# Math 6 Notes: Expressions, Equations and Inequalities

## Expressions

A **numerical expression** is simply a name for a number. For example, 4 + 6 is a numerical expression for 10, and $400 \bullet 4$ is a numerical expression for 1,600, and $5-3+2$ is a numerical expression for 4. Numerical expressions include numbers and operations and do <u>not</u> include an equal sign and an answer. In English, we use expressions such as "Hey", "Awesome", "Cool", "Yo". Notice they are not complete sentences.

In this unit we will begin working with variables. **Variables** are letters and symbols we use to represent one or more numbers. An expression, such as 100 × *n*, that contains a variable is called a **variable expression**.

Students have been working with variables in problems like $2\times\square=8$ and $5+\_\_\_\_=9$, where a box, or a circle, or a line represents the missing value. Now we will begin to replace those symbols with letters, so we will see 2*m* = 8 and 5 + y = 9. To avoid some initial confusion it is important to stress that the variables represent different values that make each statement true. Some students want to make an incorrect connection such as *a* = 1, *b* = 2, *c* = 3, …

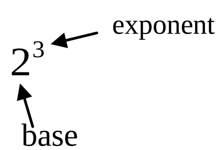
When working with variables, as opposed to numerical expressions, we omit the multiplication sign so 100 × *n* is written 100*n*. The expression *x* × *y* is written *xy*. Hopefully you can see the confusion that could be caused using variables with the "×" multiplication sign. Stress to students the need to omit the use of the "×" sign with variables, but allow them time to adapt. Remind them in a numerical expression for a product such as $100\times 4$, we **must** use a multiplication sign to avoid confusion. This may also be a great opportunity to begin to have students use the **raised dot** as a multiplication sign, so $100\times 4$ may be written $100\bullet 4$.

We **simplify** or **evaluate** numeric expressions when we replace it with its simplest name. For example, when we simplify the expression 4 + 6 we replace it with its simplest name, 10.

**NVACS 6.EE.A.1** *Write and evaluate numerical expressions involving whole number exponents.*

### Exponents

An ***exponent*** is the superscript which tells how many times the ***base*** is written as a factor.




In the number $2^3$, read "2 to the third power" or "2 cubed", the 2 is called the *base* and the 3 is called the *exponent*.

# Math 6 Notes: Expressions, Equations and Inequalities

***Examples:*** $2^3 = 2 \times 2 \times 2$

$6^4 = 6 \times 6 \times 6 \times 6$

$1^{11} = 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1$

$2^3 \bullet 5^2 = 2 \bullet 2 \bullet 2 \bullet 5 \bullet 5$

Numbers written using exponents are called **exponentials** or **powers**. This form of writing is called **exponential notation**.

***Examples:*** Write the following expressions using exponents.

$5 \bullet 5 \bullet 5 \bullet 5 \bullet 5 \bullet 5 \bullet 5 = 5^7$

$4 \bullet 4 \bullet 4 \bullet 4 \bullet 4 \bullet 4 = 4^6$

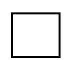
$8 \bullet 8 \bullet 8 \bullet 8 \bullet 8 = 8^5$

$20 \bullet 20 \bullet 20 \bullet 20 = 20^4$

$7 \bullet 7 \bullet 7 \bullet 9 \bullet 9 \bullet 9 \bullet 9 = 7^3 \bullet 9^4$

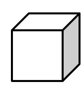
The second and third powers of a number have special names. The second power is called the **square** of the number and the third power is called the **cube**. One way to get kinds to associate these terms is to connect this to the idea of 2 dimensional and 3 dimensional figures

$5^2$ is read "five squared"



a square has 2 dimensions – length and width

$7^3 = 7 \times 7 \times 7$ is read "seven cubed"



a cube has 3 dimensions – length, width and height

To simplify or write an exponential in **standard form**, you compute the products.

$5^2 = 5 \times 5 = 25$

***Example*:** Simplify the following exponentials.

$3^3 = 3 \bullet 3 \bullet 3 = 27$

$5^4 = 5 \bullet 5 \bullet 5 \bullet 5 = 625$

$3^4 = 3 \bullet 3 \bullet 3 \bullet 3 = 81$

$7^2 = 7 \bullet 7 = 49$

$1^6 = 1 \bullet 1 \bullet 1 \bullet 1 \bullet 1 \bullet 1 = 1$

$2^5 = 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 = 32$

$2^4 \times 5^2 = 2 \bullet 2 \bullet 2 \bullet 2 \bullet 5 \bullet 5 = 16 \bullet 25 = 400$

# Math 6 Notes: Expressions, Equations and Inequalities

As we begin to examine powers, we find a special case with powers of 10…Base 10!!!

**Special Case**

***Examples:***

$10^1 = 10$

$10^2 = 100$

$10^3 = 1,000$

$10^4 = 10,000$

Check for Understanding: What is the value of $\mathbf{10^5}$? $\mathbf{10^6}$? Etc?

What pattern allows you to find the value of an exponential with base 10 quickly?

*Answer*: The number of zeroes is equal to the exponent!

**Caution: If a number does not have an exponent showing, it is understood to have an exponent of ONE!**

***Example:***

$$\begin{aligned} 4^2 \bullet 5 &= 4^2 \bullet 5^1 \\ &= 4 \bullet 4 \bullet 5 \\ &= 16 \bullet 5 \\ &= 80 \end{aligned}$$

***Example:***

$$\begin{aligned} &2^3 \bullet 3 \bullet 5^2 = \\ &2 \bullet 2 \bullet 2 \bullet 3 \bullet 5 \bullet 5 = \\ &8 \bullet 3 \bullet 25 = \\ &200 \bullet 3 = \\ &600 \end{aligned}$$

Let's look at a pattern that will allow you to determine the values of exponential expressions with exponents of 1 or 0. Use of this concept development technique will allow students a method to remember the rules for expressions with exponents of 1 or 0, rather than relying just on memorization of a rule.

$2^4 = 2 \bullet 2 \bullet 2 \bullet 2 = 16$ ÷2
$2^3 = 2 \bullet 2 \bullet 2 = 8$ ÷2
$2^2 = 2 \bullet 2 = 4$
$2^1 = ?$ $2^1 = 2$
$2^0 = ?$ $2^0 = 1$

$3^4 = 3 \bullet 3 \bullet 3 \bullet 3 = 81$ ÷3
$3^3 = 3 \bullet 3 \bullet 3 = 27$ ÷3
$3^2 = 3 \bullet 3 = 9$
$3^1 = ?$ $3^1 = 3$
$3^0 = ?$ $3^0 = 1$

$4^4 = 4 \bullet 4 \bullet 4 \bullet 4 = 256$ ÷4
$4^3 = 4 \bullet 4 \bullet 4 = 64$ ÷4
$4^2 = 4 \bullet 4 = 16$
$4^1 = ?$ $4^1 = 4$
$4^0 = ?$ $4^0 = 1$

**Any number to the power of 1 is equal to the number. That is, $n^1 = n$.**

$7^1 = 7$ $\quad 11^1 = 11$ $\quad \left(\frac{1}{2}\right)^1 = \frac{1}{2}$ $\quad \left(\frac{3}{4}\right)^1 = \frac{3}{4}$ $\quad 0.56^1 = 0.56$

**Any number to the power of 0 is equal to one. That is, $n^0 = 1$.**

$9^0 = 1$ $\quad 14^0 = 1$ $\quad \left(\frac{2}{3}\right)^0 = 1$ $\quad (1.43)^0 = 1$

# Math 6 Notes: Expressions, Equations and Inequalities

Remind students that if there is no exponent, the exponent is always 1.

**Writing Numbers in Exponential Form**

***Example:*** Write 81 with a base of 3.

$$81 = 3^{?}, \quad 81 = 9 \text{ x } 9 = 3 \times 3 \times 3 \times 3, \text{ therefore } 81 = 3^4$$

***Example:*** Write 125 with a base of 5.

$$125 = 5^{?}, \quad 125 = 25 \times 5 = 5 \times 5 \times 5, \text{ therefore } 125 = 5^3$$

**SBAC Example:**

***Standard: 6.EE.A.1,*** ***DOK: 1*** ***Item Type:Equation/Numeric***

**TM1**
**Stimulus:** The student is presented with a numerical expression with exponents.

**Example Stem:** Enter the value of $3^3 \bullet 7^2 - 8 \div 4$.

**Rubric:** (1 point) Student enters the correct value for the expression (e.g., 1321).

**Response Type:** Equation/Numeric

**NVACS 6.EE.A.2c** *Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations).*

## Order of Operations

The Order of Operations is just an agreement to compute problems the same way so everyone gets the same result, like wearing a wedding ring on the left ring finger or driving (in the US) on the right side of the road.

**<u>Order of Operations</u>** (**PEMDAS** or **P**lease **e**xcuse **m**y **d**ear **A**unt **S**ally's **l**oud **r**adio)*

1. Do all work inside the grouping symbols and/or **P**arentheses.

   Grouping symbols include $[\ ]$, $(\ )$, and $\frac{x}{y}$.

2. Evaluate **E**xponents.

3. **M**ultiply/**D**ivide from left to right.*

4. **A**dd/**S**ubtract from left to right.*

   *Emphasize that it is NOT always multiply then divide, but rather which ever operation occurs first (going from left to right). Likewise, it is NOT always add-then subtract, but which of the two operations occurs first when looking from left to right.

***Example:*** Simplify the following expression.

$$4+5\times 6-7$$

$4+\underline{5\times 6}-7$ Underline the first step.

$\underline{4+30}-7$ Simplify underlined step, and then underline next step.

$\underline{34-7}$ Repeat; simplify and underline next step until finished.

$27$ Note: each line is simpler than the line above it.

***Example:*** Simplify the following expressions.

(a) $3+5\times 2$

Work:

$3+\underline{5\times 2}=$

$3+10=$

$13$

(b) $4+24\div 6\times 2+1$

Work:

$4+\underline{24\div 6}\times 2+1=$

$4+\underline{4\times 2}+1=$

$\underline{4+8}+1=$

$12+1=$

$13$

(c) $8\div(1+3)\times 5^2-8$

Work:

$8\div\underline{(1+3)}\times 5^2-8=$

$8\div 4\times \underline{5}^2-8=$

$\underline{8\div 4}\times 25-8=$

$\underline{2\cdot 25}-8=$

$50-8=$

$42$

# Math 6 Notes: Expressions, Equations and Inequalities

***Example:*** **Evaluate** $\frac{3 \bullet 6-2}{4}$

Work:

$$\frac{3 \bullet 6-2}{4}=\frac{18-2}{4}=\frac{16}{4}=4$$

Be sure to point out to students that the numerator must be simplified before they can divide by 4.

***Example:*** $(5+3)\times(9-2)+9$

Work:

$$(5+3)\times(9-2)+9=$$
$$8 \times 7 +9=$$
$$56 +9=$$
$$65$$

***Example:*** Evaluate $\mathbf{9^2 \bullet 2^5 \bullet 0^8 =}$

$\mathbf{81 \bullet 32 \bullet 0 =}$

$\mathbf{0}$

***Example:*** Evaluate $\mathbf{5^2 \bullet 10^3 =}$

$\mathbf{25 \bullet 1{,}000 =}$

$\mathbf{25{,}000}$

It would be more efficient to use the Commutative Property and place the zero factor first to minimize the need for work, since 0 times any number(s) is zero.

***Example:*** Evaluate $\mathbf{7+9 \bullet 2^2 =}$

$\mathbf{7+9 \bullet 4 =}$

$\mathbf{7+36 =}$

$\mathbf{43}$

***Example:*** Evaluate $\mathbf{5+5 \bullet 8 \div 2^2 =}$

$\mathbf{5+5 \bullet 8 \div 4 =}$

$\mathbf{5+40 \div 4 =}$

$\mathbf{5+10 =}$

$\mathbf{15}$

***Example:*** Evaluate $7\left[5(6-1)+3(2+3)\right]=$

$$7\left[5(6-1)+3(2+3)\right]=$$
$$7\left[\ 5(5) \quad + \ 3(5)\right]=$$
$$7\left[\ \ 25 \quad + \ 15\right]=$$
$$7\left[40\right]=$$
$$280$$

# Math 6 Notes: Expressions, Equations and Inequalities

***Example:*** Which is the greatest?

| | |
|---|---|
| A. | $2^5$ |
| B. | $3^4$ |
| C. | $4^3$ |
| D. | $5^2$ |

A. $2^5 = 25$ B. $3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$ C. $4^3 = 4 \cdot 4 \cdot 4 = 64$ D. $5^2 = 5 \cdot 5 = 25$

So the answer is B.

**NVACS 6.EE.A.2** *Write, read and evaluate expressions in which letters stand for numbers.*

**NVACS 6.EE.A.2c** *Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (**Order of Operations**).*

To evaluate an algebraic expression, substitute a given value for the variable, then follow the Order of Operations to evaluate the arithmetic expression.

***Example:*** If $w = 3$, evaluate $w + 5$.

$$w + 5$$
$$= 3 + 5$$
$$= 8$$

***Example:*** If $h = 6$, evaluate the algebraic expression $h \div 2$.

$$h \div 2$$
$$= 6 \div 2$$
$$= 3$$

***Example:*** Find the value of $2b + 4$, if $b = 3$.

$$2b + 4$$
$$= 2(3) + 4$$
$$= 6 + 4$$
$$= 10$$

***Example:*** If $x = 3$, evaluate $x^2 - 3$.

$$x^2 - 3$$
$$= 3^2 - 3$$
$$= 9 - 3$$
$$= 6$$

***Example:*** Evaluate $2a + 3b - 4c$ when $a = 5$, $b = 10$ and $c = 7$.

$2a + 3b - 4c$
$= 2(5) + 3(10) - 4(7)$
$= 10 + 30 - 28$
$= 40 - 28$
$= 12$

***Example:*** Evaluate $4s$, when $s = 8$.

$4s$
$= 4 \cdot 8$
$= 32$

***Example:*** Evaluate $2l + 2w$, when $l = 10$ and $w = 5$.

$2l + 2w$
$= 2 \cdot 10 + 2 \cdot 5$
$= 20 + 10$
$= 30$

***Standard: 6.EE.A.1, 6.EE.A.2*** ***DOK: 2*** ***Difficulty: High*** ***Item Type:CR Constructed Response***

Write an expression that is equivalent to 64 using each of the following numbers and symbols once in the expression.

7
7
7
$^2$ (exponent of 2)
+
÷
( )

*Key and Distractor Analysis or Scoring Rubric for Multi-Part Items:*

key: $(7 \div 7 + 7)^2$

**NVACS 6.EE.A.3** *Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression* $3(2 + x)$ *to produce the equivalent expression* $6 + 3x$*; apply the distributive property to the expression* $24x + 18y$ *to produce the equivalent expression* $6(4x + 3y)$*; apply properties of operations to* $y + y + y$ *to produce the equivalent expression 3y.*

# Math 6 Notes: Expressions, Equations and Inequalities

## Properties of Real Numbers

The properties of real numbers are rules used to simplify expressions and compute numbers more easily.

| Property | Operation | Algebra | Numbers | KeyWords/Ideas |
|---|---|---|---|---|
| Identity | + | $a+0=a$ | 5+0=5 | **Adding zero** |
| Identity | $\times$ | $b \bullet 1 = b$ | $7 \times 1 = 7$ | **Multiplying by 1** |
| Zero Property of Multiplication | $\times$ | $a \bullet 0=0$ | $5 \times 0 = 0$<br>$0 \times 5 = 0$ | **Multiply by 0** |
| **Co**mmutative (**c**hange **o**rder) | + | $a+b=b+a$ | $3+4=4+3$ | **C**hange **O**rder |
| **Co**mmutative | $\times$ | $a \times b = b \times a$ | $5 \times 2 = 2 \times 5$ | **C**hange **O**rder |
| Associative | + | $(a+b)+c=a+(b+c)$ | $(4+9)+6=4+(9+6)$ | Change Grouping |
| **Property** | **Operation** | **Algebra** | **Numbers** | **KeyWords/Ideas** |
| Associative | $\times$ | $(a \times b) \times c = a \times (b \times c)$ | $9 \times (5 \times 2) = (9 \times 5) \times 2$ | Change Grouping |
| Distributive | With respect to addition | $a(b+c)=ab+ac$ | 5(23) =5(20+3)<br>= 5(20) + 5(3)<br>=100 + 15<br>=115 | Passing out |
| | With respect to subtraction | *ab-ac=a(b-c)* | $29 \bullet 7 - 29 \bullet 5 = 29(7-5)$<br>$= 29(2)$<br>$= 58$ | Passing in |

***Commutative Property of Addition***

$a+b=b+a$
$b+c+d=d+b+c$

Order does not matter!!

***Commutative Property of Multiplication***

$a \bullet b = b \bullet a$
$abc = cba$

***Examples:*** 4 + 5 = 5 + 4
3+4+7=3+7+4
10 $\times$ 7 = 7 $\times$ 10
2$\times$3$\times$4$\times$5=2$\times$5$\times$3$\times$4
$2(4+7)=(4+7)2$

# Math 6 Notes: Expressions, Equations and Inequalities

***Associative Property of Addition*** $(a+b)+c=a+(b+c)$

***Associative Property of Multiplication*** $(a\times b)\times c=a\times(b\times c)$

Groupings change but order remains the same

***Examples:*** $(7 + 8) + 2 = 7 + (8 + 2)$
$(13 \times 25) \times 4 = 13 \times (25 \times 4)$

***Distributive Property*** $a(b+c)=a\times b+a\times c$ (distribute <u>over</u> add/sub)

(passing out)

***Example:*** $5 \times 23 = 5 \times (20 + 3)$
$= 5 \times 20 + 5 \times 3$
$= 100 + 15$
$= 115$

(passing in)

***Example:*** $7(7) + 7(2) = 7(7 + 2)$
$= 7(9)$
$= 63$

***Example:*** $25 \times 19 = 25 \times (20 - 1)$
$= 25 \times 20 - 25 \times 1$
$= 500 - 25$
$= 475$

***Example:*** $83(9) - 83(5) = 83(9-5)$
$= 83(4)$
$= 332$

***Example:*** $3(x+2)=3\bullet x+3\bullet 2$
$=3x+6$

***Example:*** $4x+6y=2(2x+3y)$

**Using the Distributive Property we can write equivalent expressions.**

***Example:***

$2x+3x=x(2+3)=x(5)=5x$

$y+y+y=1y+1y+1y=(1+1+1)y=3y$

$5m-3m=(5-3)m=2m$

# Math 6 Notes: Expressions, Equations and Inequalities

***SBAC Example:***

Two expressions are shown below:

$P: \; 2(3x-9)$

$Q: \; 6x-9$

Part A
Apply the distributive property to write an expression that is equivalent to expression P.

Part B
Explain whether or not expressions P and Q are equivalent for any value of x.

**Solution:**

**Part A:**
$6x-18$

**Part B:**
**P and Q are not equivalent since the distributive property was not applied correctly. The first terms of *P* and *Q*, *6x*, are equivalent, but the second terms of *P* a**

***Standard: 6.EE.A.3 DOK: 1 Difficulty: Medium Item Type: SR(Selected Response)***

Select Yes or No to indicate whether the pairs are equivalent expressions.

1a. Are $4(3x-y)$ ***and*** $12x-4y$ equivalent expressions?
◯s ◯

1b. Are $32+16y$ ***and*** $8(4+2y)$ equivalent expressions?
Y◯ ◯

1c. Are $3(x+2y)$ ***and*** $3x+2y$ equivalent expressions?
Y◯ ◯

*Key and Distractor Analysis:*

Key: A correct YYN response to this item will receive 1 point.

*NVACS 6.EE.A.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity.*

A ***variable*** is defined as a letter or symbol that represents a number that can change.
*Examples:* *a, b, c,* _, △, □, …

# Math 6 Notes: Expressions, Equations and Inequalities

An ***algebraic*** *(variable)* ***expression*** is an expression that consists of numbers, variables, and operations.

*Examples:* $a+b,\ 4x,\ 5+z$

*A* ***constant*** is a quantity that does not change, like the number of cents in one dollar.

*Examples: 5, 12*

***Terms*** of an expression are a part or parts that can stand alone or are separated by the + (or – ) symbol. (In algebra we talk about monomials, binomials, trinomials, and polynomials. Each term in a polynomial is a monomial.)

*Example: The expression* ***9+a*** *has 2 terms 9 and a.*

*Example: The expression* ***3ab*** *has 1 term.*

*Example: The expression* $7\boldsymbol{a}^2\boldsymbol{b}-2\boldsymbol{a}$ *has 2 terms,* $7a^2b$ *and 2a.*

*Example: The expression* $\mathbf{2}\boldsymbol{a}\ +\mathbf{3}\boldsymbol{b}\ -\mathbf{4}\boldsymbol{c}$ *has 3 terms, 2a, 3b and 4c.*

A ***coefficient*** is a number that multiplies a variable.

*Example: In the expression* ***3ab,*** the coefficient is *3*

*Example: In the expression* $-2ab+1$**,** the coefficient is $-2$**.** (Note: 1 is a constant.)

*Example: In the expression* $\frac{\boldsymbol{x}}{\mathbf{2}}$**,** the coefficient is $\frac{1}{2}$.

In review, in the algebraic expression $x^2+6x+2y+8$

- the variables are *x* and *y*.
- there are 4 terms, $x^2$, $6x$, $2y$ and 8.
- the coefficients are 1, 6, and 2 respectively.
- There is one constant term, 8.
- This expression shows a sum of 4 terms.

## Word Translations

**NVACS 6.EE.A.2a** *Write expressions that record operations with numbers and with letters standing for numbers.*

Words/Phrases that generally mean:

| | |
|---|---|
| ADD : | total, **sum**, add, in all, altogether, more than, increased by |
| SUBTRACT: | **difference**, less, less than, minus, take away, decreased by, words ending in "er" |
| MULTIPLY: | times, **product,** multiplied by, |
| DIVIDE: | **quotient**, divided by, one, per, each, goes into |

***Examples***:

| Operation | Verbal Expression | Algebraic Expression |
|---|---|---|

# Math 6 Notes: Expressions, Equations and Inequalities

| | | |
|---|---|---|
| Addition + | a number plus 7 | $n + 7$ |
| Addition + | 8 added to a number | $n + 8$ |
| Addition + | a number increased by 4 | $n + 4$ |
| Addition + | 5 more than a number | $n + 5$ |
| Addition + | the sum of a number and 6 | $n + 6$ |
| Addition + | Tom's age 3 years from now | $n + 3$ |
| Addition + | two consecutive integers | $n, n+1$ |
| Addition + | two consecutive odd integers | Let $x = 1$st odd, $x + 2 = 2$nd odd |
| Addition + | 2 consecutive even integers | Let $x = 1$st even, $x + 2 = 2$nd even |
| Subtraction – | a number minus 7 | $x - 7$ |
| Subtraction – | 8 subtracted from a number* | $x - 8$ |
| Subtraction – | a number decreased by 4 | $x - 4$ |
| Subtraction – | 4 decreased by a number | $4 - x$ |
| Subtraction – | 5 less than a number* | $x - 5$ |
| Subtraction – | the difference of a number and 6 | $x - 6$ |
| Subtraction – | Tom's age 3 years ago | $x - 3$ |
| Subtraction – | separate 15 into two parts* | $x, 15 - x$ |
| Multiplication • ( ) | 12 multiplied by a number | $12n$ |
| Multiplication • ( ) | 9 times a number | $9n$ |
| Multiplication • ( ) | the product of a number and 5 | $5n$ |
| Multiplication • ( ) | Distance traveled in $x$ hours at 50 mph | $50x$ |
| Multiplication • ( ) | twice a number | $2n$ |
| Multiplication • ( ) | half of a number | $\frac{n}{2}$ or $\frac{1}{2}n$ |
| Multiplication • ( ) | number of cents in $x$ quarters | $25x$ |
| Division ÷ | a number divided by 12 | $\frac{x}{12}$ |
| Division ÷ | the quotient of a number and 5 | $\frac{x}{5}$ |
| Division ÷ | 8 divided into a number | $\frac{x}{8}$ |

*Be aware that students have difficulty with some of these expressions. For example, "five less than a number" is often incorrectly written as $5 - n$, and should be written $n - 5$.

***Example:*** Write a word translation for the expression $\mathbf{5n + 2}$.

***5 times a number increased by 2***

***Example:*** Write the expression for ***seven more than the product of fourteen and a number x*** $7 + 14x$

***Example:*** Write the expression for ***3 times the sum of a number and 9.***

$\mathbf{3(n + 9)}$

# Math 6 Notes: Expressions, Equations and Inequalities

***Example:*** Write the expression for ***5 less than the product of 2 and a number x*** and evaluate it when ***x = 7.***

$$\begin{aligned} &2x - 5 \\ &= 2 \cdot 7 - 5 \\ &= 14 - 5 \\ &= 9 \end{aligned}$$

## *SBAC Example:*

***Standard: 6.EE.A.1, 6.EE.A.2a*** ***DOK: 1*** ***Item Type:Equation/Numeric***

**TM2**

**Stimulus:** The student is presented with a verbal numerical expression with exponents or verbal algebraic expression with or without exponents.

**Example Stem 1:** Enter a numerical expression that represents the sum of eight squared and thirty-two.

**Example Stem 2:** Enter an algebraic expression that represents eight times the sum of $y$ squared and twenty-eight.

**Rubric:** (1 point) Student enters a correct numerical/algebraic expression for the given verbal expression (e.g., $8^2 + 32$; $8(y^2 + 28)$).

**Response Type:** Equation/Numeric

# Math 6 Notes: Expressions, Equations and Inequalities

***Standard: 6.EE.B.6,*** ***DOK: 1*** ***Difficulty: Medium Item*** ***Type:TE Technology Enhanced***

> Let *b* represent a number.
>
> Click and drag the objects (numbers, operation symbols, letter) to the line below to create an expression that represents the following:
>
> "5 more than the product of 3 and the number *b*"
>
> Not all objects will be used.
>
> | 3 | 5 | *b* | + | − | x | ÷ |
>
> ______________________________

> *Key:*
>
> 3*b*+5 or 3x*b*+5 or *b*x3+5 or 5+3*b* or 5+3x*b* or 5+*b*x3
>
> See TE Information

***Example:*** Write an expression for ***3 times the sum of a number and 9,*** and simplify the expression.

$$3(n+9) = \begin{array}{r} n+9 \\ n+9 \\ +\ n+9 \\ \hline \end{array} = 3 \cdot n + 3 \cdot 9 = 3n+27$$

- Direct translation: $3(n+9)$
- Repeated addition – 3 groups of $n+9$
- Using the Distributive Property: $3 \cdot n + 3 \cdot 9$

***Example:*** Write possible equivalent expressions for $24x+18y$.

Solution:
$$24x+18y=2(12x+9y)$$
$$24x+18y=3(8x+6y)$$
$$24x+18y=6(4x+3y)$$

***SBAC Example:***

***Standard: 6.EE.A.4, DOK: 2 Difficulty: Medium Item Type:TE***

Identify each expression as either equal to 12x + 36y or **not** equal to 12x + 36y. Drag each expression to the appropriate box below.

$(10x+36y)+(2x+y)$ $\qquad 6(2x+6y)$

$3(4x+5y)+7(3y)$ $\qquad 5x+5y+x+y+6x+6y$

| **Expressions Equivalent to 12*x* + 36*y*** | **Expressions Not Equivalent to 12*x* + 36*y*** |
|---|---|
| | |

*Key:*

Expressions equivalent to $12x+36y$ : $6(2x+6y)$, $3(4x+5y)+7(3y)$

Expressions not equivalent to $12x+36y$ : $(10x+36y)+(2x+y)$, $5x+5y+x+y+6x+6y$

# Math 6 Notes: Expressions, Equations and Inequalities

*TE Information:*

**Item Code:** MAT.06.TE.1.000EE.E.690

**Template:** Classification

**Interaction Space Parameters:**

A. The 2 sections of the table: expressions equivalent to $12x+36y$, expressions not equivalent to $12x+36y$

B. The following 4 expressions: $(10x+36y)+(2x+y)$, $6(2x+6y)$, $3(4x+5y)+7(3y)$, $5x+5y+x+y+6x+6y$

**Scoring Data:**

{1=BC,2=AD}=1

## Solving One-Step Equations

**NVACS 6.EE.B.7** *Solve real-world and mathematical problems by writing and solving equations of the form $x+p=q$ and $px=q$ for cases in which p, q, and x are all **nonnegative rational numbers.***

**NVACS 6.EE.A.5** *Understand solving an equation or inequality as a process of answering the question: which values from a specified set, if any, make the equations or inequalities true? Use substitution to determine whether the given number in a specified set makes an equation or inequality true.*

An **equation** is a mathematical statement using the equal sign between two mathematical expressions naming the same number. For example, $3 + 5 = 8$, $x+3=12$ or $3x - 1 = 17$ are equations.

**Solving Equations –** finding the value(s) of *x* which make the equation a true statement.

**Strategy for Solving Equations**: To solve linear equations, put the variable terms on one side of the equal sign, and put the constant (number) term on the other side. To do this, use OPPOSITE (or INVERSE) OPERATIONS.

Let's look at a gift wrapping analogy to better understand this strategy. When a present is wrapped, it is placed in a box, the cover is put on, the box is wrapped in paper, and finally a ribbon is added to complete the project. To get the present out of the box, everything would be done in reverse order, performing the OPPOSITE (INVERSE) OPERATION. First we take off the ribbon, then take off the paper, next take the cover off, and finally take the present out of the box.

To **solve equations in the form of $x + b = c$**, we will "undo" this algebraic expression to isolate the variable. To accomplish this, we will use the opposite operation to isolate the variable.

***Example:*** Solve for $x$ in the equation $x - 5 = 8$.

$$x - 5 = 8$$
$$+5 = +5$$
$$x = 13$$

To isolate the $x$ term, undo "subtracting 5" by "adding 5" to both sides.

$$13 - 5 = 8 \checkmark$$

Check to see that the answer is a solution.

It is also common practice to show the work this way:

$$x - 5 = 8$$
$$x - 5 + 5 = 8 + 5$$
$$x = 13$$

***Example:*** Solve: $x+7=16$.

$$x+7=16$$
$$-7=-7$$
$$x=9$$

To isolate the $x$ term, undo "adding 7" by "subtracting 7" from both sides.

$9+7=16$ ✓ Check to see that the answer is a solution.

It is also common practice to show the work this way:

$$x+7=16$$
$$x+7-7=16-7$$
$$x=9$$

***Example:*** Solve: $3x=27$.

$$3x=27$$
$$\frac{3x}{3}=\frac{27}{3}$$
$$x=9$$

To isolate the $x$ term, undo "multiplying by 3" by "dividing both sides by 3".

$3(9)=27$ ✓ Check to see that the answer is a solution

***Example:*** Solve: $\frac{x}{4}=12$.

$$\frac{x}{4}=12$$
$$(4)\frac{x}{4}=12(4)$$
$$x=48$$

To isolate the $x$ term, undo "dividing by 4" by "multiplying both sides by 4".

$\frac{48}{4}=12$ ✓ Check to see that the answer is a solution

**Solving Equations Involving Rational Numbers**

**Strategy for Solving**: You solve equations containing nonnegative rational numbers the same as you do with whole numbers; the strategy does not change. Subtraction and addition are inverse operations. If an equation contains addition, solve it by subtracting from both sides to undo the addition. When an equation contains subtraction, use addition to undo the subtraction.

***Example:*** Solve: $x-\frac{1}{3}=\frac{2}{5}$.

$$x-\frac{1}{3}=\frac{2}{5}$$
$$x-\frac{1}{3}+\frac{1}{3}=\frac{2}{5}+\frac{1}{3}$$
$$x=\frac{6}{15}+\frac{5}{15}$$
$$x=\frac{11}{15}$$

Undo subtracting one-third by adding one-third to both sides of the equation; make equivalent fractions with a common denominator of 15; add.

Check:
$$\frac{11}{15}-\frac{1}{3}=\frac{2}{5}$$
$$\frac{11}{15}-\frac{5}{15}=\frac{6}{15}$$
$$\frac{6}{15}=\frac{6}{15}\ \checkmark$$

# Math 6 Notes: Expressions, Equations and Inequalities

***Example*:** Solve: $4\frac{2}{5} = x + \frac{9}{10}$

$$4\frac{2}{5} = x + \frac{9}{10}$$
$$4\frac{2}{5} - \frac{9}{10} = x + \frac{9}{10} - \frac{9}{10}$$
$$4\frac{4}{10} - \frac{9}{10} = x$$
$$3\frac{14}{10} - \frac{9}{10} = x$$
$$3\frac{5}{10} = x$$
$$3\frac{1}{2} = x$$

Undo adding $\frac{9}{10}$ by subtracting $\frac{9}{10}$ from both sides.
Make equivalent fractions with a common denominator of 10, regroup and subtract.
Simplify.

Check: $4\frac{2}{5} = 3\frac{1}{2} + \frac{9}{10}$
$4\frac{4}{10} = 3\frac{5}{10} + \frac{9}{10}$
$4\frac{4}{10} = 3\frac{14}{10} = 4\frac{4}{10}$ ✓

***Example*:** Solve: $x + 0.6 = 1.5$

$$x + 0.6 = 1.5$$
$$x + 0.6 - 0.6 = 1.5 - 0.6$$
$$x = 0.9$$

Undo adding 0.6 by subtracting 0.6 from both sides. Simplify.

Check: $0.9 + 0.6 = 1.5$ ✓
$1.5 = 1.5$

***Example*:** Solve: $x - 1.2 = 2.3$

$$x - 1.2 = 2.3$$
$$x - 1.2 + 1.2 = 2.3 + 1.2$$
$$x = 3.5$$

Undo subtracting 1.2 by adding 1.2 from both sides.
Simplify.

Check: $3.5 - 1.2 = 2.3$
$2.3 = 2.3$ ✓

**Strategy for Solving**: You solve equations containing nonnegative rational numbers the same as you do with whole numbers; the strategy does not change. To solve an equation that contains multiplication, use division to undo the multiplication. To solve an equation involving division, use multiplication to undo the division.

# Math 6 Notes: Expressions, Equations and Inequalities

***Example***: Solve: $\frac{x}{5}=\frac{2}{3}$

$$\frac{x}{5}=\frac{2}{3}$$
$$\left(\frac{5}{1}\right)\left(\frac{x}{5}\right)=\frac{2}{3}\left(\frac{5}{1}\right)$$
$$x=\frac{10}{3} \text{ or } 3\frac{1}{3}$$

To undo division by 5, multiply both sides of the equation by 5. Put the answer in simplest form.

$$\frac{3\frac{1}{3}}{5}=\frac{2}{3}$$

Check: $3 \bullet 3\frac{1}{3}=5 \bullet 2$
$10=10$ ✓

***Example***: Solve: $\frac{2}{5}x=3$

$$\frac{2}{5}x=3$$
$$\frac{2}{5}x \div \frac{2}{5}=3 \div \frac{2}{5}$$
$$\frac{2}{5}x \cdot \frac{5}{2}=3 \cdot \frac{5}{2}$$
$$\frac{2}{5}x \cdot \frac{5}{2}=\frac{3}{1} \cdot \frac{5}{2}$$
$$x=\frac{15}{2} \text{ or } 7\frac{1}{2}$$

To undo multiplication by $\frac{2}{5}$, divide both sides of the equation by $\frac{2}{5}$.

Dividing by $\frac{2}{5}$ is the same as multiplying by the reciprocal $\frac{5}{2}$.

Simplify the answer as needed.

Check: $\frac{\overset{1}{2}}{\underset{1}{5}} \bullet \frac{\overset{3}{15}}{\underset{1}{2}}=3$
$3=3$ ✓

***Example:*** Solve: $4x=\frac{6}{7}$

$$4x=\frac{6}{7}$$
$$4x \cdot \frac{1}{4}=\frac{{}^{3}6}{7} \cdot \frac{1}{4_2}$$
$$x=\frac{3}{14}$$

To undo multiplying by 4, we divide by 4. Dividing by 4 is the same as multiplying by the reciprocal $\frac{1}{4}$.

Cancel and simplify.

Check: $\frac{4}{1} \bullet \frac{3}{14}=\frac{6}{7}$
$\frac{6}{7}=\frac{6}{7}$ ✓

***Example:*** Solve: $0.4x=1.6$

$$0.4x=1.6$$
$$\frac{0.4x}{0.4}=\frac{1.6}{0.4}$$
$$x=4$$

To undo multiplying by 0.4, we divide both sides by 0.4. Simplify.

Check: $0.4 \bullet 4=1.6$
$1.6=1.6$ ✓

# Math 6 Notes: Expressions, Equations and Inequalities

***Example:*** Solve: $\frac{x}{2.5} = 6$

$$\frac{x}{2.5} = 6$$
$$\frac{x}{2.5} \bullet 2.5 = 6 \bullet 2.5$$
$$x = 15.0 = 15$$

To undo dividing by 2.5, we multiply both sides by 2.5. Simplify.

Check: $\frac{15}{2.5} = 6$

$6 = 6$ ✓

**OnCore examples:**

***Example:*** Which is the solution to the equation $x - 5 = 12$?

| | |
|---|---|
| A. | $x = 7$ |
| B. | $x = 8$ |
| C | $x = 17$ |
| D. | $x = 18$ |

***Example:*** Solve $p + 2\frac{4}{5} = 4\frac{1}{2}$

| | |
|---|---|
| **A.** | $p = 1\frac{3}{10}$ |
| **B.** | $p = 1\frac{7}{10}$ |
| **C.** | $p = 2\frac{3}{10}$ |
| **D.** | $p = 2\frac{7}{10}$ |

***Example:*** Solve $7.2h = 57.6$

| | |
|---|---|
| **A.** | $h = 0.8$ |
| **B.** | $h = 8$ |
| **C.** | $h = 50.4$ |
| **D.** | $h = 80$ |

Now we use our skills in translating from words to math expressions to form equations to help us solve word problems. Look for the key word "is" "the result is", "equals", "will be", 'was", etc. to help place the "=" symbol.

# Math 6 Notes: Expressions, Equations and Inequalities

***Example:*** When 15 is subtracted from a number, the result is 56. Write an equation that can be used to find the original number. Then find the original number.

Let $x$ represent the original number.

$x - 15 = 56$ is translated from "when 15 is subtracted from a number"

$$x - 15 = 56$$
$$+15 = +15$$
$$x = 71$$

The original number is 71.

***Standard: 6.EE.B.5, DOK: 2 Difficulty: Medium Item Type:SR Selected Response***

Select the equation(s) where x = 5 is a solution. Click all that apply.

Ⓐ 2x + 4 = 14

Ⓑ 5x = 55

Ⓒ 6x + 3 = 14

Ⓓ 8 + 3x = 23

Ⓔ 6x = 30

Ⓕ 5x = 1

*Key and Distractor Analysis or Scoring Rubric for Multi-part Items:*

**2 points:** The student shows a thorough understanding of evaluating equations at specific values. Chooses A, D, and E ONLY.

**1 point:** The student shows partial understanding of evaluating equations at specific values. Misses only 1 of the correct answers.

**0 points:** The student shows inconsistent or no understanding of evaluating equations at specific values or solving equations.

# Math 6 Notes: Expressions, Equations and Inequalities

*TE information:*

**Template: Select and Order**
**Item ID:** MAT.06.TE.1.000EE.F.170
**Interaction Space Parameters:**

A. An image with seven squares containing the two digits, one variable, and four operators 3, 5, *b*, +, −, x, ÷ respectively.

B. Five images of squares.

| 3 | 5 | *b* | + | − | x | ÷ |
|---|---|---|---|---|---|---|

**Scoring Data:**

{1=A, 2=F, 3=C, 4=D, 5=B} = 1
{1=C, 2=F, 3=A, 4=D, 5=B} = 1
{1=B, 2=D, 3=A, 4=F, 5=C} = 1
{1=B, 2=D, 3=C, 4=F, 5=A} = 1

***Standard: 6.EE.B.5, DOK: 2 Difficulty: High Item Type:ER Enhanced Response***

***Part A***

Ana is saving to buy a bicycle that costs $135. She has saved $98 and wants to know how much more money she needs to buy the bicycle.

The equation $135 = x + 98$ models this situation, where $x$ represents the additional amount of money Ana needs to buy the bicycle.

- When substituting for $x$, which value(s), if any, from the set $\{0, 37, 98, 135, 233\}$ will make the equation true?
- Explain what this means in terms of the amount of money needed and the cost of the bicycle.

***Part B***

Ana considered buying the $135 bicycle, but then she decided to shop for a different bicycle. She knows the other bicycle she likes will cost more than $150.

This situation can be modeled by the following inequality.

$$x + 98 > 150$$

- Which values, if any, from $-250$ to 250 will make the inequality true? If more than one value makes the inequality true, identify the least and greatest values that make the inequality true.
- Explain what this means in terms of the amount of money needed and the cost of the bicycle.

*Sample Top-Score Response:*

***Part A***
37 is the only value in the set that makes the equation true.

This means that Ana will need exactly $37 more to buy the bicycle.

***Part B***
The values from 53 to 250 will make the inequality true.

This means that Ana will need from $53 to $250 to buy the bicycle.

*Scoring Rubric:*

Responses to this item will receive 0–3 points, based on the following:

**3 points:** The student shows a thorough understanding of equations and inequalities in a contextual scenario, as well as a thorough understanding of substituting values into equations and inequalities to verify whether or not they satisfy the equation or inequality. The student offers a correct interpretation of the equality and the inequality in the context of the problem. The student correctly states that 37 will satisfy the equation and that the values from 53 to 250 will satisfy the inequality.

**2 points:** The student shows a thorough understanding of substituting values into equations and inequalities to verify whether or not they satisfy the equation or inequality but limited understanding of equations or inequalities in a contextual scenario. The student correctly states that 37 will satisfy the equation and that the values from 53 to 250 will satisfy the inequality, but the student offers an incorrect interpretation of the equality or the inequality in the context of the problem.

**1 point:** The student shows a limited understanding of substituting values into equations and inequalities to verify whether or not they satisfy the equation or inequality and a limited understanding of equations and inequalities in a contextual scenario. The student correctly states that 37 will satisfy the equation, does not state that the values from 53 to 250 will satisfy the inequality, and offers incorrect interpretations of the equality and the inequality in the context of the problem. **OR** The student correctly states that the values from 53 to 250 will satisfy the inequality, does not state that 37 satisfies the equation, and offers incorrect interpretations of the equality and the inequality in the context of the problem.

**0 points:** The student shows little or no understanding of equations and inequalities in a contextual scenario and little or no understanding of substituting values into equations and inequalities to verify whether or not they satisfy the equation or inequality. The student offers incorrect interpretations of the equality and the inequality in the context of the problem, does not state that 37 satisfies the equation, and does not state that the values from 53 to 250 will satisfy the inequality.

# Math 6 Notes: Expressions, Equations and Inequalities

***Standard: 6.EE.B.5,*** ***DOK: 2*** ***Item Type:Multiple Choice Multiple correct response***

**TM1d**
**Example Stem:** Consider the inequality $x > 7$.

Determine whether each value of $x$ makes this inequality true. Select Yes or No for each value.

| *x* | Yes | No |
|---|---|---|
| 22 | | |
| -7 | | |
| 13 | | |
| 5 | | |
| -39 | | |

**Rubric:** (1 point) Student correctly determines whether all five values make the inequality true (e.g., Y, N, Y, N, N).

**Response Type:** Matching Tables

**NVACS 6.EE.A.6** *Use variables to represent numbers and write expression when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.*

***Example:*** Between the hours of 6 a.m. and 9 p.m., 8 buses that were filled to capacity left the terminal. Since the capacity of each bus is the same and 392 tickets were sold, how many passengers were on each bus?

*Solution:* *Let p = the number of passengers on each bus*
*We know the total is 392 tickets.*
*There were 8 buses .*

$$8p = 392$$
$$\frac{8p}{8} = \frac{392}{8}$$
$$p = 49$$
$$49 \textit{ passengers per bus}$$

***Example:*** The area of a rectangle is 42 square meters. Its width is 7 meters. What is its length?

Let $l$ represent the length.

$A = l \times w$ is translated from "the area of the rectangle is 42" and "width is 7"

$42 = l \times 7$

$$42 = l \times 7$$
$$\frac{42}{7} = \frac{l \times 7}{7}$$
$$6 = l$$

The length of the rectangle is 6 meters.

Emphasize to students that they need to include the label "meters".

***Example:*** In a fish tank, $\frac{8}{11}$ of the fish have a red stripe on them. If 16 of the fish have red stripes, how many total fish are in the tank?

| | |
|---|---|
| A. | 20 fish |
| B. | 21 fish |
| C. | 22 fish |
| D. | 26 fish |

***Example:*** You are working for as an assistant to a chef. The chef made $\frac{3}{4}$ of a recipe and used 20 cups of milk. He wrote the following equation to show how many cups of milk he would use if he made the whole recipe. $\frac{3}{4} = 20x$ Did he write the equation correctly? If not, write the equation as it should be.

***Example:*** Black rhinos usually live to be 50 years old. A black rhino at the zoo is 23 years old. Write an equation and solve it to show how many years longer the black rhino will most likely live.

# Math 6 Notes: Expressions, Equations and Inequalities

***Example:*** Two stacks of books together are $7\frac{1}{2}$ inches tall. If one stack is $4\frac{3}{4}$ inches tall, how tall is the other? Write an equation and solve it.

***Example:*** Sami is inviting her friends to the roller-skating rink for her birthday party.

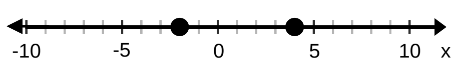
**Part A:** If the skating rink charges \$6 per person and the birthday party will cost \$72, how many skaters will be at Sami's party? Use an equation to solve and check your answer.
**Part B:** What will be the new cost of the birthday party if each skater receives \$2 spending money? Check your answer.

***NVACS 6.EE.B.8*** *Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.*
**NVACS 6.EE.A.5** *Understand solving an equation or inequality as a process of answering the question: which values from a specified set, if any, make the equations or inequalities true? Use substitution to determine whether the given number in a specified set makes an equation or inequality true.*

***Example:*** Write an inequality for the two positive numbers on the number line below:


In addition to being able to write 3 < 9 and 9 > 3, students should be able to state that 3 is less than 9, 3 is located to the left of 9, and/or 9 is greater than 3 and 9 is located to the right of 3.

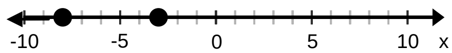
***Example:*** Write an inequality for the numbers on the number line below.




In addition to being able to write $-2 < 4$ and $4 > -2$, students should be able to state that 2 is less than 4, $-2$ is located to the left of 4, and/or 4 is greater than $-2$ and 4 is located to the right of $-2$.

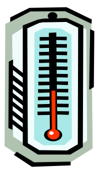
***Example:*** Write an inequality for the numbers on the number line below.

# Math 6 Notes: Expressions, Equations and Inequalities




In addition to being able to write $-8 < -3$ and $-3 > -8$, students should be able to state that $-8$ is less than $-3$, $-8$ is located to the left of $-3$, and/or $-3$ is greater than $-8$ and $-3$ is located to the right of $-8$.

***Example:***



Given 2 thermometer readings (or vertical number lines) students should be able to interpret which temperature is colder/lower, write statements that compare the temperatures, and write inequalities to represent the situation.

For example, given two thermometers, one reading $-5°$ Fahrenheit and the second reading $3°$ Fahrenheit, students should be able to state $-5 < 3$, $3 > -5$, $3°$ is warmer than $-5°$ and $-5°$ is colder than $3°$

***Example:*** What statement is true?

| | |
|---|---|
| ***A.*** | $-13 > 11$ |
| ***B.*** | $-8 < -2$ |
| ***C.*** | $-1 < -19$ |
| ***D.*** | $-5 > 16$ |

Solution: B

***Example:*** The level of the top of the water in the ocean is considered to be an altitude of zero (0) feet. The ocean floor at a particular dive site is $-25$ feet. A diver at the site is located at $-8$ feet.

Write an inequality that represents the relationship between the location of the diver to the dive site.

Solution: $-8 > -25$ *or* $-25 < -8$

Interpret/Explain in words the location of the diver to the dive site.

Solution: Although answers may vary, several possible solutions are:

The diver is closer to the top of the water than the dive site is to the top of the water. **OR**
The dive site is below the diver. **OR**
The diver is above the dive site. **OR**
The diver is descending to the dive site.

# Math 6 Notes: Expressions, Equations and Inequalities

## Solving One-Step Inequalities

We use inequalities in real life all the time. Let's say you are going to purchase a $2 candy bar and you do not have to use exact change. How would you list all the amounts of money that are enough to buy the item? You might start a list: $3, $4, $5, $10; quickly you would discover that you could not list all possibilities. However, you could make a statement like "any amount of money $2 or more" and that would describe all the values.
In algebra, we use inequality symbols to compare quantities when they are not equal, or compare quantities that may or may not be equal.

| This symbol | means | and can be disguised in word problems as |
|---|---|---|
| $<$ | is less than | below, fewer than, less than |
| $>$ | is greater than | above, must exceed, more than |
| $\leq$ | is less than or equal to | at most, cannot exceed, no more than |
| $\geq$ | is greater than or equal to | at least, no less than |

An ***inequality*** is a mathematical sentence that shows the relationship between quantities that are *not* equal. For example, m>5, $2x < 8$ and $4x-7 \geq 35$ are inequalities.

Our strategy to solve inequalities will be to isolate the variable on one side of the inequality and numbers on the other side by using the opposite operation (same as equations).

***Example:*** Solve the inequality for *x*: $3x \leq 27$.

Isolate the variable by dividing both sides by 3 of the inequality

$$3x \leq 27$$

$$\frac{3x}{3} \leq \frac{27}{3}$$

$$x \leq 9$$

***Example:*** Solve the inequality for *y*: $y + 6 > 10$.

Isolate the variable by subtracting 6 from both sides of the inequality.

$$y + 6 > 10$$

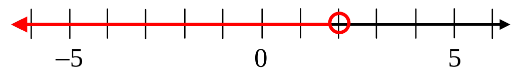
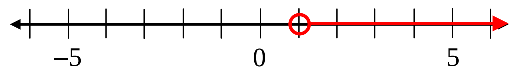
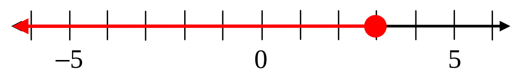
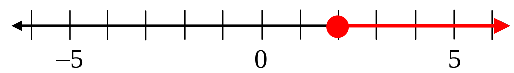
$$y + 6 - 6 > 10 - 6$$

$$y > 4$$

## Graphing Solutions of Equations and Inequalities in One Variable

The *solution* of an inequality with a variable is the set of all numbers that make the statement true. You can show this solution by graphing on a number line.

# Math 6 Notes: Expressions, Equations and Inequalities

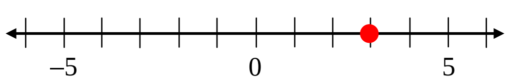
| inequality | in words | graph |
|---|---|---|
| $x < 2$ | all numbers less than two |  |
| $x > 1$ | all numbers greater than one |  |
| $x \le 3$ | all numbers less than or equal to three |  |
| $x \ge 2$ | all numbers greater than or equal to two |  |

*Note that an open circle ○ is used in the "is less than" or "is greater than" graphs, indicating that the number* <u>*is not*</u> *included in the solution. A closed circle ● is used in the "is greater than or equal to" or "is less than or equal to" graphs to indicate that the number* <u>*is*</u> *included in the solution.*

We can solve linear inequalities the same way we solve linear equations. We use the Order of Operations in reverse, using the opposite operation. Linear inequalities look like linear equations with the exception they have an inequality symbol ($<$, $>$, $\le$, or $\ge$) rather than an equal sign. **Note: we will limit our equation answers and the open or closed circles to whole number values (we will show the less than inequality graphs continuing into negative values).**

***Example:*** Linear Equation:

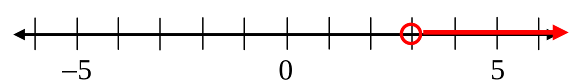
$$x - 2 = 1$$
$$x - 2 + 2 = 1 + 2$$
$$x = 3$$



Notice the graph on the left only has the one point representing 3 plotted; one solution. That translates to $x = 3$.

Linear Inequality:

$$x - 2 > 1$$
$$x - 2 + 2 > 1 + 2$$
$$x > 3$$

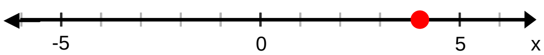


Notice the graph on the right has a dot on 3 which is not shaded because 3 is not part of the solution. Also notice that there is a solid line to the right of the open dot representing all values greater than 3 are part of the solution.

***Example:***

Linear Equation:

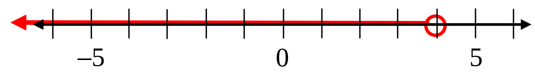
$$x+3=7$$
$$x+3-3=7-3$$
$$x=4$$




The graph on the left has a point plotted at 4 indicating there is only one solution. That translates to *x* = 4.

Linear Inequality:

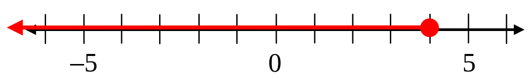
$$x+3<7$$
$$x+3-3<7-3$$
$$x<4$$




The graph on the right has a dot on 4, which is *not* shaded because 4 is not included as part of the solution. Also notice that there is a solid line to the left of the open dot, representing all the values less than 4 are part of the solution set.

If the last example contained the symbol "≤", $x+3\le 7$, then everything would be done the same in terms of solving the inequality, but the solution and graph would look a little different. It would include 4 as part of the solution set (closed/shaded circle) and all the values less than 4 are part of the solution set.

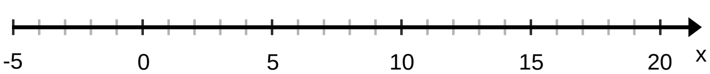
The solution $x \le 4$.




***Example:*** Solve the inequality for $t$: $t-6<9$ and graph the solution.

Solution: Isolate the variable by adding 6 to both sides of the inequality.

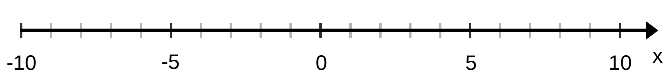
$$t-6<9$$
$$t-6+6=9+6$$
$$t<15$$




***Example:*** Solve the inequality for $z$: $z+8\le 9$ and graph the solution.

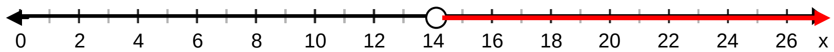
Solution: Isolate the variable by subtracting 8 to both sides of the inequality.

$$z+8\le 9$$
$$z+8-8=9-8$$
$$z\le 1$$



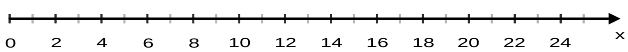
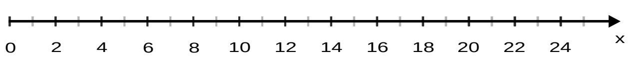
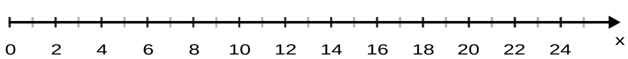
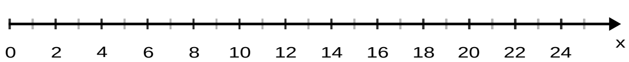
# Math 6 Notes: Expressions, Equations and Inequalities

***Example:*** Write an inequality that represents the graph below if the endpoint is moved 5 units to the left.




| | |
|---|---|
| A. | $x > 14$ |
| B. | $x < 14$ |
| C. | $x > 9$ |
| D. | $x > 17$ |

***Example:*** Dominica made more than \$8 babysitting yesterday. Her sister Tamara made \$13 more than Dominica did. Write an inequality to represent the amount of money Tamara could have made. Graph the inequality on the number line. Could \$15 be the amount of money Tamara made?

| | |
|---|---|
| A. | $T > 8$ <br>  <br> Yes; 15 is greater than 8. |
| B. | $T > 13$ <br>  <br> Yes; 15 is greater than 13. |
| C. | $T > 21$ <br>  <br> No; 15 is not greater than 21. |
| D. | $T \geq 21$ <br>  <br> No; 15 is not greater than or equal to 21. |

# Math 6 Notes: Expressions, Equations and Inequalities

***Standard: 6.EE.B.8,   DOK: 1   Difficulty: Medium   Item Type:SR Selected Response***

An inequality is shown.

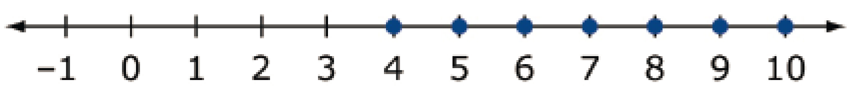
X > 4

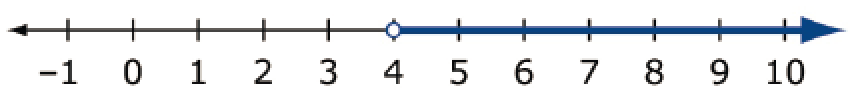
Select the statement(s) and number line(s) that can be represented by the inequality. Click all that apply.

Ⓐ The temperature increased by 4° Fahrenheit.

Ⓑ The value of a number substituted for *x* is greater than 4.

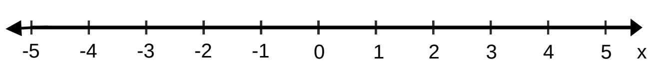
Ⓒ Marcus drinks more than 4 glasses of water every day.

Ⓓ 

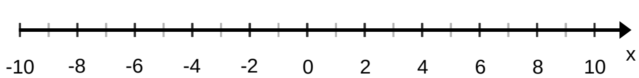

Ⓔ 


Key: B,C,E

***Example:*** Graph the solutions to $y < 2.7$ on the number line below.




***Example:*** Choose the inequality shown by the graph below.




| | |
|---|---|
| A. | $x < 7$ |
| B. | $x \le 7$ |
| C. | $x > 7$ |
| D. | $x \ge 7$ |

***Example:*** There are at least 37 visitors to the counselor's office each day at school. Which inequality represents the number of visitors to the counselor's office?

| | |
|---|---|
| A. | $v < 37$ |
| B. | $v \leq 37$ |
| C. | $v > 37$ |
| D. | $v \geq 37$ |

**NVACS 6.EE.B.9** *Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.*

## Writing Expressions or Equations from Tables

## Arithmetic Sequence

A ***sequence*** is a set of numbers in a particular order. Each number is called a ***term*** of the sequence.

***Example:*** {1, 3, 6, 10, 15} is a sequence with 5 terms.

An ***arithmetic sequence*** *is a* sequence in which the terms change by the **same amount** each time. We are often asked to find the next term(s) in the sequence or identify the pattern. To find the next term in an arithmetic sequence, subtract any two consecutive terms, then add that difference to the last term to find the next term. (Some exams will refer to finding a ***pattern*** in lieu of using the word "sequence".)

***Example:*** Find the next term in the following arithmetic sequence:
{5, 10, 15, 20, …}

- Students know this pattern by heart so it is probably a good place to begin. They know they are skip counting by 5, or increasing by 5 each time so the next term in the sequence is 25.

***Example:*** Find the next term in the following arithmetic sequence:
{8, 19, 30, 41, …}

- Now this one requires more thought. Choose two consecutive terms (8, 19) or (19, 30) and find their difference:

30 – 19 = **11**.

- Add the difference (11) to the last term.
  41 + 11 = 52.

52 is the next term in the sequence.

Using a table is a useful way to see the pattern and identify the next 3 terms.

| Position | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Value of the Term | 8 | 19 | 30 | 41 | ? | ? | ? |

+11 +11 +11 +11

The next three terms would be 52, 63 and 74 because 41 + 11 = 52, 52 + 11 = 63, and 63 + 11 = 74.

***Example:*** Identify the pattern and find the next 3 terms: {4, 10, 8, 14, 12, …}

| Position | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Value of the Term | 4 | 10 | 8 | 14 | 12 | ? | ? | ? |

+6 -2 +6 -2

The 6th term in the sequence would be 18 (which we obtain by adding 6 to the 12).
The 7th term in the sequence would be 16 (which we obtain by subtracting 2 from 18).
The 8th term in the sequence would be 22 (which we obtain by adding 6 to the 16).

***Example:*** Identify the pattern and find the next term: {2, 5, 8, 11, 14, …}

| Position | 1 | 2 | 3 | 4 | 5 | 6 | … | $n^{th}$ |
|---|---|---|---|---|---|---|---|---|
| Value of the Term | 2 | 5 | 8 | 11 | 14 | ? | ? | ? |

+3 +3 +3 +3

The next term in the sequence would be 17 (which we obtain by adding 3 to the 14).

***Example:*** Identify the pattern and find the next term: {5, 10, 15, 20, …}

This arithmetic sequence has an infinite number of terms. The terms are obtained by adding 5 to the previous term. The next term would be 25.

Using a table:

| Position | 1 | 2 | 3 | 4 | 5 | … | $n^{th}$ |
|---|---|---|---|---|---|---|---|
| Value of the Term | 5 | 10 | 15 | 20 | ? | | |

+5 +5 +5 +5

The pattern can be determined by looking for a relationship between the *position* and the *value of the term*. We can see that in each case the position is multiplied by 5 to obtain the value; hence, the $n^{th}$ term would be **5 times *n***, or **5*n***.

***Example:*** Make a table or chart to display the data. Write an expression for the rule.

*The height of a painting is 4 times its width.*

Create and complete a table.

| Width | 1 | 2 | 3 | 4 | 5 | 6 | … | $n^{th}$ |
|---|---|---|---|---|---|---|---|---|
| Height | ? | ? | ? | ? | ? | ? | ? | ? |

**1•4 = 4**

**2•4 = 8**

1 **3•4 = 12**

**4•4 = 16**

**5•4 = 20**

**6•4 = 24**

| Width | 1 | 2 | 3 | 4 | 5 | 6 | … | $n^{th}$ |
|---|---|---|---|---|---|---|---|---|
| Height | 4 | 8 | 12 | 16 | 20 | 24 | ? | |

×4 ×4 ×4 ×4

This pattern is simple – the **height is four times the width** so the rule is **4× width**, **4× *n*** or **4*n*.**

***Example:*** Write an expression for the rule for the following data.

| *x* | *y* |
|---|---|
| 1 | 8 |
| 2 | 9 |
| 3 | 10 |

| | |
|---|---|
| 4 | 11 |
| 5 | 12 |
| 6 | |
| … | |
| $n^{th}$ | |

In this pattern both the *x* and *y* terms are increasing by one each time. Upon further examination we see the *x* increase by 7 each time to equal the *y* value or the difference between each pair of *x* and *y* terms shows an increase of 7 each time so the rule is **add 7** or ***n* + 7**.

**BEWARE: NOT all sequences follow this simple format for finding the rule.**

***Example:*** Identify the pattern and find the next term: {2, 5, 8, 11, 14, …}

| Position | 1 | 2 | 3 | 4 | 5 | 6 | … | $n^{th}$ |
|---|---|---|---|---|---|---|---|---|
| Value of the Term | 2 | 5 | 8 | 11 | 14 | ? | ? | ? |

+3 +3 +3 +3

The next term in the sequence would be 17 (which we obtain by adding 3 to the 14).
The pattern for this one is not so easily determined—simple addition or multiplication does not work. We would have to try some combination of multiplication and addition/subtraction. In this case, the $n^{th}$ term would be $3n-1$. (That is, take term "1", multiply by 3 and subtract 1 and you arrive at the value "2". Take term "4" and test it: multiply 4 by 3 and then subtract 1 and you arrive at the value "11").

***Example:*** John needs to purchase gas for his vehicle. Complete the table shown below and write an expression that can be used to find the total cost for "*n*"gallons of gasoline.

| Number of gallons of gasoline | Total Cost ($) |
|---|---|
| 1 | $2.75 |
| 2 | $5.50 |
| 3 | $8.25 |
| 4 | ? |
| *n* | ? |

# Math 6 Notes: Expressions, Equations and Inequalities

***Example:*** Write an expression to describe the relationship in the following table and find the next term.

| $1^{st}$ column | $2^{nd}$ column |
|---|---|
| 1 | 4 |
| 2 | 5 |
| 3 | 6 |
| 4 | 7 |
| 5 | ? |

Examining the $1^{st}$ column we see the values are sequential and increasing by 1's. Examining the second column, the numbers seem to be increasing by 1, so the next term would be 8. Looking at the first and second rows, it appears the number in the second column is always 3 more than the number in the first column. Let $x$ represent a number in the first column. The algebraic expression that describes the relationship between the first column and the second column is $x + 3$.

***Example:*** Using the information in the table, choose the expression for the missing value.

| Marx's Age | Juan's Age |
|---|---|
| 10 | 14 |
| 11 | 15 |
| 12 | 16 |
| 13 | 17 |
| $m$ | ? |

| | |
|---|---|
| A. | $m+1$ |
| B. | $m+2$ |
| C. | $m+4$ |
| D. | $m+17$ |

***Example:*** Write an expression for the sequence {3, 5, 7, 9, 11, ?, …}and find the next term.

First, create a table that summarizes the given information:

| Term ($n^{th}$) | $1^{st}$ | $2^{nd}$ | $3^{rd}$ | $4^{th}$ | $5^{th}$ | $6^{th}$ |
|---|---|---|---|---|---|---|
| Value of term | 3 | 5 | 7 | 9 | 11 | ? |

Examining the second row, the numbers seem to be increasing by 2, so the next term would be 13.

Looking for a pattern to describe the relationship between the first and second rows is not obvious, so try *trial & error* – referred to as guess & check.

If I multiply the first row by 2, that does not give me the corresponding number in the second row. But, if I add one to the doubling, that gives me 3 and that works for the value of the first term. Try multiplying each term by 2 and then adding one. That seems to be working. Let $n$ represent a value in the “Term” row. The algebraic expression for the above sequence is $2n+1$.

Caution: A common error is for students to write an expression that compares the *term* to the *term* or the *value of the term* to the *value of the term* (rather than correctly comparing the *term* to the *value of the term*).

For example, in the above problem a common error would be to incorrectly write $n+2$ for the rule (comparing the second row values to each other) rather than comparing row 1 to row 2 (to obtain $2n+1$).
When you can find the next term in a sequence by adding the same number (constant) to the preceding term, you can use a formula to find the algebraic expression for that sequence. *(The following material is for teacher reference—it is not necessarily intended to be part of your lesson plan with your Math 6 classes.)*

In the previous example, the value of the next term was found by always adding 2. After you know the first value of the term, how many times will you add 2 to get to value of the second term, the third term, fourth term, fifth term and sixth term?

The answer is you will always add the constant one less time than the value of the term you are trying to find. So, the second term, you will add 2 once. For the fifth term, you will add 2 four times to the first term. For the *n*th term, you will add the constant $(n-1)$ times.

To find the expression, start with the value of the first term, which is 3, then I add the constant $(n-1)$ times.

$$\begin{aligned} 3+(n-1)2 &= 3+2n-2 \\ &= 2n+1 \end{aligned}$$

***Example:*** Write an expression for the following sequence described in the table and find the missing term and the 101$^{st}$ term.

| Term ($n^{th}$) | 1$^{st}$ | 2$^{nd}$ | 3$^{rd}$ | 4$^{th}$ | 5$^{th}$ | 6$^{th}$ | 7$^{th}$ |
|---|---|---|---|---|---|---|---|
| Value of term | 6 | 11 | 16 | 21 | 26 | 31 | ? |

The numbers in the sequence (numbers in the second row) seem to be increasing by 5. So the next term would be 36.

Now try to find an expression by trial & error, or, since I am adding the same number over again to find the next term, ask how many times the constant is being added **to the first term** to get to the nth term? Answer: $(n-1)$.

$$6+(n-1)5=6+5n-5$$
$$=5n+1$$

Use the expression $5n+1$ to find the $101^{st}$ term: 5(101) + 1 = 506. The $101^{st}$ term is 506.

***Standard: 6.EE.B.7, DOK: 1 Difficulty: Medium Item Type:TE Technology Enhanced***

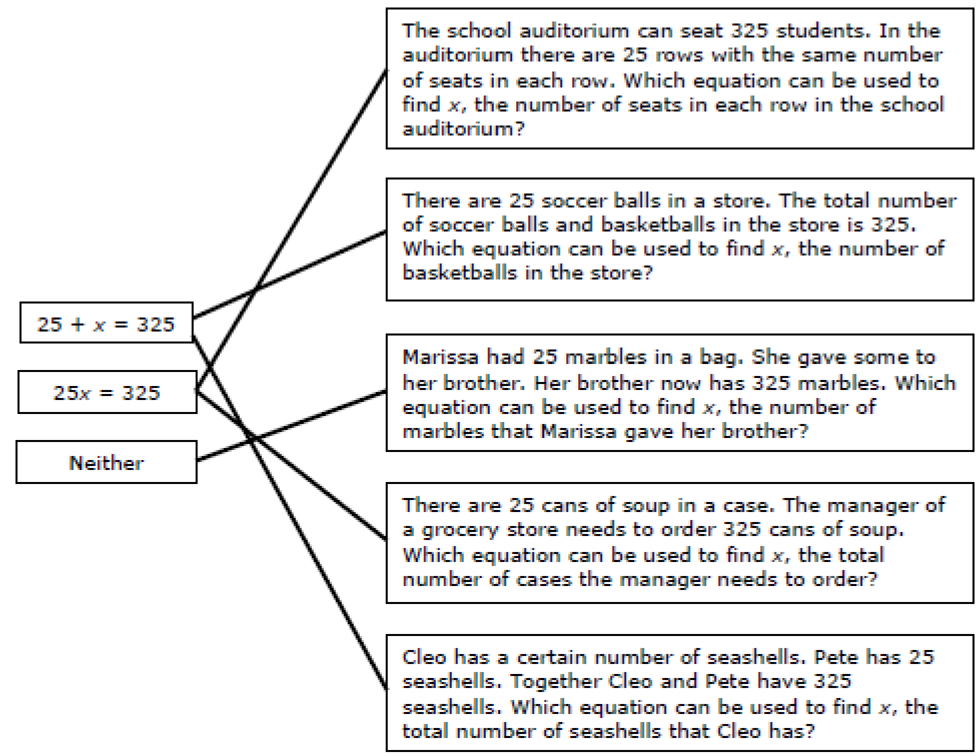
Read each of the following problem situations. Label each situation according to the equation that would answer the question. If neither equation works, select "Neither." The labels may be used more than one time.

[To connect two objects, click the first object and then the second object. A line will be automatically drawn between the two objects.]

# Math 6 Notes: Expressions, Equations and Inequalities

| | |
|---|---|
| | The school auditorium can seat 325 students. In the auditorium there are 25 rows with the same number of seats in each row. Which equation can be used to find $x$, the number of seats in each row in the school auditorium? |
| $25 + x = 325$ | There are 25 soccer balls in a store. The total number of soccer balls and basketballs in the store is 325. Which equation can be used to find $x$, the number of basketballs in the store? |
| $25x = 325$ | Marissa had 25 marbles in a bag. She gave some to her brother. Her brother now has 325 marbles. Which equation can be used to find $x$, the number of marbles that Marissa gave her brother? |
| Neither | There are 25 cans of soup in a case. The manager of a grocery store needs to order 325 cans of soup. Which equation can be used to find $x$, the total number of cases the manager needs to order? |
| | Cleo has a certain number of seashells. Pete has 25 seashells. Together Cleo and Pete have 325 seashells. Which equation can be used to find $x$, the total number of seashells that Cleo has? |

*Sample Top-Score Response:*




*Scoring Rubric:*

Responses to this item will receive 0-2 points, based on the following:

**2 points:** The student shows a thorough understanding of identifying equations that match a given real-world scenario and chooses $25x = 325$, $25 + x = 325$, Neither, $25x = 325$, $25 + x = 325$

**1 point:** The student shows a limited understanding of identifying equations that match a given real-world scenario and misidentifies one of the equations buy using addition instead of multiplication for the variable, or multiplication instead of addition for the variable, or uses "Neither" in place of where an equation could have been utilized.

**0 points:** The student shows little or no understanding of identifying equations that match a given real-world scenario and misidentifies two or more of the equations buy using addition instead of multiplication for the variable, and/or multiplication instead of addition for the variable, and/or uses "Neither" in place of where an equation could have been utilized.

# Math 6 Notes: Expressions, Equations and Inequalities

*TE information:*

**Template: Connections**
**Item ID:** MAT.06.TE.1.000EE.F.177
**Interaction Space Parameters:**

A. Three images in the first region:

| | |
|---|---|
| $25 + x = 325$ | A |
| $25x = 325$ | B |
| Neither | C |

B. Five images in the second region:

| | |
|---|---|
| 1 | The school auditorium can seat 325 students. In the auditorium there are 25 rows with the same number of seats in each row. Which equation can be used to find *x*, the number of seats in each row in the school auditorium? |
| 2 | There are 25 soccer balls in a store. The total number of soccer balls and basketballs in the store is 325. Which equation can be used to find *x*, the number of basketballs in the store? |
| 3 | Marissa had 25 marbles in a bag. She gave some to her brother. Her brother now has 325 marbles. Which equation can be used to find *x*, the number of marbles that Marissa gave her brother? |
| 4 | There are 25 cans of soup in a case. The manager of a grocery store needs to order 325 cans of soup. Which equation can be used to find *x*, the total number of cases the manager needs to order? |
| 5 | Cleo has a certain number of seashells. Pete has 25 seashells. Together Cleo and Pete have 325 seashells. Which equation can be used to find *x*, the total number of seashells that Cleo has? |

**Scoring Data:**
{A-2, A-5, B-1, B-4, C-3} {0 errors=2} {1 error=1}

# Math 6 Notes: Expressions, Equations and Inequalities

***Standard: 6.EE.A.1, 6.EE.A.2b*** ***DOK: 1*** ***Item Type:Multiple Choice Multiple correct response***

**TM3a:**

**Stimulus:** The student is presented with a numerical or algebraic expression.

**Example Stem:** Select **all** the statements that correctly describe the expression $4^2 \bullet (8w - 3)$.

A. The expression contains four terms.
B. A term in the expression has a coefficient of 8.
C. The expression shows the quotient of $8w - 3$ and $4^2$.
D. The expression shows a product of two factors.

**Answer Choices:** Answer choices should be statements that include the following vocabulary: sum, term, product, factor, quotient, and coefficient. Distractors will include confusing the meaning of sum, term, product, factor, quotient, and coefficient. At least two statements must be correct.

**Rubric:** (1 point) Student selects all the correct statements (e.g., B and D).

**Response Type:** Multiple Choice, multiple correct response