

Unit 6: Use Functions to Model Relationships Between Quantities

Clusters: Define, evaluate, and compare functions.
Use functions to model relationships between quantities.

Nevada Academic Content Standard

What does this standard mean that a student will know and be able to do? (adapted from North Carolina 8th Grade Standards, *Unpacked Content*)

8.F.A.3

Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

8.F.3 Students understand that linear functions have a constant rate of change between any two points. Students use equations, graphs and tables to categorize functions as linear or non-linear.

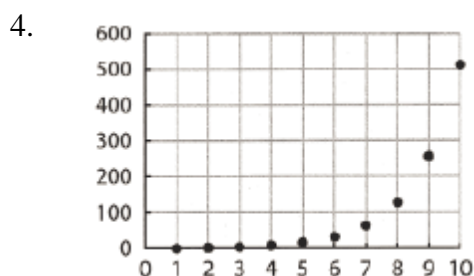
Example 1:

Determine if the functions listed below are linear or non-linear. Explain your reasoning.

1. $y = -2x^2 + 3$
2. $y = 0.25 + 0.5(x - 2)$

3.

| x | y |
|-----|-----|
| 1 | 12 |
| 2 | 7 |
| 3 | 4 |
| 4 | 3 |
| 5 | 4 |
| 6 | 7 |



Solution:

1. Non-linear; there is not a constant rate of change
2. Linear; there is a constant rate of change
3. Non-linear; there is not a constant rate of change
4. Non-linear; the graph curves indicating the rate of change is not constant.

8.F.4 Students identify the rate of change (slope) and initial value (y-intercept) from tables, graphs, equations or verbal descriptions to write a function (linear equation). Students understand that the equation represents the relationship between the x -value and the y -value; what math operations are performed with the x -value to give the y -value. Slopes could be undefined slopes or zero slopes.

Tables:

Students recognize that in a table the y -intercept is the y -value when x is equal to 0. The slope can be determined by finding the ratio $\frac{y}{x}$ between the change in two y -values and the change between the two corresponding x -values.

8F.B.4

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading

these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Example 1:

Write an equation that models the linear relationship in the table below.

| x | y |
|-----|-----|
| -2 | 8 |
| 0 | 2 |
| 1 | -1 |

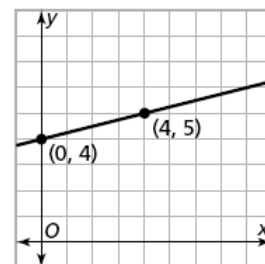
Solution: The y -intercept in the table above would be $(0, 2)$. The distance between 8 and -1 is 9 in a negative direction $\rightarrow -9$; the distance between -2 and 1 is 3 in a positive direction. The slope is the ratio of rise to run or $\frac{y}{x}$ or $\frac{-9}{3} = -3$. The equation would be $y = -3x + 2$.

Graphs:

Using graphs, students identify the y -intercept as the point where the line crosses the y -axis and the slope as the $\frac{\text{rise}}{\text{run}}$.

Example 2:

Write an equation that models the linear relationship in the graph.



Solution: The y -intercept is 4. The slope is $\frac{1}{4}$, found by moving up 1 and right 4 going from $(0, 4)$ to $(4, 5)$.

The linear equation would be $y = \frac{1}{4}x + 4$.

Equations:

In a linear equation the coefficient of x is the slope and the constant is the y -intercept. Students need to be given the equations in formats other than $y = mx + b$, such as $y = ax + b$ (format from graphing calculator), $y = b + mx$ (often the format from contextual situations), etc.

Point and Slope:

Students write equations to model lines that pass through a given point with the given slope.

Example 2:

A line has a zero slope and passes through the point $(-5, 4)$. What is the equation of the line?

Solution: $y = 4$

Example 3:

Write an equation for the line that has a slope of $\frac{1}{2}$ and passes through the point $(-2, 5)$.

Solution: $y = \frac{1}{2}x + 6$

Students could multiply the slope $\frac{1}{2}$ by the x -coordinate -2 to get -1. Six (6) would need to be added to get to 5, which gives the linear equation.

Students also write equations given two ordered pairs. **Note that point-slope form is not an expectation at this level.** Students use the slope and y -intercepts to write a linear function in the form $y = mx + b$.

Contextual Situations:

In contextual situations, the y -intercept is generally the starting value or the value in the situation when the independent variable is 0. The slope is the rate of change that occurs in the problem. Rates of change can often occur over years. In these situations it is helpful for the years to be “converted” to 0, 1, 2, etc. For example, the years of 1960, 1970, and 1980 could be represented as 0 (for 1960), 10 (for 1970) and 20 (for 1980).

Example 4:

The company charges \$45 a day for the car as well as charging a one-time \$25 fee for the car’s navigation system (GPS). Write an expression for the cost in dollars, c , as a function of the number of days, d , the car was rented.

Solution: $C = 45d + 25$

Students interpret the rate of change and the y -intercept in the context of the problem. The rate of change is 45 (the cost of renting the car) and that initial cost (the first day charge) also includes paying for the navigation system. Classroom discussion about one-time fees vs. recurrent fees will help students model contextual situations.

8.F.B.5

Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

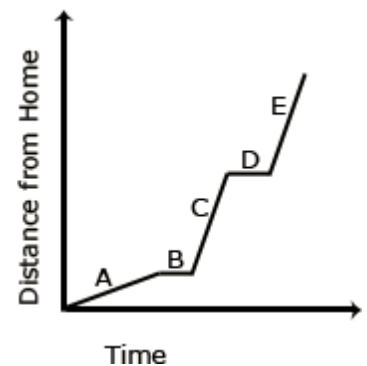
8.F.5 Given a verbal description of a situation, students sketch a graph to model that situation. Given a graph of a situation, students provide a verbal description of the situation.

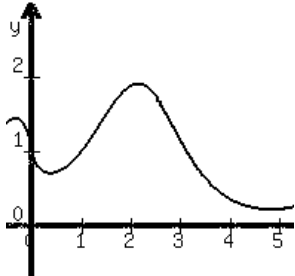
Example 1:

The graph below shows a John’s trip to school. He walks to his Sam’s house and, together, they ride a bus to school. The bus stops once before arriving at school. Describe how each part A – E of the graph relates to the story.

Solution:

- A John is walking to Sam’s house at a constant rate.
- B John gets to Sam’s house and is waiting for the bus.
- C John and Sam are riding the bus to school. The bus is moving at a constant rate, faster than John’s walking rate.
- D The bus stops.
- E The bus resumes at the same rate as in part C.



| | | |
|--|------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| | <p><i>Example 2:</i></p>  | <p>Describe the graph of the function between $x = 2$ and $x = 5$.</p> <p><i>Solution:</i></p> <p>The graph is non-linear and decreasing.</p> |
|--|------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|

Approximate Time Frame: 2- 3 weeks

Terms:

- | | | |
|--------------|------------------|--------------|
| ✓ linear | ✓ y-intercept | ✓ increasing |
| ✓ non-linear | ✓ rate of change | ✓ decreasing |
| ✓ function | ✓ initial value | ✓ interval |
| ✓ slope | ✓ constant | ✓ relation |

Resources

- | | |
|---------------------------------------------------------------|---------------------------------------------|
| MGH – McGraw Hill, <i>Glencoe Math</i> (2015) | NY – <i>Engage New York</i> |
| ML – McDougal Littell, <i>Pre-Algebra Book</i> ; Larson, 2005 | IL – <i>Illinois Model Math Curriculum</i> |
| EX – <i>Explorations in Core Math</i> (Holt McDougal) | MAP – <i>Math Assessment Project (MARS)</i> |

| | <i>Suggested Topics for Lessons</i> | <i>Possible Resources</i> |
|----------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 8.F.B.4 | <p>Construct Functions</p> <p>Write Linear Equations</p> <p>SBAC Evidence:</p> <ul style="list-style-type: none"> ➤ The student constructs a function to model a linear relationship between two quantities. ➤ The student determines the rate of change and initial value of a function, either from a description of a relationship or from two (x, y) values, including reading the rate of change and/or the value of the function from a table or a graph. ➤ The student interprets features of a linear function, such as rate of change and initial value, in terms of the situation it models, its graph, or a table of values. | <ul style="list-style-type: none"> ➤ MGH 3-6 <i>Writing Linear Equations</i> (page 221) ➤ MGH 4-6 <i>Construct Functions</i> (page 319) ➤ EX 8-1 <i>Graphing Linear Equations</i> (page 295) ➤ EX 8-3 <i>Using Slopes and Intercepts</i> (page 305) ➤ IL Lesson 3: Constructing Functions ➤ IL 8F4 Activity (click on Assessments #13): Function Matching ➤ Rice.edu Lesson: Fun & Sun Rent-A-Car ➤ NY Module 6 Lesson 1: Modeling Linear Relationships ➤ NY Module 6 Lesson 2: Interpreting Rate of Change and Initial Value ➤ NY Module 6 Lesson 3: Representations of a Line ➤ Learn Zillion Lesson Plan: Interpret initial value and rate of change in terms of the situation it models ➤ Learn Zillion Lesson Plan: Determine rate of change and initial value from a table by examining values in the table ➤ Learn Zillion Lesson Plan: Determine rate of change and initial value from a graph or an equation by inspection ➤ Learn Zillion Video Lessons: Construct functions, determine slope and initial value, and interpret in terms of a situation ➤ NC Lessons for Learning (page 29): <i>Bow Wow Barkley</i> ➤ NC Lessons for Learning (page 45): <i>Sandy’s Candy Corporation</i> |

| | | |
|-----------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>8.F.A.3</p> | <p>Linear vs Non-Linear Functions</p> <p>SBAC Evidence:</p> <ul style="list-style-type: none"> ➤ The student recognizes and gives examples of functions that are not linear. | <ul style="list-style-type: none"> ➤ MGH 4-7 Linear and Nonlinear Functions (page 327) ➤ ML 12.6 part of Quadratic Functions (page 680) ➤ EX 9-3 Linear Functions (page 355) ➤ IL Lesson 2 Segment 2: Linear and Non-Linear Functions ➤ Learn Zillion Video Lessons: Interpret the Equation $y = mx + b$ as defining a linear Function ➤ Learn Zillion Video Lessons: Interpret the Equation $y = mx + b$ ➤ Illustrative Math Task: Introduction to Linear Functions ➤ NY Module 5 Lesson 8: Graphs of Simple Non-Linear Functions ➤ NC Lessons for Learning (page 36): Nonlinear Functions |
| <p>8.F.B.5</p> | <p>Qualitative Graphs</p> <ul style="list-style-type: none"> ➤ The student qualitatively describes the functional relationship between two quantities by analyzing a graph (e.g., whether the function is increasing or decreasing, or whether the graph is linear or nonlinear.) ➤ The student draws a graph that exhibits the qualitative features of a function that has been described in writing. | <ul style="list-style-type: none"> ➤ MGH 4-9 Qualitative Functions (page 347) ➤ EX 2-3 Interpreting Graphs (page 55) ➤ MAP: Interpreting Distance-Time Graphs ➤ PBS Learning Media: Location Graphs ➤ NY Module 6 Lessons 4-5: Increasing and Decreasing Functions ➤ Directions and videos to use: Graphing Stories ➤ NC Lessons for Learning (page 53): The Case of the Vase |