It's Not Greek to Me – Grade 11

Ohio Standards Connection

Geometry and Spatial Sense

Benchmark A
Use trigonometric relationships to verify and determine solutions in problem situations.

Indicator 4
Use trigonometric relationships to determine lengths and angle measures; i.e., Law of Sines and Law of Cosines.

Mathematical Processes

Benchmarks
B. Construct logical verifications or counter-examples to test conjectures and to justify or refute algorithms and solutions to problems. Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.
F. Use formal mathematical language and notation to represent ideas, to demonstrate relationships within and among representation systems, and to formulate generalizations.

Lesson Summary:
In this lesson students discover, prove and apply the Law of Sines, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$. *Students create sample problems that apply to the Law of Sines to problem situations.*

Estimated Duration: *Three hours*

Commentary:
This lesson is designed as an introduction to the Law of Sines. It does not include the study of the ambiguous case.

The title of the lesson refers to the early history of trigonometry through the contributions of Greek mathematicians. As an extension of this lesson have students research the history of trigonometry and Greek contributions to trigonometry.

Pre-Assessment:

- Conduct a verbal review of basic trigonometric (trig) ratios as a class. Remind students that the pneumonic SOH-CAH-TOA can be used to recall the trigonometric ratios Sine (sin) = opposite/hypotenuse, Cosine (cos) = adjacent/hypotenuse and Tangent (tan) = opposite/adjacent.
- Distribute *It's Not Greek to Me Pre-Assessment*, Attachment A, and have students complete the problems individually. Allow students to use a calculator or Trig Table to approximate the value of the angles.
- Observe the students as they work and mentally note prior knowledge of the use of trigonometric functions sine, cosine and tangent to solve right triangle problems.
- Have students share answers to the questions. Discuss any questions or errors students demonstrate on the use of the basic trigonometric functions. Ask follow-up questions of various students in response to each answer:
  a. What trigonometric ratio did you use to solve this problem? (sine, cosine, tangent depending on problem)
  b. What equation did you use?

c. How could you find the measure of the other angle in this triangle? (Use another trigonometric ratio, or by subtracting the angle measure from 90°.)
d. How else could you find the measure of the angle? (Use a different trigonometric function.)
e. How could you find the measure of the second or third side of the triangle? (Trigonometry or Pythagorean Theorem)

Instructional Tip:
Use the follow-up questions as needed depending upon your informal assessment of student recall of these ideas.

Scoring Guidelines:
Answers to the problems on Attachment A can be found on *It's Not Greek to Me Pre-Assessment Answer Key,* Attachment B. Use students' informal notes and responses during discussion of pre-assessment questions to determine class readiness to proceed. Carefully monitor for students who need additional practice solving right triangle problems as they complete this pre-assessment.

Post-Assessment:
- Distribute *It's Not Greek to Me Post-Assessment,* Attachment C.
- Instruct students to work in pairs to create two problems to solve problems involving non-right triangles using the Law of Sines. Explain to students that one of the problems is to be an application or from a real-life situation.
- Have students share their problems with others. Provide opportunities for students to exchange and solve each others problems and use the scoring guidelines to evaluate the work.

Scoring Guidelines:

Meets Expectations	• Develops two original problems involving non-right triangles, at least one is an application problem. • Correctly uses the Law of Sines to solve each problem.
Approaches Expectations	• Develops two problems involving non-right triangles which may contain some aspects of examples used in class. • Correctly uses the Law of Sines to solve at least one problem. The solution may contain minor mathematical errors.
Intervention Needed	• Develops one problem involving non-right triangles which closely resembles class examples. • Attempts to use the Law of Sines, but with significant errors.

Instructional Procedures:
Part One
1. Distribute *Diagram of a Hit,* Attachment D to each student. Present scenarios similar to the one given in the attachment by substituting local landmarks and geographic features.
2. Have students brainstorm with partners and share ideas for solving the problem. Record the ideas on the board. If students need assistance, use guiding questions such as,

- Could we use our right triangle methods to solve this problem?
- How is this triangle different from the ones we have worked with previously?

Inform students that they will be investigating relationships in non-right triangles to develop a method to solve these types of problems.

3. Assign the following journal prompt and allow five minutes for writing a response:
   - What kind of a relationship might exist between the sides and angles of this triangle? Write a ratio or formula that demonstrates any relationships you find.

Instructional Tip:
After the next part of the lesson and the Law of Sines has been derived, return to the journal and have students compare their relationships to the methods used in the proof.

4. Use the same scenario and the triangle created to have students draw an altitude from the 143^o angle to the opposite side forming two right triangles. Use *h* as the altitude and *y* as the side between the 143^o angle and where the ball landed.
5. Ask the students to set up two trigonometric ratios to find the length of *h* ($\sin 27^o = \frac{h}{100}$ and $\sin 10^o = \frac{h}{y}$).
   a. Have the students solve for *h* obtains the equations ($h = 100 \sin 27^o$ and $h = y \sin 10^o$).
   b. Have the students set the equations equal to each other so $100 \sin 27^o = y \sin 10^o$ then dividing by $100y$ gives the equation $\frac{\sin 27}{y} = \frac{\sin 10}{100}$. This provides a method of finding *y* from the original triangle.

Instructional Tip:
Determine whether to proceed with Attachment E or provide another example to demonstrate that the proportional equation above works for multiple triangles. Provide rulers and protractors to measure to see if the measurements make sense if needed (use a scale of cm or inches = the number of feet).

6. Distribute *Proving the Law of Sines*, Attachment E to each student. Have students work with a partner to complete the problems using the altitudes. Circulate, as students work, carefully monitoring student progress. Have students share their results and write an equation for the Law of Sines
7. Distribute the triangles from *Practicing the Law of Sines*, Attachment F to pairs of small groups. Choose to assign all six triangles to each pair of students or two triangles to selected groups and then have groups share with each other.
8. Have students present the measures for the unknown side and angle measurements for each of the six triangles to the class. Answers for the unknown measurements are found on *Practicing the Law of Sines Answer Key*, Attachment G. After completing the presentations, have students summarize their understanding of the Law of Sines in a journal.

Part Two

9. Distribute *Applying the Law of Sines,* Attachment H. Have students complete the problems, share responses with each other and discuss any discrepancies in their work and solutions. Note students who show evidence of not understanding the application or formula for the Law of Sines and provide immediate assistance. Answers for the problems can be found on *Applying the Law of Sines Answer Key,* Attachment I.
10. Assign a journal entry to review the Law of Sines. Have students answer questions such as the following as an informative assessment. Have students share and clarify responses before assigning the post-assessment task.
    - Can you clear the fraction?
    - What can you multiply by to clear the fraction?
    - To look more like the law of Sines, what should remain in the numerator of each fraction?
    - What could you divide the first equation by to leave this numerator? the second equation? the third equation?
    - Could you divide each equation by *abc*? Try it and see what happens.

Differentiated Instructional Support:

Instruction is differentiated according to learner needs, to help all learners either meet the intent of the specified indicator(s) or, if the indicator is already met, to advance beyond the specified indicator(s).

- *Proving the Law of Sines,* Attachment E has many uses. Use to lead a small group of students step-by-step through the proof.
- Encourage students to complete the proof without Attachment E and then use the attachment to verify their proofs.
- Create a large triangle on the floor or chalkboard. Ask students to find the missing lengths using the *Law of Sines* and verify by measuring.
- Direct students demonstrating an understanding of the Law of Sines to explore the ambiguous case and create a poster with moving parts, a side length and an angle measure, used to demonstrate the two possible solutions in a particular situation.

Extension:

Prove the Law of Sines in other ways. (e.g. using triangle area formula)

Home Connection:

Find the distance from your home to a distant object or landmark using the Law of Sines.

Materials and Resources:

The inclusion of a specific resource in any lesson formulated by the Ohio Department of Education should not be interpreted as an endorsement of that particular resource, or any of its contents, by the Ohio Department of Education. The Ohio Department of Education does not endorse any particular resource. The Web addresses listed are for a given site's main page, therefore, it may be necessary to search within that site to find the specific information required for a given lesson. Please note that information published on the Internet changes over time,

therefore the links provided may no longer contain the specific information related to a given lesson. Teachers are advised to preview all sites before using them with students.

For the teacher: Calculator, compass, ruler, landmarks for local version of the baseball problem

For the student: Calculators, scrap paper, rulers and compasses (if teacher decided on measuring)

Vocabulary:
- Ambiguous
- Law of Sines
- Sextant

Technology Connections:
- Geometry exploration software enhances the exploration of the ambiguous case. Fix two sides and an angle while the third side pivots to form two different angles. Students can create and easily find angle and side measures of triangles and focus their efforts on creating application problems.
- Trigonometry is used in manufacturing, space exploration and map-making. Allow students to explore these careers and others to make connections to the real-world and to understand the importance of trigonometry.

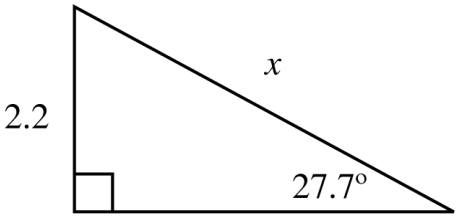
Attachments:
Attachment A, *It's Not Greek to Me Pre-Assessment*
Attachment B, *It's Not Greek to Me Pre-Assessment Answer Key*
Attachment C, *It's Not Greek to Me Post-Assessment*
Attachment D, *Diagram of Hit*
Attachment E, *Proving the Law of Sines*
Attachment F, *Practicing the Law of Sines*
Attachment G, *Practicing the Law of Sines Answer Key*
Attachment H, *Applying the Law of Sines*
Attachment I, *Applying the Law of Sines Answer Key*

It's Not Greek to Me – Grade 11

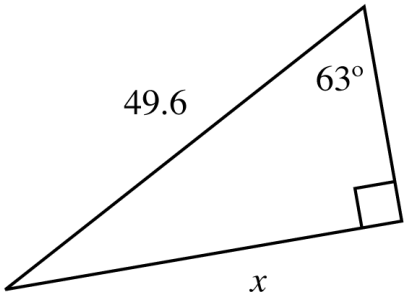
Attachment A
It's Not Greek to Me Pre-Assessment

Directions: Find the value of the variable in each problem below. Approximate the answers to two decimal places.

1.



2.



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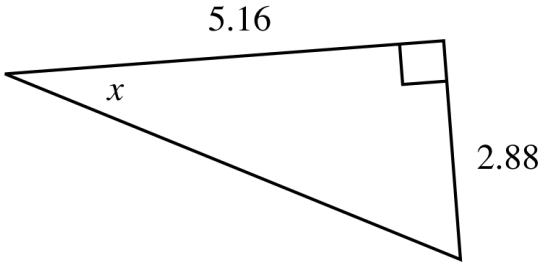
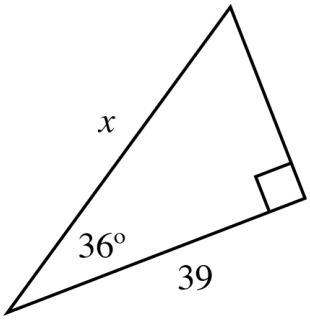


4.



5. While in Washington D.C. late one afternoon, Muriel notices that she can see the top of the Washington Monument from where she is standing and that the angle of the sun to the ground is 36°. Her guidebook states that the monument is 555 feet tall. How far away from the monument, is Muriel?

6. Give three values of θ between 0° and 540° so that the $\sin \theta = \sin 50°$.

Attachment B
It's Not Greek to Me Pre-Assessment Answer Key

1. 4.73	2. 44.19
3. 29.17°	4. 48.21
5. 763.89 feet	6. 130°, 410°, 490°

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Attachment C
It's Not Greek to Me Post-Assessment

Name _____ **Date** _____

Directions: Now that you have discovered, proved and applied the Law of Sines it's time to show what you know. Your task is to write and solve two problems requiring the Law of Sines for their solution. Write at least one problem that is a real world example of how you might use the Law of Sines. You may invent measurements for your problem such as the height of a particular building or the distance between two objects or locations, but your invented measurements should be reasonable. Assess your problems using the scoring guidelines provided. Read it carefully before you begin and again after you finish to be sure you included all the necessary requirements.

Scoring Guidelines:

Meets Expectations	• Develops two original problems involving non-right triangles, at least one is an application problem. • Correctly uses the Law of Sines to solve each problem.
Approaches Expectations	• Develops two problems involving non-right triangles which may contain some aspects of examples used in class. • Correctly uses the Law of Sines to solve at least one problem. The solution may contain minor mathematical errors.
Intervention Needed	• Develops one problem involving non-right triangles which closely resembles class examples. • Attempts to use the Law of Sines, but with significant errors.

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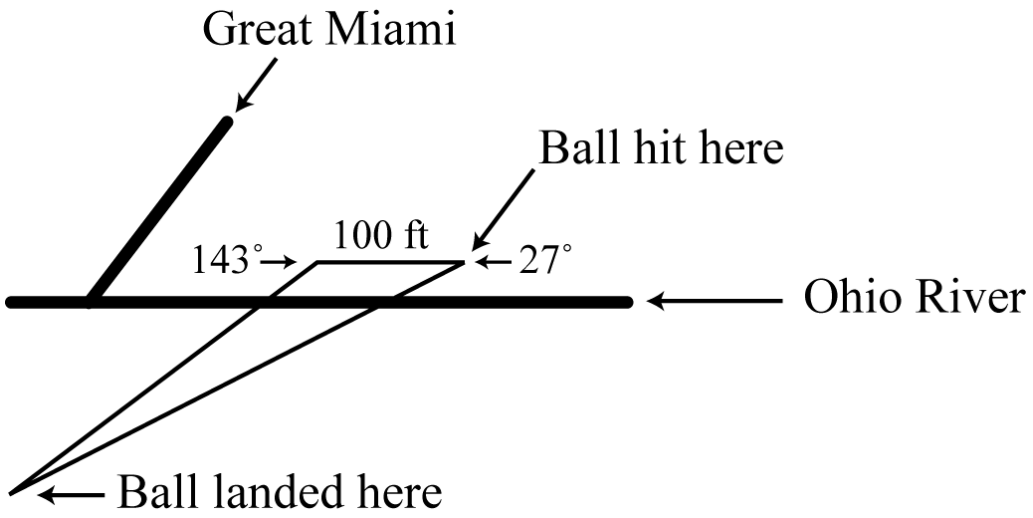
Attachment D
Diagram of a Hit

Name _____ **Date** _____

A ball field sits on the bank of the Ohio River, near its confluence with the Great Miami River. One day Cookie Hillman and her friends were playing ball when Cookie unleashed a hit that carried the ball across the Ohio River. Cookie's friends were amazed and wanted to report her incredible feat to the local newspaper. They decided that the story would be even more exciting if they knew exactly how far the ball had traveled. Fortunately, Cookie's friend Mia had a sextant and a tape measure with her. They were able to make the following measurements. How far did Cookie's incredible hit travel?

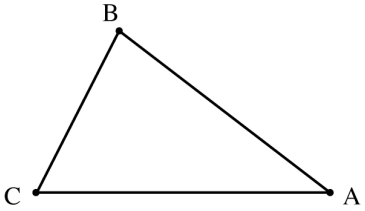


Attachment E
Proving the Law of Sines

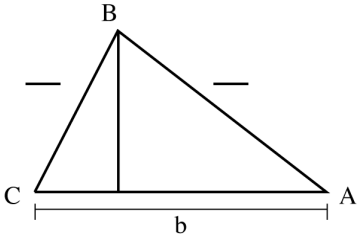
Name _____ **Date** _____

Directions: Follow the steps below on your own paper to prove the Law of Sines.

1. Draw a triangle in your journal like the one shown.



2. Draw an altitude from point B to the base of triangle ABC. Label the side of the original triangle across from angle A as *a* and the side of the original triangle across from C as *c*, and the height *h*.



3. Complete the equation $\sin C = \frac{?}{?}$, then solve for *h*.

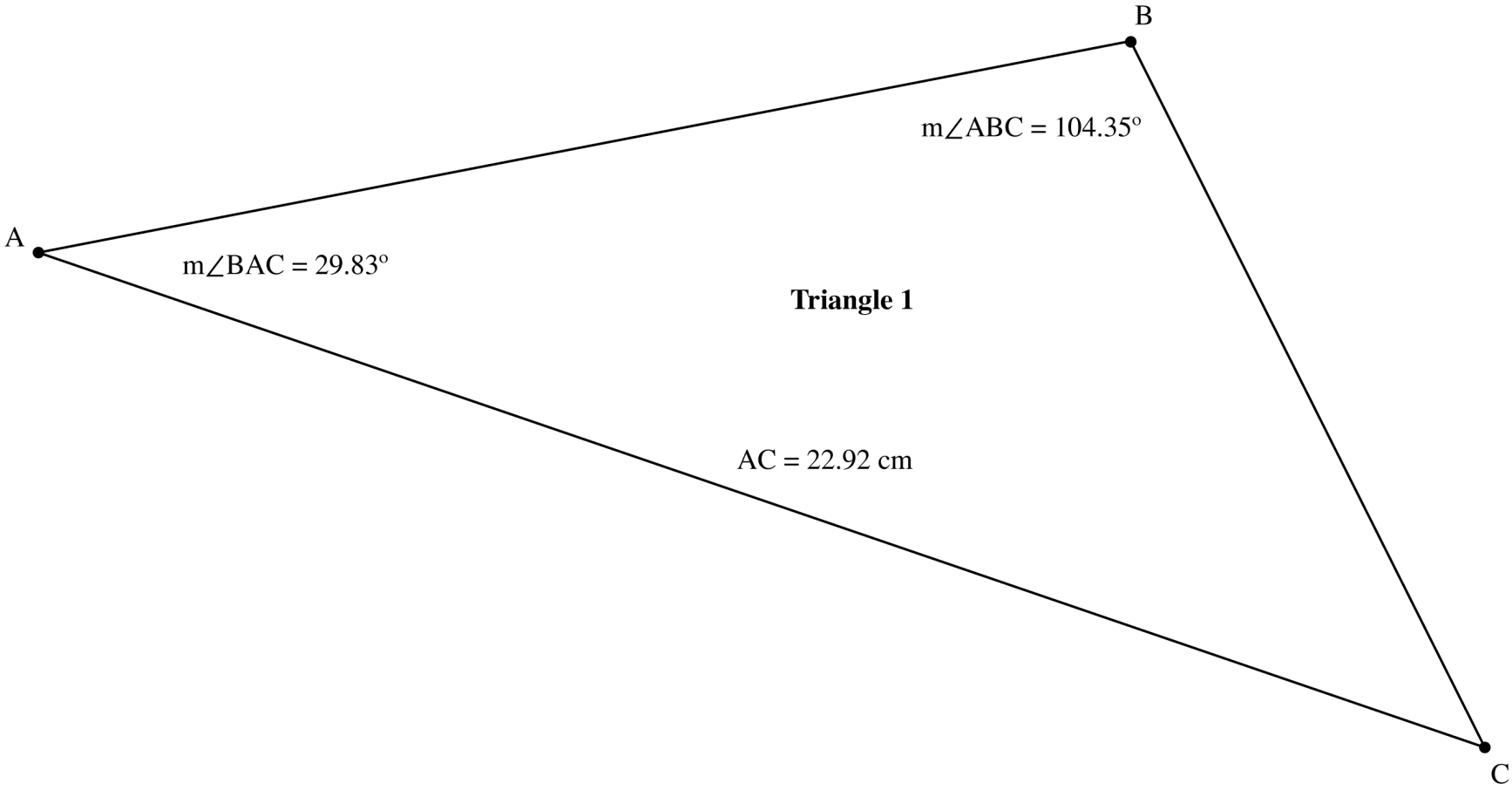
4. Complete the equation $\sin B = \frac{?}{?}$, then solve for *h*.

5. Since each is equal to h set the two expressions equal to each other and divide by the lengths of the sides.

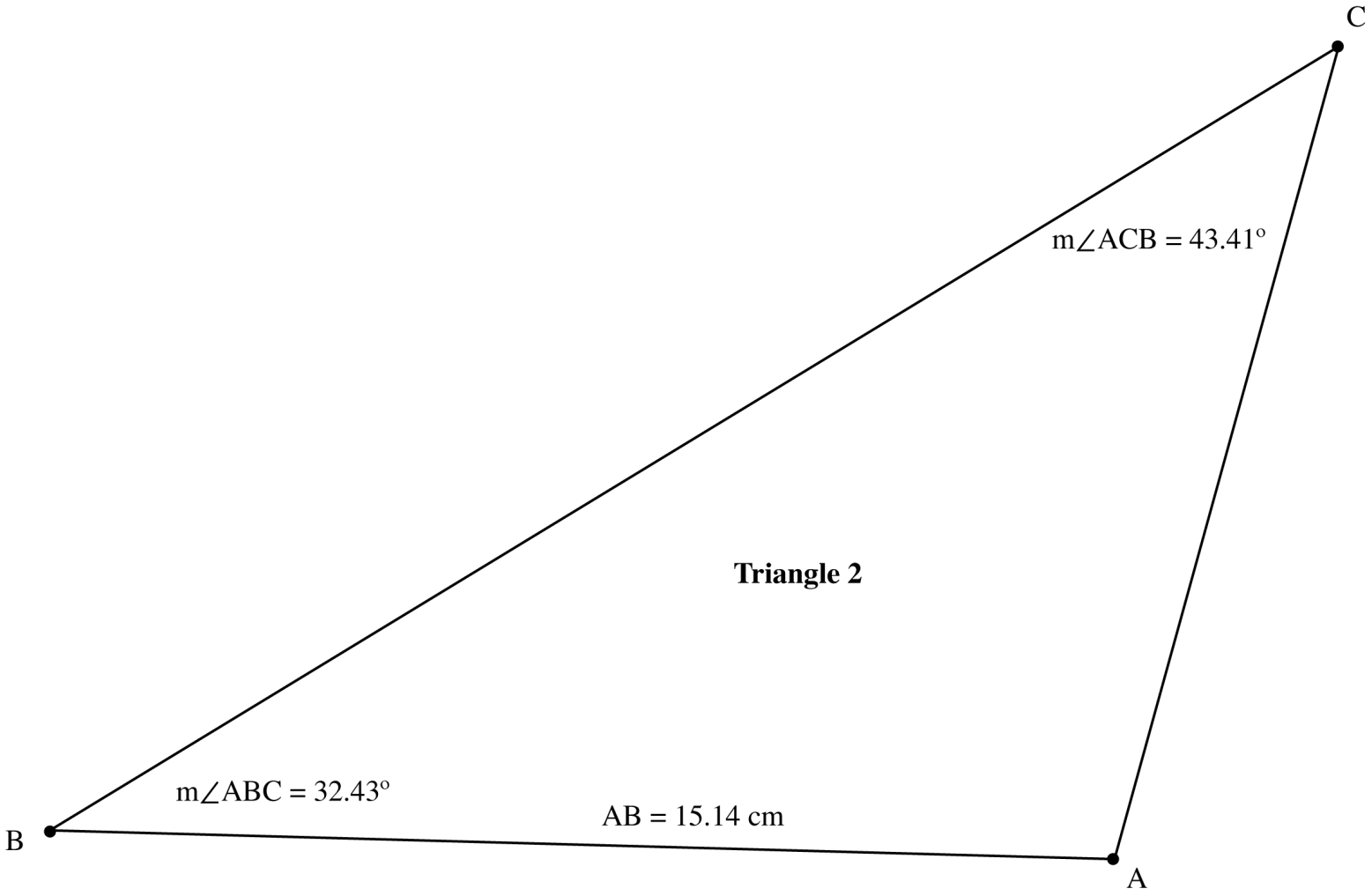
6. Repeat steps 2 through 5 after drawing an altitude from points A and then C.

7. After completing these activities write the equation for the law of Sines. Summarize your results.

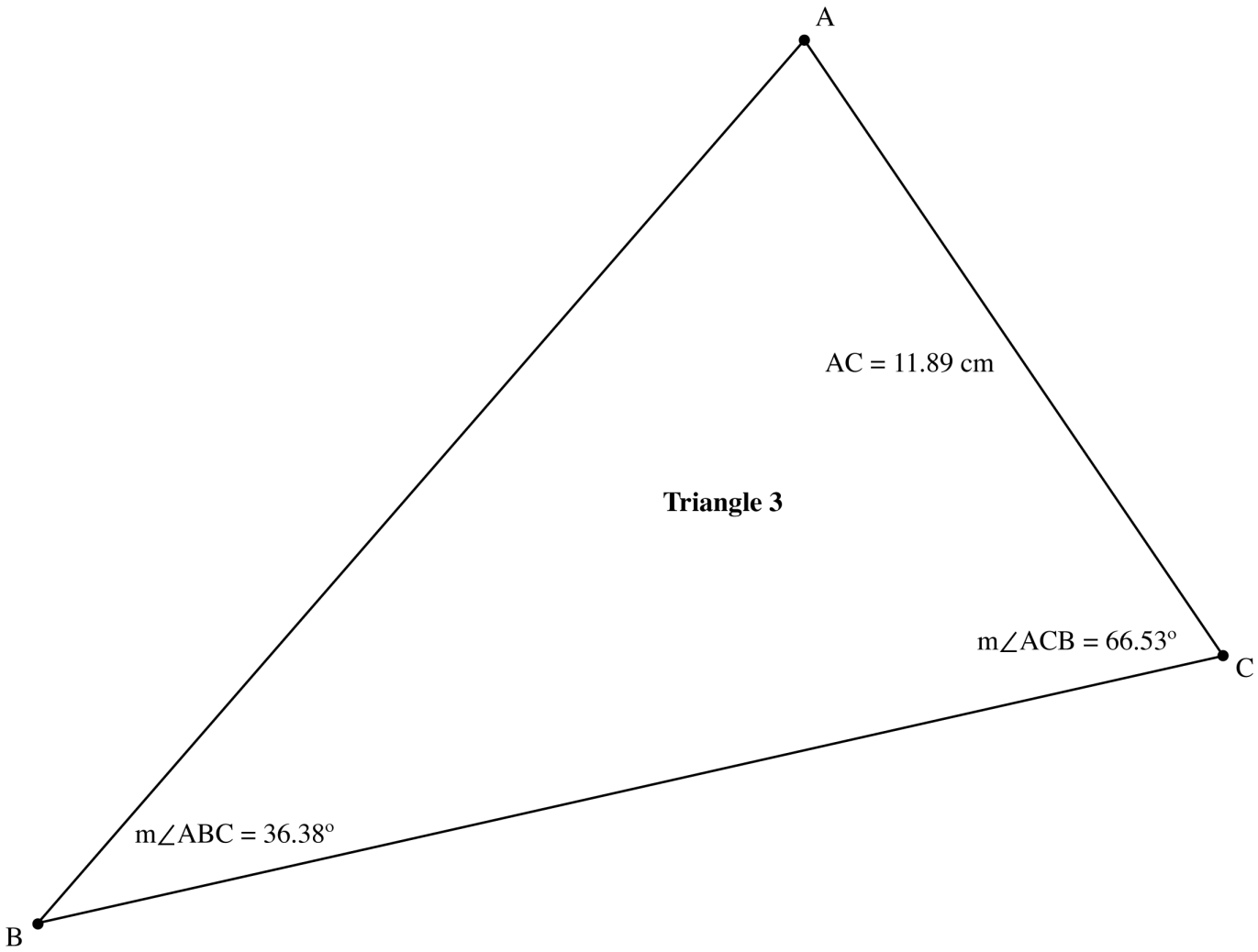
Attachment F
Practicing the Law of Sines



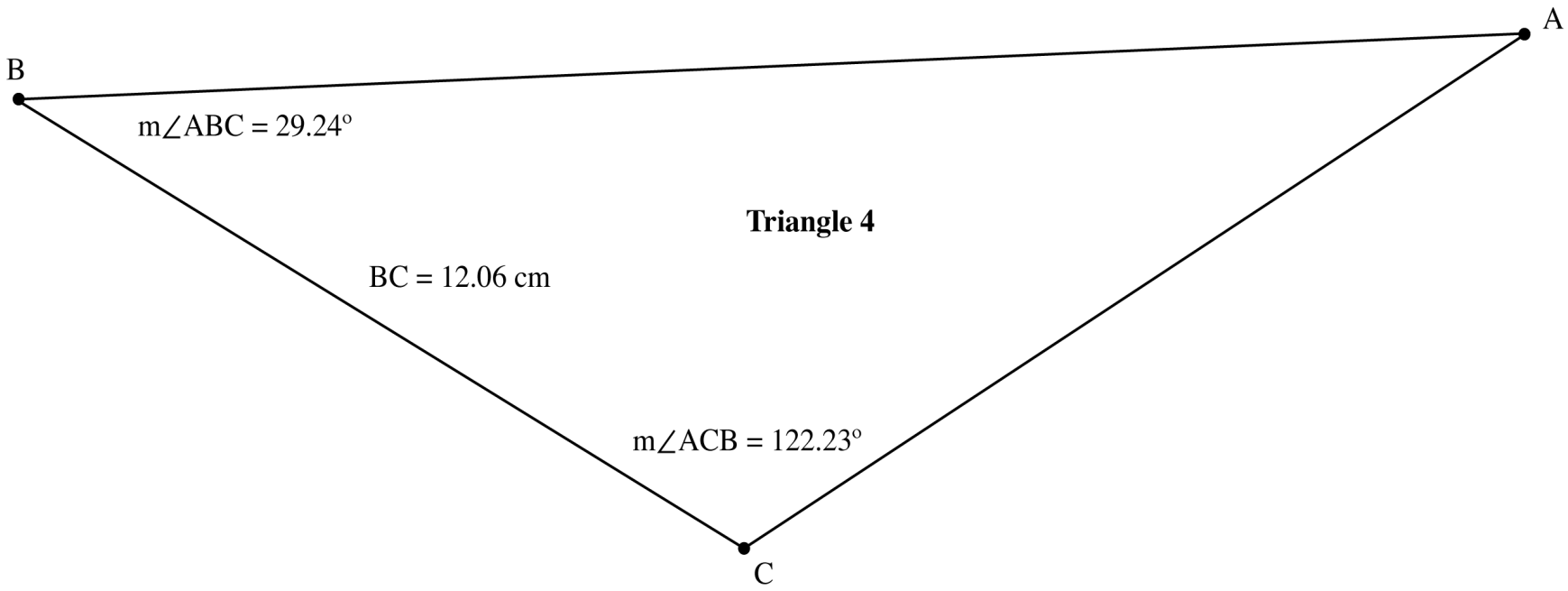
Attachment F (continued)
Practicing the Law of Sines



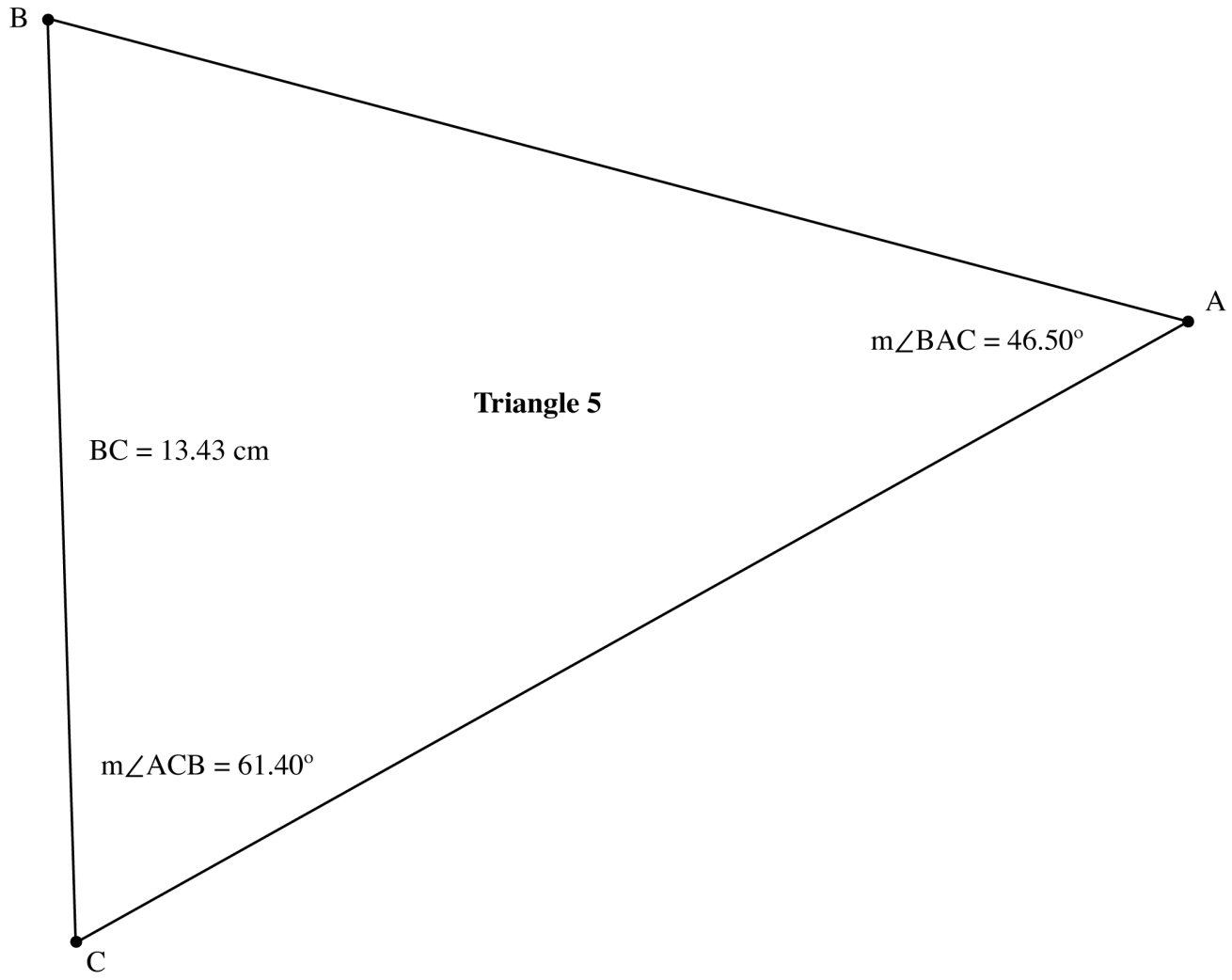
Attachment F (continued)
Practicing the Law of Sines



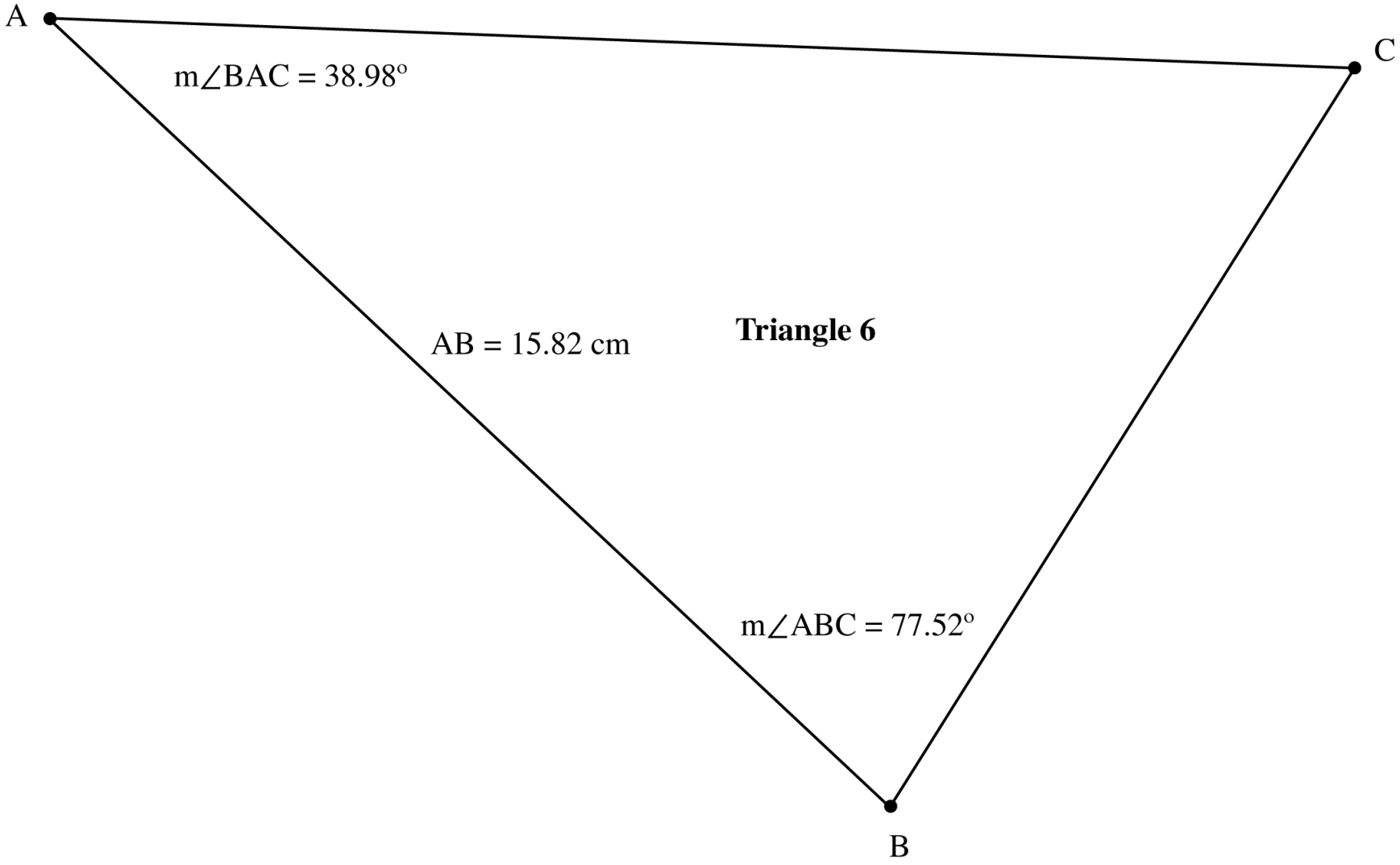
Attachment F (continued)
Practicing the Law of Sines



Attachment F (continued)
Practicing the Law of Sines



Attachment F (continued)
Practicing the Law of Sines



Attachment G
Practicing the Law of Sines Answer Key

Triangle One

AB = 16.97 cm
BC = 11.77 cm
m∠ACB = 45.82°

Triangle Two

BC = 21.36 cm
AC = 11.81 cm
m∠BAC = 104.16°

Triangle Three

AB = 18.38 cm
BC = 19.53 cm
m∠BAC = 77.09°

Triangle Four

AB = 21.36 cm
AC = 12.34 cm
m∠BAC = 28.53°

Triangle Five

AB = 16.26 cm
AC = 17.62 cm
m∠ABC = 72.10°

Triangle Six

AC = 17.26 cm
BC = 11.12 cm
m∠ACB = 63.49°

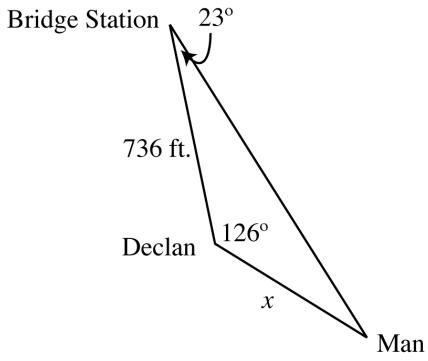
Attachment H
Applying the Law of Sines

Directions: Use the Law of Sines to solve each of the problems below. Sketch a picture if none is given.

1. In Δ DEF, $m\angle D=36°$, $m\angle E=42°$ and EF=15. Find the length of DF.

2. In Δ QRS, $m\angle Q=96°$, QS= 25 and RS=27 Find the $m\angle R$.

3. Declan is a park ranger at Niagara Falls. One day during his shift a man rides over the falls in a barrel. Declan knows that the distance from his observation tower station to the ranger station near the top of the falls is 736 feet. The man is at an angle of 126° with a line connecting the two stations. The ranger at the bridge station reports that the angle from his station to the man is 23°. How far is the man from Declan?



Attachment I
Applying the Law of Sines Answer Key

Solutions:

1. $\frac{\sin 36}{15} = \frac{\sin 42}{x}$ DF=17.1

2. $\frac{\sin 96}{27} = \frac{\sin R}{25}$ $m\angle R = 67°$

3. $\frac{\sin 31}{736} = \frac{\sin 23}{x}$ Distance to man = 558.4 ft