could say 14 is NOT a factor of 197 and 197 is NOT a multiple of 14.

To be quite frank, you already know some of them. For instance, if I asked you to determine if a number is divisible by two, would you know the answer? Sure you do, if the number is even, then it's divisible by two. Can you tell if a number is divisible by 10? How about 5?

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Because you are familiar with those numbers, chances are you know if a number is divisible by 2, 5 or 10. We could look at more numbers to see if any other patterns exist that would let you know what they are divisible by, but we don't have that much time or space. So, if you don't mind, I'm just going to share some rules of divisibility with you first, and then I will give the examples.

Rules of Divisibility

A number is divisible:

- by 2, if the number ends in 0, 2, 4, 6 or 8. In other words the number must be even.
- by 5, if the number ends in 0 or 5.
- by 10, if the number ends in 0.
  Notice here we are suggesting you teach these rules grouped together since they all involve just "looking at the one's digit".

- by 3, if the sum of the digits is a multiple of 3.
- by 9, if the sum of the digits is a multiple of 9.
- by 6, if the number ends in 0, 2, 4, 6 or 8 AND the sum of the digits is a multiple of 3.
  (In other words, if the number is divisible by 2 and 3, then it is by 6.)
  Notice here we are suggesting you teach these 3 rules grouped together since they are similar – they all involve "finding a sum of the digits".

- by 4, if the last 2 digits of the number is divisible by 4. (Remember $4 = 2^2$)
- by 8, if the last 3 digits of the number is divisible by 8. (Remember $8 = 2^3$)
  Again we grouped these similar rules together.

Others that are important and quick to teach are the rules for 20, 25, 50 and 100.

- by 20, if the number ends in 00, 20, 40, 60 or 80.
- by 25, if the number ends in 00, 25, 50 or 75.
- by 50, if the number ends in 00 or 50.
- by 100, if the number ends in 00.

Now come the examples…

Divisibility by 3

Example: Is 111 divisible by 3? (We could write this as 111;3 which is read "Is 111 divisible by 3?".)

The rule says to find the sum of the digits and if that sum is a multiple of 3 the number is divisible by 3. 1+1+1=3, Since $3 \div 3 = 1$ with a 0 remainder, 3 is a multiple of 3, so the number 111 is divisible by 3.

Example: 147;3 (Read, "Is 147 divisible by 3?)

Adding 1, 4, and 7 we get 12. Is 12 divisible by 3? (Is the remainder 0?) If that answer is yes, that means 471 is divisible by 3. If you don't believe it, try dividing 471 by 3.

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Example: 1,316;3 (Read, "Is 1,316 divisible by 3?)

The sum is 1+3+1+6=11. Is 11 a multiple of 3? (3, 6, 9, 12, 15, 18, 21, 24, 27, 30, …)
No, then 1,316 is NOT divisible by 3.

<u>Divisibility by 9</u>

Example: Is 111 divisible by 9? (We could write this as 111;9 which is read "Is 111 divisible by 9?".)

The rule says to find the sum of the digits and if that sum is a multiple of 9 the number is divisible by 9. 1+1+1=3, 3 is not a multiple of 9, so the number 111 is NOT divisible by 9.

Example: 5,247;9 (Read, "Is 5,247 divisible by 9?)

5+2+4+7=18 18 is a multiple of 9 so yes, 5,247 is divisible by 9.

Example: 254,145;9 (Read, "Is 254,145 divisible by 9?)

2+5+4+1+4+5=21 21 is NOT a multiple of 9, so the number is NOT divisible by 9.

<u>Divisibility by 6</u>

Example: Is 21,306 divisible by 6? (We could write this as 21,306;6 which is read "Is 21,306 divisible by 6?".)

The rule has two parts - the number ends in 0, 2, 4, 6 or 8 yes
<u>AND</u>
- the sum of the digits is a multiple of 3. 2+1+3+0+6=12 yes

So, yes, 21,306 is divisible by both 2 and 3 so it is divisible by 6.

Example: 746; 6? (This is read "Is 746 divisible by 6?".)

The rule has two parts - the number ends in 0, 2, 4, 6 or 8 yes
<u>AND</u>
- the sum of the digits is a multiple of 3. 7+4+6=17 no

So 746 is divisible by 2 but NOT by3 so it is NOT divisible by 6.

Example: 48,761;6 (This is read "Is 21,306 divisible by 6?".)

The rule has two parts - the number ends in 0, 2, 4, 6 or 8 no
<u>AND</u>
- the sum of the digits is a multiple of 3.

Since it is not divisible by 2, it is NOT divisible by 6.

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Divisibility by 4

Example: Is 12,316 divisible by 4? (We could write this as 12,316;4 which is read "Is 12,316 divisible by 4?".)

Using the rule for 4, we look at the last 2 digits. In this example te last two digits are 16, since 16 is divisible by 4. Then 12,316 is divisible by 4.

Example: 87,502;4 (This is read "Is 12,316divisible by 4?".)

We look at the last 2 digits, in this case 02, which is NOT divisible by 4.

Example: 961,500;4 (This is read "Is 961,500divisible by 4?".)
divisible by 4

Using the rule for 4, we look at the last 2 digits. In this example they are 00, since 4 goes into 00 zero times it is divisible by 4.

Divisibility by 8

Example: Is 456,040 divisible by 8? (We could write this as 456,040;8 which is read "Is 456,040 divisible by 8?".)

Using the rule for 8, we look at the last 3 digits. In this example the last 3 digits are 040, since it is divisible by 8, then 456,040 is divisible by 8.

Example: 85,600;8 (This is read "Is 85,600 divisible by 8?".)

We look at the last 3 digits, in this case 600, and we would probably have to divide (still easier to divide $600 \div 8$ compared to $85{,}600 \div 8$). Once we divide we see $600 \div 8 = 75$ with no remainder. So yes 85,600 is divisible by 8.

Example: 612,513;8 (This is read "Is 612,513 divisible by 8?".)

Using the rule for 8, we look at the last 3 digits. In this example 513, since it is not even an even number we know it is NOT divisible by 8.

A fun rule for testing divisibility is divisibility by 11. Use it to challenge some of your students. The rule is "wordy" but the process is simple once learned.

- by11, if the sum of the 1st, 3rd, 5th, …digits and the sum of the 2nd, 4th, 6th, …digits are equal <u>OR</u> differ by a multiple of 11.

Divisibility by 11

Example: Is 186,021 divisible by 11?

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1. the sum of the 1st, 3rd, 5th, …digits 1+6+2=9
2. the sum of the 2nd, 4th, 6th, …digits 8+0+1=9
3. the sums are equal so ……186,021 is divisible by 11.

Example: Is 806,124 divisible by 11?

1. the sum of the 1st, 3rd, 5th, …digits 8+6+2=16
2. the sum of the 2nd, 4th, 6th, …digits 0+1+4=5
3. the sums are not equal so find the difference between the sums 16-5=11
4. since 11 is a multiple of 11 (11, 22, 33, 44, 55, 66, 77, 88, 99, …)
   806,124 is divisible by 11.

Example: Is 123,456 divisible by 11?

1. the sum of the 1st, 3rd, 5th, …digits 1+3+5=9
2. the sum of the 2nd, 4th, 6th, …digits 2+4+6=12
3. the sums are not equal so find the difference between the sums 12-9=3
4. since 3 is NOT a multiple of 11 (11, 22, 33, 44, 55, 66, 77, 88, 99, …)
   then, 123,456 is NOT divisible by 11.

Example: Using the rules of divisibility, determine if the following numbers are divisible by 2, 3, 4, 5, 6, 8, 9 or 10. Place a check in each box that the number IS divisible by.

#	By 2	By 3	By 4	By 5	By 6	By 8	By 9	By 10
756								
9,045								
16,701								
86,400								
720,000								

Example: Write a 5 digit number that is divisible by 3 and 4.

Example: Write the smallest 6 digit number that is divisible by 3and 5.

Example: What single digit value or values might the "?" represent in each of the following?

a. 12,30? ; 4 and 6 b. 145,6?0; 3 and 5 c. 6,1?2: 4 and 9

Solutions: ?=0 ?=2, 5 or 8 ?=9

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Examples: True or False?

All even numbers are divisible by 2. True
All odd numbers are divisible by 3. False
Some even numbers are divisible by 5. True
Every number divisible by 2 is also divisible by 4. False
Every number divisible by 8 is also divisible by 4. True
Every number divisible by 9 is also divisible by 3. True

Examples: If a number is divisible by 3 and 5, it must be a multiple of what other number?
15

If a number is divisible by 12, then it is divisible by what other numbers?
2, 3, 4, and 6

Find a number that is divisible by 2, 3, 4, 5 and 6 but not by 9.
Possible answer: 120

Prep for 6.NS Primes and Composites

Prime Number: A number that has **only two** factors - one and itself.

Examples: We could say 2, 3, 5, 7 and 11 have only two factors - 1 and itself - or we can say they are only divisible by 1 and itself.

Composite number: A number that is has **more than two** factors.

Examples: 4, 6, 8, 9, and 10 all have 3 or more factors OR we can say they are divisible by three or more numbers. For instance, 8 is divisible by 1, 2, 4 and 8.

The numbers 0 and 1 are neither prime nor composite numbers.

As students begin to identify primes it may be helpful to use a Sieve of Eratosthenes. Really all it is, is a 100's chart like below.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

As you begin to talk about whether a number is prime or composite always **go back to the definitions to verify.**

1. Starting with the number 1, ask students how many factors the number 1 has. Since it has only 1 factor it is neither prime nor composite. Choose a color – I will use aqua - and shade in the one's box.
2. Next we see the number 2. Ask students how many factors the number 2 has. They should state that 2 has only 2 factors – 1 and itself. That makes it prime and we will color the primes boxes yellow.

   Next we think about all the multiples of 2. Since every multiple of 2 will have at least 3 factors - 1, itself, and at least 2 (since they are even), the remaining even numbers in the chart are composites.

   For example: 4 has $1 \bullet 4$ and $2 \bullet 2$, so 3 factors.
   6 has $1 \bullet 6$ and $2 \bullet 3$, so 4 factors.

   Students may skip count 4, 6, 8, 10, 12, … remind them these are multiples of 2.
   We begin to color all the multiples of 2 – I will use gray. Your chart should look like the following.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Next we see the number 3. Ask students how many factors the number 3 has. They should state the number 3 has only 2 factors – 1 and itself. That makes it prime and we will color the 3 box yellow.

Once again we will see that the multiples of 3 have more than 2 factors.

For example: 3x2 or 6 has 1∙6 and 2∙3, so 4 factors.
3x3 or 9 has 1∙9 and 3∙3, so 3 factors. Etc.

Students may skip count, 6, 9, 12, 15, … remind them these are multiples of 3.

We begin to color all the multiples of 3 in gray. Your chart should look like the following. (Students should notice every other multiple is already shaded….WHY?)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

3. 5 is the next number unshaded in the chart. Ask students how many factors the number 5 has. They should state that 5 has only 2 factors – 1 and itself. That makes it prime and we will color the 5 box yellow.

Again, any multiple of 5 will have more than 2 factors.

For example: 5x2 or 10 has 1∙10 and 2∙5, so 4 factors.
5x3 or 15 has 1∙15 and 3∙5, so 4 factors. Etc.

Teacher and students may skip count, 10, 15, 20, 25, … remind them these are multiples of 5 and begin to color all the multiples of 5 gray. Your chart should look like the following. (Students should notice every other multiple is already shaded….WHY?)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

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4. 7 is the next unshaded number in the chart. Ask students how many factors the number 7 has. They should state that 7 has only 2 factors – 1 and itself. That makes it prime and we will color the 7 box yellow.

   Again any multiple of 7 will have more than 2 factors.
   For example: 7x2 or 14 has 1 • 14 and 2 • 7, so 4 factors.
   7x3 or 21 has 1 • 21 and 3 • 7, so 4 factors. Etc.

   Teacher and students may skip count, 14, 21, 28, 35, … remind them these are multiples of 7 and begin to color all the multiples of 7 gray. Your chart should look like the following. (Note: Only 49, 77 and 91 were unshaded and need to be shaded.)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

5. 11 is the next unshaded number in the chart. Ask students how many factors the number 11 has. They should state that 11 has only 2 factors – 1 and itself. That makes it prime and we will color the 11 box yellow.

   Again any multiple of 11 will have more than 2 factors. They are 22, 33, 44, 55, 66, 77, 88, and 99 but they are all already shaded.

6. 13 is the next number we should check – it is prime. Its multiple would be composite and checking those 26, 39, 52, 65, 78, and 91 they are all already shaded too.

   At this point we find the sieve did its job - it filtered out numbers that are composite or not prime. Shade the remaining unshaded boxes in yellow.

   Now we see all the prime numbers less than 100 highlighted in yellow.

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1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Take a moment to ask students what they notice. They should be able to state info like:

- 1 is not prime or composite.
- 2 is the ONLY even prime number.
- 5 is the ONLY prime number ending in a 5.
- 97 is the only number in the 90's that is prime.
- There are 25 prime numbers less than 100.

- Patterns - 23 and 29 and (thirty away) 53 and 59 and (thirty away) 83 and 89
- Patterns - 31 and 37 and (thirty away) 61 and 67 and (thirty away) 97 - not 91 since $7 \bullet 13 = 91$
- Patterns – 41, 43 and 47 and (thirty away) 71,73, and 79

Most students will know/learn the primes less than 10 or 20, challenge them to extend this knowledge further. Let them talk with a shoulder buddy or pair/share with a close neighbor and have one student state the primes less than 100 to the other. Then reverse roles. Have a contest to see who can name all 25 primes less than 100.

Remember to point out to students when they "think" a number is prime that is not, e.g. 51, that 51 is not prime because 5+1=6 so it is divisible by 3 which tells them it has more than 2 factors. Use the knowledge they learned in divisibility to support or refute if a number is prime or not.

In addition to identifying if specific numbers are prime or composite be sure to ask questions like:

Examples: Explain why 2 is the only even prime number. In addition to the number 1 and the number itself, all other even numbers have 2 as a factor. So they have MORE than 2 factors.

Explain why the sum of two prime numbers greater than 2 can never be a prime number. Prime numbers greater than 2 are odd numbers. The sum of two odd numbers is an even number. All even numbers have 2 as a factor.

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Examples: True or False?

All odd numbers are prime. False
All primes are odd numbers. False

Prep for 6.NS Prime Factorization

We just learned about prime numbers and we know factors are numbers that are multiplied to find a product. Now we move on to ***prime factorization***: Rewriting a number as a product of prime numbers.

Example: Write the prime factorization of 12.

$12 = 4 \times 3$. That's a product, but 4 is not prime. So rewrite 4 as 2×2.

$$12 = 4 \times 3$$
$$12 = 2 \times 2 \times 3$$
$$12 = 2^2 \times 3$$

Factor Trees

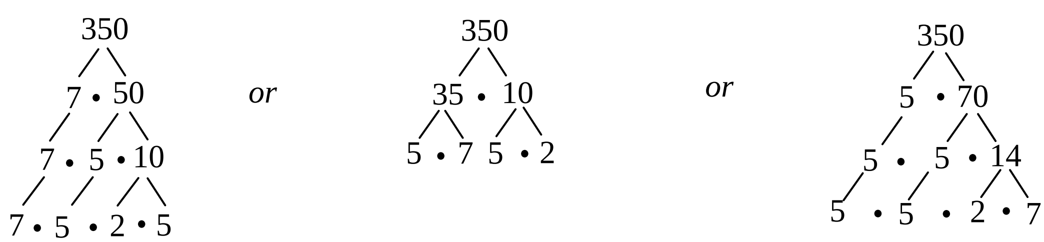
The use of a factor tree or a ladder diagram is a systematic way of finding the prime factorization of larger numbers one step at a time.

A composite number has exactly one prime factorization, however, the factor tree that leads to that prime factorization may look different.

Example: Write the prime factorization of 350. (NOTE: Many other variations of factor trees for 350 could have been shown here.)

Method 1

Using a factor tree



Looking at the factor trees above, the prime factorization for 350 is $2 \times 5 \times 5 \times 7$ or $2 \times 5^2 \times 7$. The standard convention for writing a number as prime factors is to write the factors from smallest to largest. However, it is not wrong if you do not. The preferred way to write 350 as a product of primes is $2 \times 5^2 \times 7$. But it could have been written as $7 \times 5^2 \times 2$.

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Method 2

Using a ladder diagram

Keep dividing by <u>prime</u> factors until the quotient is prime.

3 | 36
3 | 12
2 | 4
2

or

2 | 36
3 | 18
3 | 6
2

The prime factorization of 36 is $2^2 \bullet 3^2$.

CCSS 6.NS.4 ***Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1 – 100 with a common factor as a multiple of a sum of two whole numbers with no common factor.***

Greatest Common Factor (GCF)

Common factor: A number that is a factor of two or more nonzero numbers.
Example: Find common factors of 18 and 24.

Factors of 18: **1**, **2**, **3**, **6**, 9, 18
Factors of 24**:** **1**, **2**, **3**, 4, **6**, 8, 12, 24

1, 2, 3, and **6** are the common factors of 18 and 24.

Greatest Common Factor (GCF): Factors shared by two or more numbers are called *common factors*. The **largest** of the common factors is called the *greatest common factor*.

In the last example, the greatest common factor, GCF, of 18 and 24 is 6.

There are a number of ways of finding the GCF.

<u>STRATEGY 1</u>

To find the GCF**, list all the factors** of each number. Look at the common factors and the largest one is the GCF.

Example: Find the GCF of 24 and 36.

Factors of 24 ⇨ 1, 2, 3, 4, 6, 8, <u>12</u>, 24
Factors of 36 ⇨ 1, 2, 3, 4, 6, 9, <u>12</u>, 18, 36

The GCF is the greatest factor that is in both lists; 12.

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STRATEGY 2

To find the GCF**, write the prime factorization** of each number and identify which factors are in each number.

Example: Find the GCF of 24 and 36.

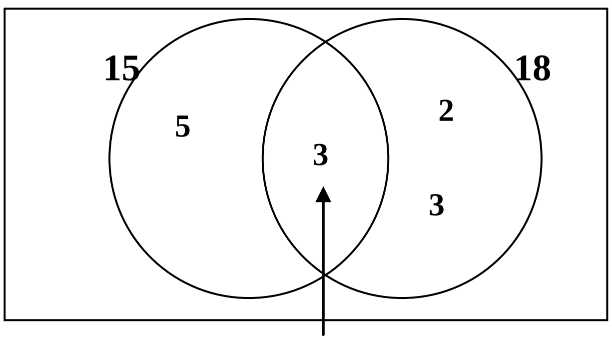
$36 = 2 \times 2 \times 3 \times 3$

$24 = 2 \times 2 \times 2 \times 3$

It might be easier for students to see if the common prime factors are circled.

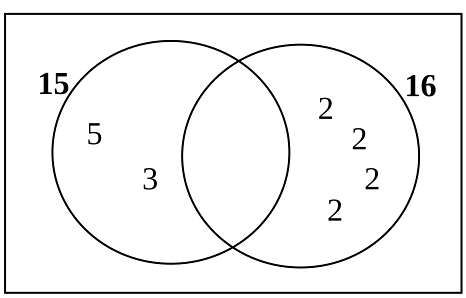
Each number has two factors of 2 and one factor of 3, therefore the GCF = $2 \times 2 \times 3 = 12$.

This can also be shown in a Venn Diagram as shown below.



The GCF is shown in the **intersection** of the diagram. So for 15 and 18, you see only the common factor 3.

Note that in this model if no numbers are listed in the intersection then the **GCF =1** and we could say the numbers are **relatively prime**. For example, if asked to find the GCF for 15 and 16, the diagram would look like this:



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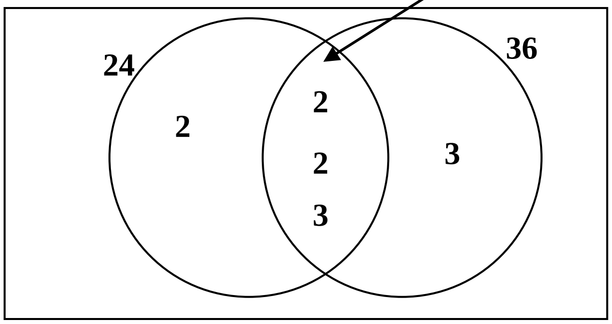
When more than one factor is displayed in the intersection, you multiply the common factors. The GCF = $2 \bullet 2 \bullet 3 = 12$.



STRATEGY 3

To find the GCF, **use a double ladder diagram**.

Begin with a factor that divides into each number. Keep dividing until the numbers have no common factors.

Example: Find the GCF of 15 and 18.

GCF = 3

3	15	18
	5	6

The GCF (15, 18) = 3

Example: Find the GCF of 24 and 36.

GCF = 2, 2, 3

2	24	36
2	12	18
3	6	9
	2	3

The GCF (24, 36) = $2 \bullet 2 \bullet 3 = 12$

Exam ' of 45 and 60.

GCF = 5, 3

5	45	60
3	9	12
	3	4

The GCF (45, 60) = $3 \bullet 5 = 15$

Example: Doris is making goodie bags for her son's birthday party. She has 24 toy cars, 32 pieces of gum and 16 candy bars. She wants each bag to have the same number of each item. What is the greatest number of bags she can make if all the items are used? How many of each item will be in each bag?

Solution: 8 bags with 3 toy cars, 4 pieces of gum and 2 candy bars in each bag

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Example: Mr. Johnson's 6th grade science class has 36 students. There are 24 girls in this class. He wants to divide the class into teams with the same number of boys on each team and the same number of girls on each team. How many different teams could he make if each person was on a team? How many girls were on each team? How many boys were on each team?

Solutions: 12 teams with 2 girls and 1 boy
6 teams with 4 girls and 2 boys
4 teams with 6 girls and 3 boys
3 teams with 8 girls and 4 boys
2 teams with 12 girls and 6 boys

Least Common Multiple (LCM)

The smallest number that is a multiple of two or more numbers is the **least common multiple (LCM)**.

There are several ways of finding the Least Common Multiple (LCM) between two numbers:

STRATEGY 1
To find the LCM, **list the multiples** of each numbers until there is a common number.

Example: Find the LCM of 12 and 10.

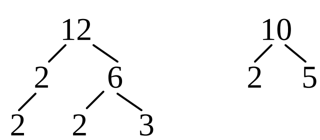
Multiples of 12: 12, 24, 36, 48, 60, 72, …
Multiples of 10: 10, 20, 30, 40, 50, 60

60 is the smallest multiple of both numbers, therefore 60 is the LCM.

STRATEGY 2

To find the LCM, **write the prime factorization** of both numbers. The LCM has to contain ALL the prime factors of both numbers using the highest exponent of each prime.

Example: Find the LCM of 12 and 10.

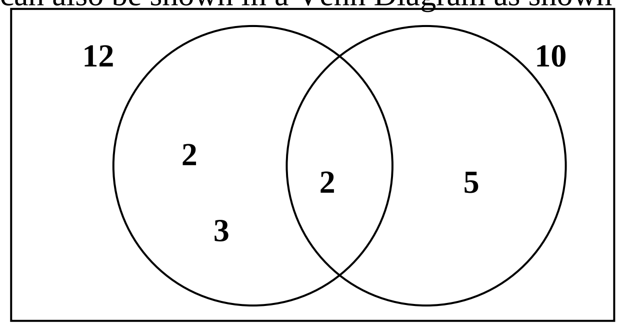


The prime factorization of 12 = $2^2 \bullet 3$
The prime factorization of 10 = $2 \quad \bullet 5$

1. Align the prime factors as shown.
2. Take the **greatest** power of <u>each</u> prime factor.
3. Multiply to get your LCM.

LCM = $2^2 \bullet 3 \bullet 5 = 4 \bullet 3 \bullet 5 = 60$

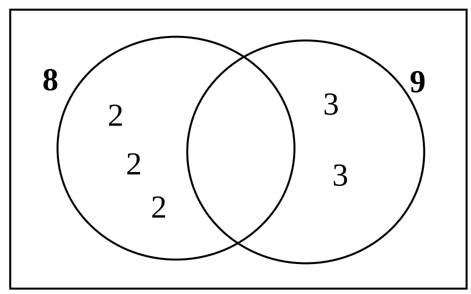
This can also be shown in a Venn Diagram as shown below.



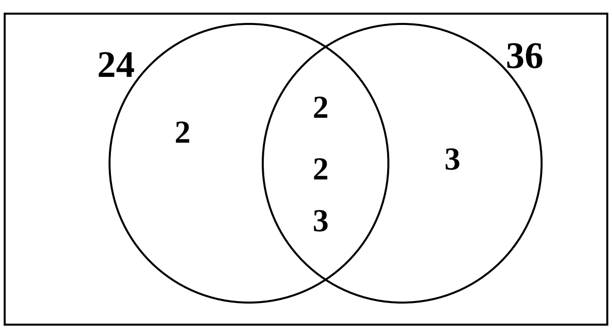
The LCM is shown in the product of the factors in the diagram. So for 12 and 10, you see $2 \cdot 3 \cdot 2 \cdot 5 = 60$.

Remember that in this model if no numbers are listed in the intersection then the **GCF =1** and we could say the numbers in this case, 8 and 9, are **relatively prime**. The fastest way to find the LCM is to simply multiply the two original numbers. So LCM (8, 9) = $8 \cdot 9 = 72$, or again we could multiply all the factors

$$\underbrace{2 \cdot 2 \cdot 2}_{8} \cdot \underbrace{3 \cdot 3}_{9} =$$
$$8 \cdot 9 =$$
$$72$$



When more than one factor is displayed in the intersection, you still multiply all the factors shown. The LCM for 24 and 36 = $2 \bullet 2 \bullet 2 \bullet 3 \bullet 3 = 72$.



STRATEGY 3

Use double ladder diagram. Begin with a factor that divides into each of the given numbers. Keep dividing until the numbers have no common factors.

Example: Find the LCM of 6 and 8.

Math 6 Part A -NOTES – Fractions

LCM =
$$\begin{array}{r|rr} 2 & 6 & 8 \\ \hline & 3 & 4 \end{array}$$

LCM = $2 \bullet 3 \bullet 4 = 24$

Example: Find the LCM of 12 and 8.

LCM =
$$\begin{array}{r|rr} 2 & 12 & 8 \\ 2 & 6 & 4 \\ \hline & 3 & 2 \end{array}$$

LCM = $2 \bullet 2 \bullet 2 \bullet 3 = 24$

Example: Find the LCM of 12 and 10.

LCM =
$$\begin{array}{r|rr} 2 & 12 & 10 \\ \hline & 6 & 5 \end{array}$$

LCM = $2 \bullet 6 \bullet 5 = 60$

<u>STRATEGY 4</u>

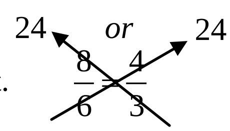
To find the LCM, write the two numbers as a fraction, simplify and cross multiply. The product is the LCM. This method is called the **Simplifying Method**.

Example: Find the LCM of 6 and 8.

Make a fraction using 6 and 8. (Order does not matter.) $\frac{6}{8}$ *or* $\frac{8}{6}$

Simplify that fraction. $\frac{8}{6} = \frac{4}{3}$

Find a cross product.



The LCM is 24.

Example: Find the LCM of 12 and 9.

Make a fraction using 12 and 9. (Order does not matter.) $\frac{12}{9}$ *or* $\frac{9}{12}$

Simplify that fraction. $\frac{12}{9} = \frac{4}{3}$

Math 6 Part A -NOTES – Fractions

Find a cross product. $\frac{12}{9} = \frac{4}{3}$ 36 *or* 36. The LCM is 36.

Example**:** Find the LCM of 12 and 10.

Make a fraction using 12 and 10. (Order does not matter.) $\frac{12}{10}$ *or* $\frac{10}{12}$

Simplify that fraction. $\frac{12}{10} = \frac{6}{5}$

Find a cross product. $\frac{12}{10} = \frac{6}{5}$ 60 *or* 60. The LCM is 60.

Application Problems Using LCM

Example: A granola bar producer puts a half off the price of one bar coupon in every 7^{th} bar and a coupon for one free bar in every 12^{th} bar. How often does the producer put both coupons in a single bar? *Solution: Every 84^{th} bar*

Example: If one car gets 18 miles per gallon and another gets 16 miles per gallon, what is the smallest whole number of gallons each car will need to travel exactly the same distance as the other car? *Solution: 8 gal and 9 gal to go 144 miles*

Example: If two kinds of brick have heights 12 cm and 20 cm respectively, what is the least number of rows of each kind that will have equal heights?

Solution: 5 rows of the 12 cm bricks; 3 rows of the 20 cm bricks

Example: Find two whole numbers whose product is 90 and whose LCM is 30.

Solution: 3 and 30; 6 and 15

Example: The MGM Hotel received a shipment of glasses packed in full cartons of 40 glasses each. The Sands Restaurant received a shipment of glasses packed in full cartons of 24 glasses each. Glasses were also shipped to the Sahara Casino, but in this shipment cartons containing 25 glasses each were used. If the hotel, the restaurant and the casino each received the same number of glasses and if none of them received more than 1,000 glasses, how many glasses were in each shipment? How many cartons were in each shipment?

Solution: 600 glasses;
MGM Hotel received 15 cartons of 40 glasses each
Sands Restaurant received 25 cartons of 24 glasses each
Sahara Casino received 24 cartons of 25 glasses each

Math 6 Part A -NOTES – Fractions

Using the Distributive Property…

to express a sum of two whole numbers 1 – 100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

Example: Rewrite 36+8= using the distributive property.

Solution: Since the GCF for 36 and is 4, we write $36+8=$ $4(9+2)$

Example: Rewrite 14+21= as a product of GCF and a sum. Solution: $14+21=$ $7(2+3)$

Example: Rewrite 27 + 36= using the distributive property. Solution: $27+36=$ $9(3+4)$

Example: Rewrite 84+28= as a product of GCF and a sum. Solution: $84+28=$ $28(3+1)$

OnCore Example: Which gives $44+100$ as a product of GCF and a sum?

A	$5(8+20)$
B	$4(40+96)$
C	$2(22+50)$
D	$4(11+25)$

Solution: D

Math 6 Part A -NOTES – Fractions

Prep for 6.NS

Decimals and Fractions

Batting averages are great examples of how decimals and fractions are related. For instance, if Joe goes up to bat 10 times and gets 3 hits, his batting average is $\frac{3}{10}$ or .300.
If Jane goes up to bat 8 times and gets 3 hits, her batting average is $\frac{3}{8}$ or .375. This example shows students that decimals or fractions can be used to represent the same number.

It is also helpful to have students learn and memorize the following common fraction/decimal conversions. Have students create flashcards for the following and refer to them throughout the year as LTMR activities. If students are able to recognize and convert these common fractions and decimals, it will increase their confidence and math skills.

Common fraction ⟺ decimal conversions:

$\frac{1}{2} = .5$

$\frac{1}{3} = .\overline{3} \text{ or } .33\frac{1}{3}$ $\frac{2}{3} = .\overline{6} \text{ or } .66\frac{2}{3}$

$\frac{1}{4} = .25$ $\frac{3}{4} = .75$ $\frac{1}{8} = .125$

$\frac{1}{5} = .2$ $\frac{2}{5} = .4$ $\frac{3}{5} = .6$ $\frac{4}{5} = .8$

$\frac{1}{8} = .125$ $\frac{2}{8} = .25$ $\frac{3}{8} = .375$ $\frac{4}{8} = .5$ $\frac{5}{8} = .625$ $\frac{6}{8} = .75$ $\frac{7}{8} = .875$

$\frac{1}{9} = .\overline{1} \text{ or } .11\frac{1}{9}$ $\frac{2}{9} = .\overline{2} \text{ or } .22\frac{2}{9}$ $\frac{3}{9} = .\overline{3} \text{ or } .33\frac{1}{3}$ $\frac{4}{9} = .\overline{4} \text{ or } .44\frac{4}{9}$ $\frac{5}{9} = .\overline{5} \text{ or } .55\frac{5}{9}$ $\frac{6}{9} = .\overline{6} \text{ or } .66\frac{2}{3}$ $\frac{7}{9} = .\overline{7} \text{ or } .77\frac{7}{9}$ $\frac{8}{9} = .\overline{8} \text{ or } .88\frac{8}{9}$

$\frac{1}{10} = .1$ $\frac{2}{10} = .2$ $\frac{3}{10} = .3$ $\frac{4}{10} = .4$ $\frac{5}{10} = .5$ $\frac{6}{10} = .6$ $\frac{7}{10} = .7$ $\frac{8}{10} = .8$ $\frac{9}{10} = .9$

<u>Converting Decimals To Fractions</u>

Algorithm for:

Converting decimals to fractions

1. Determine the denominator by counting the number of digits to the right of the decimal point. That number you counted is the power of 10 for the denominator. That place value will be the denominator.
2. The numerator is the number to the right of the decimal point.
3. Reduce.

Examples:

1) Convert .52 to a fraction. (52 hundredths)

   Since 52 hundredths has two digits to the right of the decimal, the denominator will be 10^2 or 100. The numerator is 52.

$$.52 = \frac{52}{100} = \frac{13}{25}$$

2) Convert .613 to a fraction. (613 thousandths)

$$.613 = \frac{613}{1000}$$

This fraction does not reduce.

3) Convert 8.32 to a fraction. (8 and 32 hundredths)

Since there are two digits to the right of the decimal, the denominator will be 100. The numerator is the number to the right of the decimal point, 32.

$$8.32 = 8\frac{32}{100} = 8\frac{8}{25}$$

<u>Converting Fractions to Decimals</u>

One way to convert fractions to decimals is by **making equivalent fractions**.

Example: Convert $\frac{1}{2}$ to a decimal.

Since a decimal is a special fraction whose denominator is a power of 10, look for a power of 10 that 2 will divide into with a zero remainder.

$$\frac{1}{2} = \frac{}{10} \quad (\times 5 \text{ top and bottom}) \qquad \frac{1}{2} = \frac{5}{10} = 0.5$$

Since the denominator is 10, the decimal will end in the tenths place which means there will be only one digit to the right of the decimal point. The answer is 0.5.

Example: Convert $\frac{3}{4}$ to a decimal.

Math 6 Part A -NOTES – Fractions

Again, since a decimal is a fraction whose denominator is a power of 10, look for powers of 10 that will divide into so the remainder is zero. 4 does not go into 10 with no remainder, but 4 will go into 100 with no remainder, making equivalent fractions,

$$\frac{3}{4} \overset{\times 25}{\underset{\times 25}{=}} \frac{\ }{100} \qquad \frac{3}{4} = \frac{75}{100} = 0.75$$

There are denominators that will never divide into any power of 10 without a remainder. If that happens, look for an alternative way of converting fractions to decimals. Could you recognize numbers that are not factors of powers of ten? Using your Rules of Divisibility, factors of powers of ten can only have prime factors of 2 or 5. That would mean 3, 6, 7, 9, 11, 12, whose prime numbers contain factors other than 2 and 5 would not be a factor of a power of ten. That means that 12 will never divide into a power of 10. The result of that is a fraction such as $\frac{5}{12}$ will not terminate – it will be a repeating decimal.

Because not all fractions can be written with a power of 10 as the denominator, it is necessary to look at another way to convert a fraction to a decimal. That is, to **divide the numerator by the denominator.**

Example: Convert $\frac{1}{3}$ to a decimal.

Since no power of 10 is divisible by 3, it will be a repeating decimal.

$$3\overline{)1.000} = 0.333...$$

The answer is $\frac{1}{3} = 0.\overline{3}$

Example: Convert $\frac{5}{12}$ to a decimal.

Since no power of 10 is divisible by 12, it will be a repeating decimal.

$$12\overline{)5.0000} = 0.41666...$$

The answer is $\frac{5}{12} = 0.41\overline{6}$

Math 6 Part A -NOTES – Fractions

Prep for 6.NS **Equivalent Fractions**

People have a way of describing the same thing in different ways. For instance, a mother might say her baby is twelve months old, but the father might tell somebody his baby is a year old. The age is the same but described in two different ways. Well, we do the same thing in math; or in our case, in fractions. Let's look at the two cakes.

One person might notice that 2 out of 4 pieces seem to describe the same thing as 1 out of 2 in the picture above. In other words, $\frac{1}{2} = \frac{2}{4}$.

When two fractions describe the same thing, we say they are equivalent fractions.

Equivalent fractions are fractions that have the same value.

It would be nice to be able to determine if fractions were equivalent without drawing pictures every time. Take a look at the following examples of equivalent fractions. Is there a pattern?

Examples: $\frac{1}{2} = \frac{3}{6}$ $\frac{3}{4} = \frac{6}{8}$ $\frac{2}{3} = \frac{10}{15}$ $\frac{3}{5} = \frac{30}{50}$

Is there a relationship between the numerators and denominators in the first fraction compared to the numerators and denominators in the second fraction?

Notice both the numerator and denominator is being multiplied by the same number to get the 2^{nd} fraction.

Example: $\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$ if you multiply both the numerator and denominator by 4.

Since there is a pattern, there is an algorithm or procedure for generating equivalent fractions:

To generate equivalent fractions, multiply BOTH numerator and denominator by the SAME number.

Note: In the above example, when multiplying both the numerator and denominator by the same number, we are multiplying by $\frac{4}{4}$ or 1. When a fraction is multiplied by 1, it does not change the value of the original fraction.

Example: Express $\frac{5}{6}$ as sixtieths.

Math 6 Part A -NOTES – Fractions

$\frac{5}{6} = \frac{?}{60}$ What do you multiply 6 by to get 60 in the denominator?

$\times 10$

By 10, so multiply the numerator by 10.

$\frac{5}{6} = \frac{50}{60}$

$\times 10$

Example: $\frac{2}{7}$ is equal to how many thirty-fifths?

$\frac{2}{7} = \frac{?}{35}$ What do you multiply 7 by to get 35 in the denominator? By 5, so multiply the numerator by 5.

$\times 5$

$\frac{2}{7} = \frac{10}{35}$

$\times 5$

Prep for 6.NS Mixed Numbers and Improper Fractions

Types of Fractions

1. A *proper fraction* is a fraction less than one. The numerator is less than the denominator. Example: $\frac{17}{20}$

2. An *improper fraction* is a fraction greater than one. The numerator is greater than the denominator. Example: $\frac{31}{20}$

Converting Mixed Numbers to Improper Fractions

A *mixed number* is a whole number greater than 1 and a fraction. A mixed number is used when there is more than one whole unit. For example, $1\frac{1}{4}$ is called a mixed number.

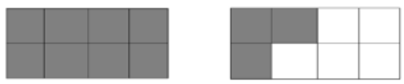
Math 6 Part A -NOTES – Fractions

From the picture above, let's say Johnny ate the entire first cake and one piece from the second cake. This could be described as eating $1\frac{1}{4}$ cakes. His mom might come home and notice Johnny ate 5 pieces of cake. Notice, eating $1\frac{1}{4}$ cakes describes the same thing as eating 5 pieces of cake.

Since we are working with fractions, 5 pieces of cake can be described as a fraction. The numerator tells how many pieces Johnny ate and the denominator tells how many equal pieces make one whole cake. In this case the fraction then is $\frac{5}{4}$.

It seems that $1\frac{1}{4}$ describes the same thing as $\frac{5}{4}$. Therefore, it can be stated that they are equivalent; $1\frac{1}{4} = \frac{5}{4}$.

Example:



The above diagram represents two cakes and the shaded regions indicate the pieces that have been eaten. It looks like an entire cake has been eaten plus $\frac{3}{8}$ of the second cake. This is represented by the mixed number $1\frac{3}{8}$ cakes.

Another way to describe the eaten pieces is to state that 11 pieces of cake have been eaten from cakes that are cut into eight equal pieces. This can be written as $\frac{11}{8}$. Therefore it can be said that $1\frac{3}{8}$ is equivalent to $\frac{11}{8}$.

Ask students if there is a way to convert mixed numbers to improper fractions without having to draw cakes for every problem.

One whole cake can be written as 1 or $\frac{8}{8}$ and the portion of the other cake is $\frac{3}{8}$.

$$1\frac{3}{8} =$$

$$1 + \frac{3}{8} =$$

$$\frac{8}{8} + \frac{3}{8} =$$

$$\frac{11}{8}$$

Math 6 Part A -NOTES – Fractions

Ask students if they can see a pattern.

$$1\frac{1}{4} = \frac{5}{4} \text{ and } 1\frac{3}{8} = \frac{11}{8}$$

Algorithm for:

Converting a mixed number to an improper fraction

1. Multiply the whole number by the denominator

2. Add the numerator to that product

3. Place that result over the original denominator

Example: Convert $2\frac{3}{4}$ to an improper fraction.

1. Multiply 2×4, which is equal to 8.
2. Add 3 (numerator) + 8, which is equal to 11.
3. Place 11 over the original denominator: $\frac{11}{4}$.

<u>Converting Improper Fractions to Mixed Numbers</u>

From previous examples:

$$\frac{5}{4} = 1\frac{1}{4} \qquad \frac{11}{8} = 1\frac{3}{8} \qquad \frac{13}{5} = 2\frac{3}{5}$$

Students are now going to find an algorithm for converting improper fractions to mixed numbers.

Example:

$$\frac{5}{4} = \frac{4}{4} + \frac{1}{4} = 1 + \frac{1}{4} = 1\frac{1}{4}$$

This means that the above improper fraction can be re-written as the sum one whole cake plus a fraction of a cake.

Example:

$$\frac{13}{5} = \frac{5}{5} + \frac{5}{5} + \frac{3}{5} = 2 + \frac{3}{5} = 2\frac{3}{5}$$

This means that the above improper fraction can be re-written as the sum of two whole cakes plus a fraction of a cake.

Math 6 Part A -NOTES – Fractions

The trick to convert an improper fraction to a mixed number seems to be to determine how many whole cakes were eaten, then write the fractional part of the cake left.

Example:

To convert $7/3$ to a mixed number, how many whole cakes were eaten? Well $7/3$ could be written as $3/3 + 3/3 + 1/3$. Two whole cakes were eaten plus a $1/3$ of another cake. Therefore $7/3$ converted to a mixed number is $2\,1/3$.

Can this be done without breaking apart the fraction?

To determine how many whole cakes there are in $7/3$, divided 7 by 3. The quotient is 2, which means there are 2 whole cakes. The remainder is 1, which means there is one piece of the last cake, or $1/3$ of a cake left. This means $7/3 = 2\,1/3$.

Algorithm for:

Converting an improper fraction to a mixed number

1. Divide the numerator by the denominator to determine the whole number

2. Write the remainder over the original denominator

Example: Convert $22/5$ to a mixed number.

1. 5 goes into 22 four times.
2. The remainder is 2.

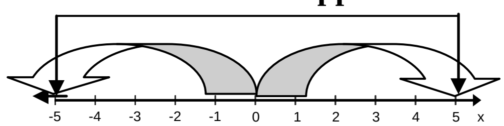
$$\frac{22}{5} = 4\frac{2}{5}$$

NVACS 6.NS.C.6a** **Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.

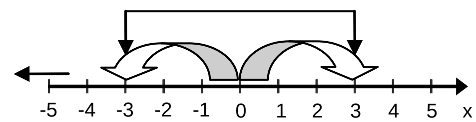
Opposites are numbers that are the <u>same distance from 0</u> on a number line but on the other side of 0. As we have seen with integers and decimal numerals, now we see opposites of fractional numbers.

Math 6 Part A -NOTES – Fractions

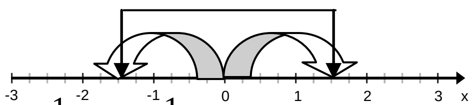
5 and -5 are **opposites**



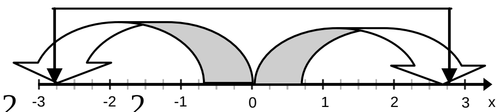
-3 and 3 are **opposites**



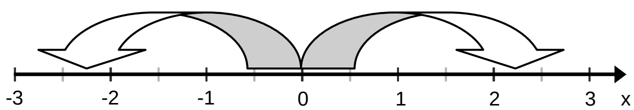
1.5 *and* -1.5 are opposites



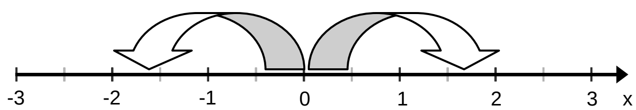
2.75 *and* -2.75 are opposites



$-2\frac{1}{3}$ and $2\frac{1}{3}$ are opposites



$1\frac{2}{3}$ and $-1\frac{2}{3}$ are opposites



What is the opposite of -7? 0.63? -1.23? $\frac{-4}{5}$? $14\frac{11}{12}$?

Answers 7 -0.63 1.235 $\frac{4}{5}$ $14\frac{11}{12}$

And now just to see if you are paying attention let me ask,…

What is the opposite of $\frac{-3}{4}$ ***?***

$- \quad \frac{-3}{4} = \frac{3}{4}$

What is the opposite of $3\frac{5}{6}$ **?**

$- \quad 3\frac{5}{6} = -3\frac{5}{6}$

"What is the opposite of the opposite of $\frac{-7}{3}$ **?" We could write this as**

$- \qquad - \quad \frac{-7}{3} = \frac{-7}{3}$

Math 6 Part A -NOTES – Fractions

What is the opposite of the opposite of $\frac{1}{7}$?" We could write this as

$$- \qquad - \qquad \frac{1}{7} = \frac{1}{7}$$

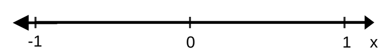
***NVACS 6.NS.C.6** Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.*

***NVACS 6.NS.C.6c** – Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.*

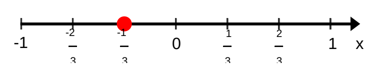
***NVACS 6.NS.C.6** – Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.*

Students need to be able to create number lines that include the negative numbers – especially fractions and decimals. As they begin to create this extended number line, we can begin to find and position these rational numbers. When graphing rational numbers on a number line diagram both horizontal and vertical orientations should be practiced. When asked to graph a number or "position" a number, we find the location of the value and shade a small circle or point at that location.

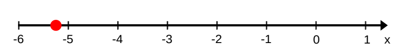
Example: *Graph $-\frac{1}{3}$ on the number line to the right.*



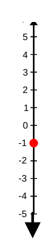
To begin, many students will need to subdivide into thirds and then plot the point(s).



Example: *Graph -5.25 on the number line to the right.*
(This is one of the places where it is advantageous for students to be flexible and use/know equivalent forms of numbers. In this case -5.25 is the same as $-5\frac{1}{4}$.)

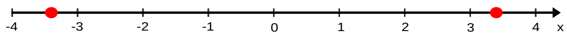


Example: *The temperature in Streeter, ND is 1 degree below zero. Graph -1 on the number line to the right*

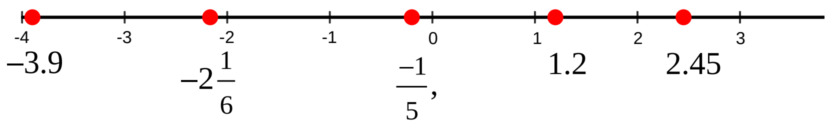


Math 6 Part A -NOTES – Fractions

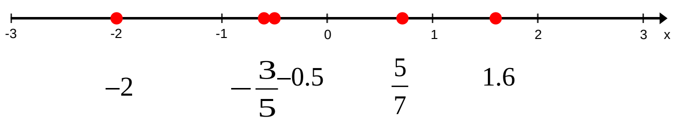
Example: *Graph* 3.4 *and its opposite on the number line.*



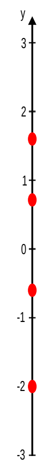
Example: *Graph the given set on the number line below.* $\{\frac{-1}{5},\ 1.2,\ -3.9,\ 2.45,\ -2\frac{1}{6}\}$



Example: *Graph the following points:* $-0.5,\ -2,\ 1.6,\ -\frac{3}{5},\ \frac{5}{7}$

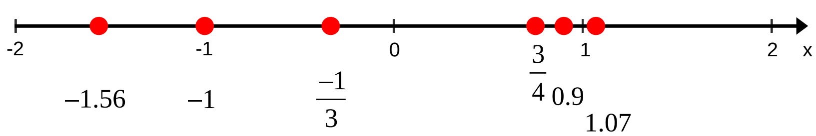


or



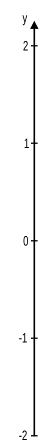
Example: *On the horizontal number line below, graph (position) the following numbers.*

$0.9,\ -1,\ -1.56,\ \frac{3}{4},\ -\frac{1}{3},\ 1.07$



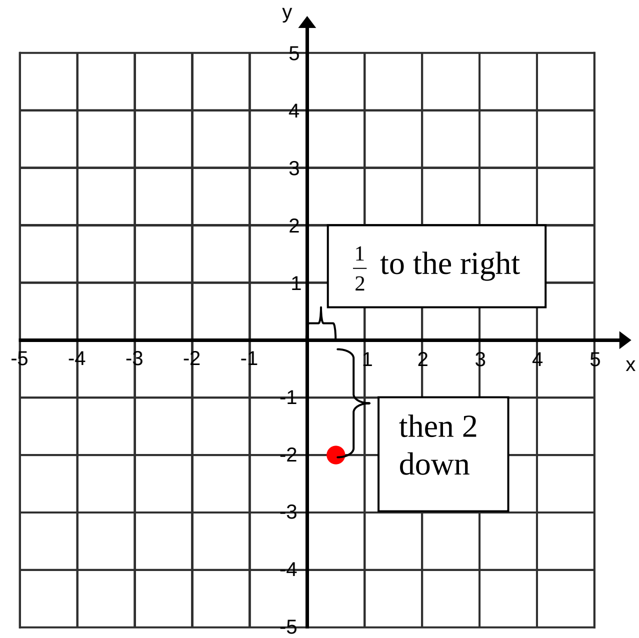
Example: *Locate the following numbers on a vertical number line.*

$-1.3,\ \frac{7}{5},\ -2,\ \frac{-4}{5},\ 1.13$

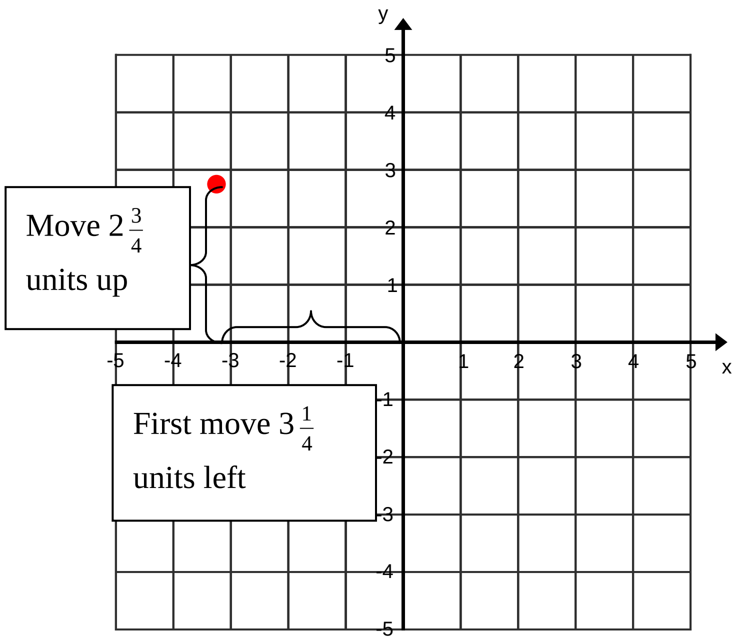


Math 6 Part A -NOTES – Fractions

Example: *Graph the* $\left(\frac{1}{2}, -2\right)$ *on the Cartesian graph below.*



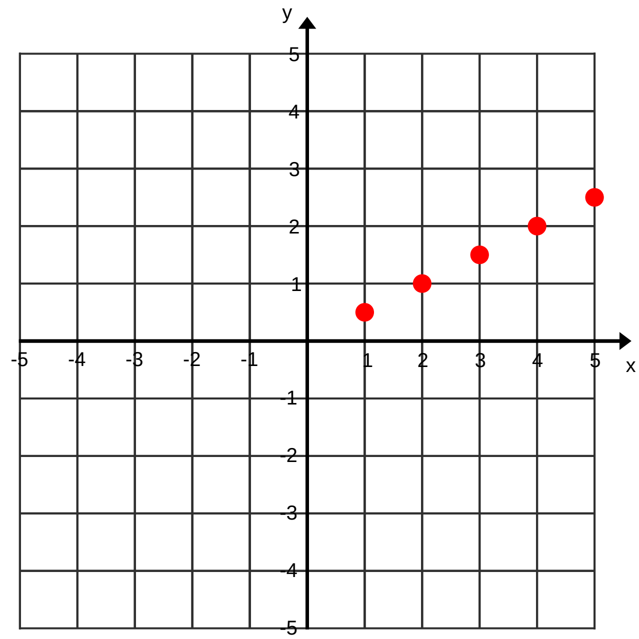
Example: *Plot the point* $\left(-3\frac{1}{4}, 2\frac{3}{4}\right)$ *on the Cartesian graph below.*



Math 6 Part A -NOTES – Fractions

Example: *Plot the given points on the Cartesian graph below.*

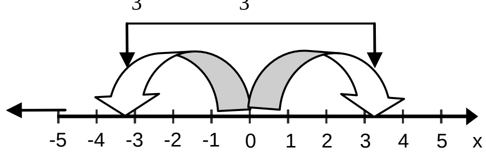
$\{(1, 0.50),(2, 1.00),(3, 1.50),(4, 2.00),(5, 2.50)\}$



***NVACS 6.NS.C.5* – Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.**

Opposites are numbers that are the same distance from 0 on a number line but on the other side of 0.

$-3\frac{1}{3}$ and $3\frac{1}{3}$ are opposites



Examples: Name the opposite of each number:

$-43\frac{2}{7}$

positive four and two-fifths

zero

Math 6 Part A -NOTES – Fractions

negative $14\frac{7}{9}$

+98.5

Examples: For each given real-world context below, identify the event that represents its opposite (makes the meaning of 0 in the situation):

Spending $4.76
Losing $8\frac{1}{2}$ pounds
Raising 3.4 million dollars for charity
Riding the elevator up 2 floors
A store credit of $10.17
A temperature of 7.2° below zero
4,234.7 miles above sea level
A penalty of $5\frac{3}{4}$ yards in football
A withdrawal of $100.94 from your savings account
$3\frac{1}{4}$ years ago
23.7 meters south of the border
70 miles west of the Appalachian Mountains
Gaining 9.3 pounds last week
Losing 0 yards in soccer
Saving $5.96
A zero charge

Examples: Using real-world contexts, identify opposite situations that represent 0.

Answers may vary*

The temperature rises 9 degrees and then falls 9 degrees.
You earn $5.75 and then you spend $5.75.
The train travelled $534\frac{3}{4}$ miles north, then returned $534\frac{3}{4}$ miles south.
A person loses ten and a half pounds then gains ten and a half pounds.
You enter an elevator on the ground floor and you go up 2 floors and then down 2 floors.
A football team gains $20\frac{2}{3}$ yards then loses $20\frac{2}{3}$ yards.
Traveling 10.6 miles west, then 10.6 miles east.
A hydrogen atom has 0 charge because its two constituents are oppositely charged. [(It has one proton (+1) and one electron (-1).]

*Other examples may include above and below sea level, credit and debits, deposits and withdrawals, etc.

Math 6 Part A -NOTES – Fractions

Example: Harriet deposits \$56.32 in the bank. Then she writes checks totaling \$56.32. Does she have more or less in her account than at the beginning? How much more or less?

Example: Marsha hiked Angel Trails that runs 5.7 miles north. She then hikes back to where she started from. Which best describes the total change in miles for the entire hike?

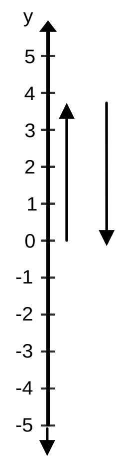
A	**11.4**
B	**5.7**
C	**0**
D	**-5.7**

Example: <u>Show</u> the meaning of 0 in each situation.

We <u>know</u> … if you only have \$3.75 and you spend \$3.75 that you are broke, you have zero dollars .

… that a gain of $3\frac{3}{4}$ yards and a loss of $3\frac{3}{4}$ yards is zero.

SHOW….We can <u>show</u> this on a number line …



Example: *A submarine dives $20\frac{1}{4}$ fathoms beneath the surface. This is shown as $-20\frac{1}{4}$. What number would show the submarine returning to the surface?*

Example: *The submarine travels north 182.4 kilometers. This is shown by +182.4. What number would show the return trip?*

Math 6 Part A -NOTES – Fractions

Comparing and Ordering Fractions

NVACS 6.NS.7 ***Understand ordering and absolute value of rational numbers.***

NVACS 6.NS.C.7a ***Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram.***

To compare fractions with different denominators, there are several methods we can use:

Method 1 – **Write in decimal form**, then compare.

Example: Order $\frac{2}{5}, \frac{1}{2}, \frac{3}{10}$ from least to greatest.

$\frac{2}{5} = 0.4$ $\frac{1}{2} = 0.5$ $\frac{3}{10} = 0.6$

Put the decimal forms in order from least to greatest.
Then convert back to the corresponding fractions given.

0.4, 0.5, 0.6

$\frac{2}{5}, \frac{1}{2}, \frac{3}{10}$

Method 2 – **Estimate the fractions** as 0, ½ or 1.

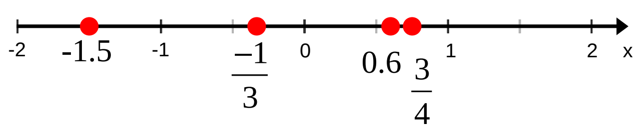
Example: Order $\frac{3}{4}, \frac{1}{3}, \frac{8}{15}$ from least to greatest.

$\frac{3}{4} \approx 1,$ $\frac{1}{3} \approx 0,$ $\frac{8}{15} \approx \frac{1}{2}$

So $\frac{1}{3}, \frac{8}{15}, \frac{3}{4},$

Method 3 - **Graph on a number line.**

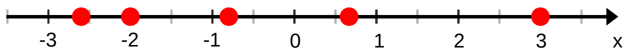
Example: Order $\frac{3}{4}, \frac{-1}{3}, -1.5, 0.6$ from least to greatest.



Solution:
$-1.5, \frac{-1}{3}, 0.6, \frac{3}{4},$

Math 6 Part A -NOTES – Fractions

Example: Order $-2.6,\ 3,\ \frac{2}{3},\ \frac{-4}{5},-2,$ from least to greatest



Solution:

$-2.6,\ -2,\frac{-4}{5},\ \frac{2}{3},\ 3$

Method 4 - **Find a common denominator**, write equivalent fractions, then compare.

Common Denominators

Let's say there are two cakes, one chocolate, the other vanilla. The chocolate is cut into thirds and the vanilla into fourths as shown below. Ashton eats one piece of chocolate cake and one piece of vanilla cake, as shown by the gray pieces.

Since two pieces of cake have been eaten, is it correct to say that Ashton ate $^2/_7$ of a cake?
Let's re-define a fraction. The numerator tells how many equal pieces have been eaten and the denominator states how many equal pieces make one whole cake. Since the pieces are not equal, $^2/_7$ of a cake does not fit the above definition of a fraction, and clearly, 7 pieces do not make one whole cake: Therefore, trying to add $^1/_4$ to $^1/_3$ and coming up with $^2/_7$ does not fit the definition of a fraction.

The key is to cut the cakes into equal pieces. The dark lines indicate additional cuts to each cake.

By making additional cuts on each cake, both cakes are now made up of 12 equal pieces. That's good news from a sharing standpoint – everyone gets the same size piece. Mathematically, the concept of a common denominator has just been introduced.

Here is the way the additional cuts were made: Cut the second cake the same way the first was cut and the first cake the same way the second was cut as shown in the picture.
Clearly, it is not convenient to make additional cuts in cakes every time there are cakes with unlike pieces. It is necessary to find a way that will allow students to determine how to make sure all the pieces are the same size.

Math 6 Part A -NOTES – Fractions

A *common denominator* is a denominator that all other denominators will divide into evenly.	**Four Methods of Finding a Common Denominator** 1. Multiply the denominators 2. Write multiples of each denominator, find a common multiple 3. Use prime factorization to find the least common denominator. 4. Use the Simplifying Method, especially for larger denominators

Cake-wise, it's the number of pieces that cakes can be cut so everyone has the same size piece.

Method 1: Find the common denominator between $\frac{1}{3}$ and $\frac{1}{4}$.

Multiply the denominators. The common denominator would be 3×4 or 12. Use this method when the denominators are prime numbers or relatively prime.

Method 2: Find the common denominator between $\frac{2}{3}$ and $\frac{1}{4}$.

List the multiples of 3 and multiples of 4. Find a multiple that is in common to both denominators. Use this method when the denominators are small or friendly numbers.

Multiples of 3: 3, 6, 9, 12, 15, 18, …
Multiples of 4: 4, 8, 12,…

Since 12 is a multiple of each denominator, 12 would be a common denominator.

Method 3: Find the common denominator between $\frac{5}{18}$ and $\frac{7}{24}$. **Use prime factorization** to find the LCD. This method is not the most expedient, so it is seldom used.
Find the prime factorization for 18 and 24, using factor trees we find:

$18 = 2 \bullet 3^2$

$24 = 2^3 \bullet 3$ Taking the greatest power of each prime factor,

the LCM $= 2^3 \bullet 3^2 = 72$

Method 4: Same problem as above, let's find the common denominator between $\frac{5}{18}$ and $\frac{7}{24}$, using **the Simplifying Method**. This method is used when the denominators are large composite numbers. Use the two denominators, 18 and 24, to write the fraction then simplify the fraction:

$$\frac{18}{24} = \frac{3}{4}$$

Now cross multiply, either 24 × 3 or 18 × 4. It does not matter which way the cross multiplication occurs, the product is 72; therefore the common denominator is 72.

Math 6 Part A -NOTES – Fractions

Notice it does not matter whether the ratio of the denominators is $\frac{18}{24} \text{ or } \frac{24}{18}$.

Simplify the fraction. $\frac{18}{24}=\frac{3}{4}$

72 *or* 72

Find a cross product. $\frac{18}{24}=\frac{3}{4}$ The LCM is 72.

Example: Find the common denominator for $\frac{5}{24}$ and $\frac{9}{42}$.

While multiplying will give you a common denominator, that product will be very large. Use method 4, place the denominators over each other and simplify.

$$\frac{24}{42}=\frac{4}{7}$$

$42\times 4 \text{ or } 24\times 7$ gives a product of 168. The common denominator is 168.

Example: Use **>** or **<** to compare the fractions. $\frac{3}{5}\square\frac{2}{3}$

Find a common denominator and make equivalent fractions, then compare the numerators:

$$\frac{3}{5}\square\frac{2}{3} \Longleftrightarrow \frac{9}{15}\square\frac{10}{15}$$

Since 9 < 10, we have $\frac{3}{5}<\frac{2}{3}$.

Example: Order the following fractions from *least to greatest*.

$$\frac{3}{4}, \frac{7}{10}, \text{and } \frac{2}{3}$$

We need to think about the methods to use for comparing 3 fractions.

Method 1 – **Write in decimal form. –** This will work OK.

Method 2 – **Estimate the fractions. –** This will <u>not</u> work because each fraction ≈ 1.

Math 6 Part A -NOTES – Fractions

Method 3 - **Find a common denominator. –** This will work OK.

$\frac{3}{4} = 0.75, \quad \frac{7}{10} = 0.7, \quad \text{and } \frac{2}{3} = 0.666...$

$So\ \frac{2}{3}, \quad \frac{7}{10}, \quad \frac{3}{4}$

or

$\frac{3}{4} = \frac{45}{60}, \quad \frac{7}{10} = \frac{42}{60}, \quad \frac{2}{3} = \frac{40}{60}$

$So\ \frac{40}{60}, \frac{42}{60}, \frac{45}{60}$

$\frac{2}{3}, \frac{7}{10}, \frac{3}{4}$

To find the LCD for 3 fractions, use a combination of the methods learned. In the problem above (with denominators 4, 10 and 3) since the denominators 4 and 3 are relatively prime we would multiply the denominators. Now we need to look for common multiples of 10 and 12. When we find the least common multiple, or LCD, make equivalent fractions using the common denominator. Then, compare the numerators to rewrite the fractions from least to greatest.

OnCore Example: Which number is the greatest?

A	$\frac{3}{20}$
B	$\frac{1}{5}$
C	$\frac{6}{15}$
D	$\frac{7}{20}$

OnCore Example: Which list is in order from least to greatest?

A	$\frac{2}{3}$, 0.52, $\frac{9}{16}$
B	0.52, $\frac{9}{16}$, $\frac{2}{3}$
C	$\frac{9}{16}$, 0.52, $\frac{2}{3}$
D	0.52, $\frac{2}{3}$, $\frac{9}{16}$

Math 6 Part A -NOTES – Fractions

OnCore Example: Which number is between $\frac{9}{20}$ and $\frac{1}{2}$?

A	$\frac{4}{9}$
B	$\frac{3}{7}$
C	$\frac{5}{11}$
D	$\frac{7}{16}$

NVACS 6.NS.7b Write, interpret, and explain statements of order for rational numbers in real-world contexts.

Example: Tom and Tim receive the same amount of allowance each week. Tom spends $\frac{5}{7}$ of his allowance going to the movies. Tim spends $\frac{3}{4}$ of his allowance on a CD. Which of the boys spent more of their allowance? Explain your work and thinking.

Solution: $\frac{3}{4} > \frac{5}{7}$ Tim spent more

Example: Darion and Elizabeth both took the same quiz. Darion answered $\frac{8}{9}$ of the questions correctly, and Elizabeth answered $\frac{7}{8}$ correctly. Who got the higher score on the test?

Solution: $\frac{8}{9} > \frac{7}{8}$ Darion

Example: Sandy made 7 baskets out of 18 shots in a basketball game, while Joan made 9 baskets out of 20 shots. Who made the greater fraction of shots?

Solution: $\frac{7}{18} < \frac{9}{20}$ Joan

Example: In archery at summer camp Chris hit the target 7 out of 12 shots. If Brett has 15 shots, how many must hit the target in order for him to have a higher fraction of successful shots? Show your thinking.

Solution: 9 shots

Math 6 Part A -NOTES – Fractions

Example: The following table shows numbers of SUVs and sedans sold by three car dealers in March. What fraction of each dealer's sales were SUVs? List the dealers from greatest to least SUV sales.

Dealer	SUVs	Sedans
Car City	15	12
Auotrama	18	14
CKL Autos	16	12

Solution: $\frac{5}{9}; \frac{9}{16}; \frac{4}{7}$
CKL Auto, Autorama, Car City

Example: Three holes need to be drilled in a piece of wood so the holes increase in size from left to right. The diameters of the holes are $\frac{1}{3}, \frac{3}{8}, and\ \frac{5}{16}$ inches. What order should the holes be drilled from left to right?

Solution: $\frac{5}{16}; \frac{1}{3}; \frac{3}{8}$

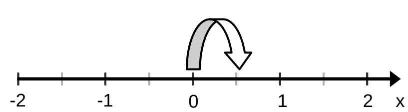
Absolute Value

NVACS 6.NS.7 Understand ordering and absolute value of rational numbers.

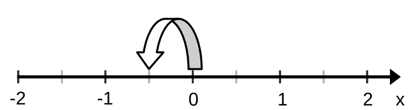
NVACS 6.NS.7c Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real world situation.

Absolute Value – is the distance from 0 on a number line.

Examples: $\left|-\frac{1}{2}\right| = \frac{1}{2}$ since $-\frac{1}{2}$ is $\frac{1}{2}$ unit to the left of 0.

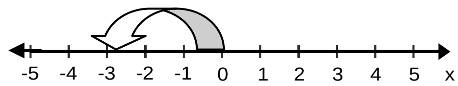


$\left|\frac{1}{2}\right| = \frac{1}{2}$ since $\frac{1}{2}$ is $\frac{1}{2}$ unit to the right of 0.

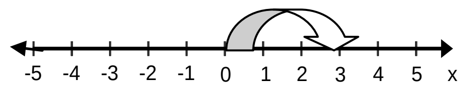


So both $\left|-\frac{1}{2}\right|$ ***and*** $\left|\frac{1}{2}\right|$ equal $\frac{1}{2}$ because the <u>distance</u> from zero is $\frac{1}{2}$ unit. It doesn't matter which direction.

Math 6 Part A -NOTES – Fractions



Examples: $|-2.9| = 2.9$ since –2.9 is 2.9 units to left of 0.



$|2.9| = 2.9$ since 2.9 is 2.9 units to the right of 0.

$\mathbf{|-19| = 19}$ $\quad |0| = 0$ $\quad |9.3| = 9.3$ $\quad \left|-\frac{5}{11}\right| = \frac{5}{11}$

$|0.152| = 0.152$ $\quad |-3.4| = 3.4$ $\quad \left|-2\frac{9}{10}\right| = 2\frac{9}{10}$ $\quad \left|-\frac{15}{4}\right| = \frac{15}{4}$

Students need to understand that if $\mathbf{|x| = \frac{6}{7}}$ then $\mathbf{x = \frac{6}{7}}$ <u>and</u> $\mathbf{x = -\frac{6}{7}}$.

We can write that as $x = \pm\frac{6}{7}$.

SBAC Examples

Standard: 6.NS.6 **DOK: 2** **Difficulty: M** **Question Type: TE** Technology Enhanced

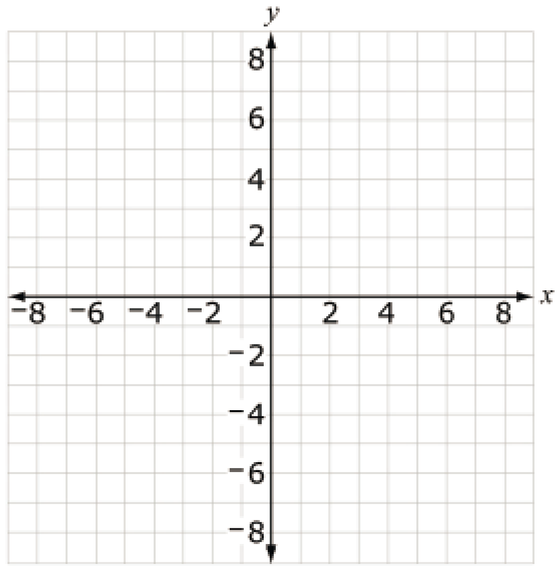
The coordinates of point *P* are $(-6, 5)$. Point *R* is a reflection of point *P* across the *x*-axis.

The coordinates of point *Q* are $(-1, 0)$. Point *T* is a reflection of point *Q* across the *y*-axis.

Part A

Plot and label points *P*, *Q*, *R*, and *T* on the coordinate plane.

Math 6 Part A -NOTES – Fractions



Part B

The coordinates of point *V* are $(7, 4)$. Point *W* is a reflection of point *V* across the *x*-axis.

In which quadrant will point *W* be located?

(A) I

(B) II

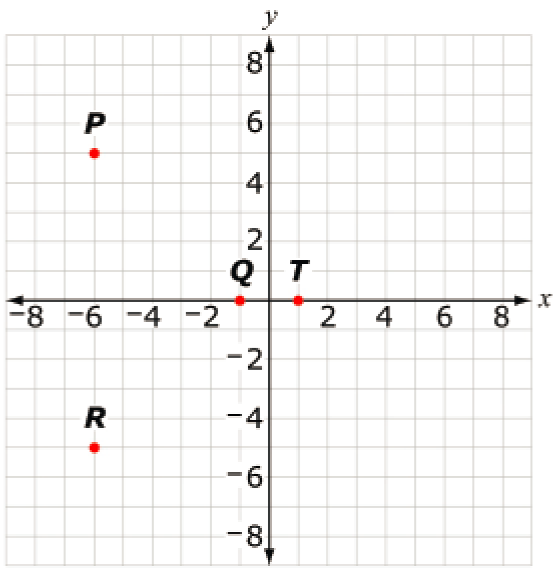
(C) III

(D) IV

Math 6 Part A -NOTES – Fractions

Sample Top-Score Response:

Part A



Part B

Quadrant IV

Scoring Rubric:

Part A 1 point for correctly plotting points P and R.
1 point for correctly plotting points Q and T.

Part B 1 point for correctly choosing D; Quadrant IV.

Math 6 Part A -NOTES – Fractions

ADD

Standard: 6.RP.A.3a ***DOK: 1, 2*** ***Question Type: Hot Spot***

Grades 6-8, Claim 2

Task Model 2

DOK Levels 1, 2

Target B: Select and use appropriate tools strategically.

Task Expectations:

- Mathematical information is presented in a table or graph or extracted from a context.
- The student is asked to solve a problem that requires strategic use of tools or formulas.

Example Item 1 (Grade 6):

Primary Target 2B (Content Domain RP), Secondary Target 1A (CCSS 6.RP.3a)

Nate waters the garden every 3 days and weeds it every 4 days.

He does both on April 2nd.

What is the next date that he will both water and weed his garden?

Select that date on the calendar.

APRIL

Sun	Mon	Tues	Wed	Thu	Fri	Sat
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Rubric: (1 point) The student selects the correct date (e.g., April 14).

Response Type: Hot Spot